Sloan School of Management — 15.010/011 - Economic Analysis for Business Decisions
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2001 Final Exam Answers: Prepared for TA Grading Purposes

1. Short Questions

1a)

1a) (i). Use the pricing formula (P-MC)/P = -1/elasticity of demand. Here

$$\frac{P-0.9}{P} = \frac{-1}{-1.5} = \frac{2}{3} \Rightarrow P = \$\ 2.70$$

1a) (ii) $\frac{\$Ads}{\$Sales} = -\frac{Ad\ Elasticity}{\text{Price}\ Elasticity}$ in profit maximizing equilibrium.

Here = – (0.5/– 1.5) = 1/3.

If sales = \$ 225 million, then optimal \$advertising is \$ 75

1b)

1b) (i). SR elasticity is price coefficient = -.40.

1b) (ii) From partial adjustment formulation of demand, LR elasticity is

$$\frac{-.4}{1-.2} = -\frac{.4}{.8} = -.5.$$

(1c)

1c) (i). Bart has no dominant strategy. Milhaus has a dominant strategy (go to class).

1c) (ii) Yes. Both go to class.

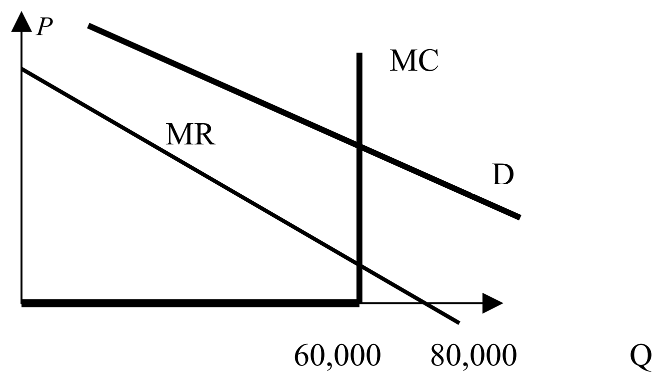
1d)

1d) (i). $Q_d = Q_s$ implies 12,000 = 300 P, so P = \$ 40. This implies Q = 11,000.

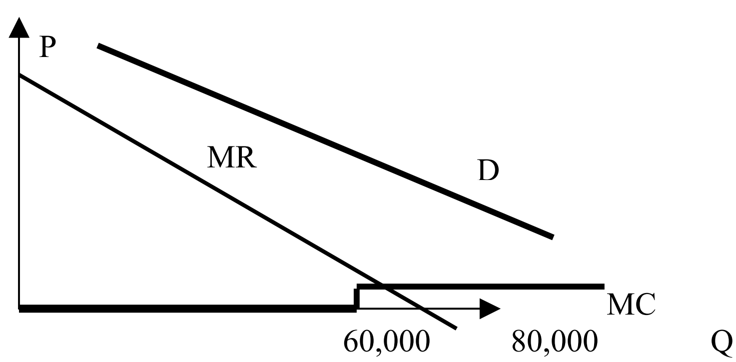
1d) (ii) At price = \$30, Q_d = 12,250, Q_s = 9,250, so Q_d - Q_s = 3,000 *shortage* results.

2. Rolling Stones

2a) a) The total cost is $1,000,000 independent of the number of seats sold (up to capacity 60,000). The marginal cost is $0 up to capacity and infinite thereafter. The inverse demand is P = 200 – 1.25 Q, which gives marginal revenue MR=200-2.5 Q. The optimal solution is found by setting MR = MC = 0, which gives Q = 200/2.5= 80, which is greater than the capacity of 60. So, we set the price to fill the 60,000 seats, which implies P = 200 – 1.25 (60) = $ 125. Profits are 125 (60,000) – 1,000,000 = 6,500,000. A picture:



2b) The MC curve becomes as depicted below (since MR = 50 at Q = 60). Beyond 60,000 seats, the MC is thus $20. Find the optimal quantity by setting MR = MC = 20. This gives 200-2.5 Q = 20 or Q = 180/2.5 = 72. To sell these 72,000 seats, we need to set price at 200-1.25*72= 110. We will pay $240,000 for the extra seats, so profits become 72,000 * 110- 1,000,000 - 240,000 = 6,680,000.



3. Scholar Rocks

3a) We have MC = 0. Since the groups are identifiable, we set MR = MC for each type of person to solve for Q, and then solve for P. For each C, MR = 10 – Q, so MR = MC implies Q = 10, and P = 5. For each S, MR = 10 – 2Q, so MR = MC implies Q = 5 and P = 5. (??!!!). So it is optimal (3^{rd} degree p.d.) to charge P = 5 for everyone. Profits are 5 (10*2,000 + 5*8000) – 50,000 = $ 250,000.

3b) We solve for the optimal two part tariff for each group separately. Since MC = 0, it is optimal to set the daily admission fee to zero for each group. The optimal membership is the consumer surplus for each group at P = 0. For C's this is T = .5 (20)(10) = $100. For S's this is T = .5 (10)(10) = $50. Profits are 100*2,000+ 50*8,000 – 50,000 = $ 550,000.

3c) Since the daily admission fee is 0, we can sell memberships to C's and S's for the S value ($50) or we can sell memberships only to C's for the C value above ($100), and we have to check which is more profitable. For C's only, profits are 100*2,000 – 5,000 = $ 150,000. For C's and S's, profits are 50*10,000 – 50,000 = $ 450,000. Therefore, it is optimal to charge $50 for a membership, with C's and S's becoming members, and profits are $450,000.

4. Ale and Cournot. This is the classic Cournot problem; to solve, we must compute reaction functions for A and B and solve them simultaneously. Revenue for A is $R_A = (12 - (Q_A + Q_B))\,Q_A$ so that marginal revenue is $MR_A = 12 - Q_B - 2\,Q_A$.

MC = 0, so MR_A = MC implies $Q_A = 6 - .5\,Q_B$ which is A's reaction function. By symmetry, B's reaction function is $Q_B = 6 - .5\,Q_A$. Solving these simultaneously gives $Q_A = 4$, $Q_B = 4$, so total supply is 8 thousand pints, and the price is $P = 4$, Profits for each firm are 4* 4 = $16,000.

5. Bolts

5a) Equilibrium is A "goes first," then "Honor" and B "Trust". If A plays "simultaneous", then game box is relevant --- A has a dominant strategy of "Skimp," given that B chooses "No Trust" and we end with the (box's) Nash Equilibrium of (5,-5). If A plays "goes first" and then "Skimp", B chooses "No Trust" and we again end up with (5, -5). If A plays "goes first" and the "Honor", then B safely plays "Trust" and we end up with payoffs (8,8). This is the highest payoff for A (and B) and so is the chosen equilibrium.

5b) By going first, A can get 8 thousand dollars, and by playing "simultaneous," A gets 5 thousand dollars. Therefore A would be willing to pay up to 3 thousand dollars to go first.

6. Tennis and Racquets. Let p be the price of a membership and r be the price of a racquet. We have $P = p + r$ in all parts of this problem. Also, for memberships, we have $MC_T = 0$, and for racquets, we have $MC_R = Q$.

6a) For p = 30, demand faced by Racquets R' Us is $r = P\text{-}30 = 150 - Q$. So, $MR_R = 150 - 2Q$, and $MR_R = MC_R$ implies $150 - 2Q = Q$ or $Q = 50$. From demand curve, price is $r = 100$. Profits for Racquets R' Us is 100*50 – TC(50) = 5,000-1,350 = $3,650. Profit to the Tennis Club is 30*50 – 500 = $ 1,000.

6b) This is the most involved question of the test. One must first solve for what Racquets R' Us will do when the Tennis Club sets a price of p. Then we must solve for the p that maximizes Tennis Club profits.

Given a price of p, Racquets R' Us faces demand $r = P\text{-}p = 180 - p - Q$. Therefore, $MR_R = 180 - p - 2Q$, and $MR_R = MC_R$ implies $180 - p - 2Q = Q$ or $Q = 60 - p/3$. This summarizes Racquet's R' Us's reaction and sets Q given p.

Tennis Club profits are $\text{Profit}_T = p\,Q - 500 = p\,(60 - p/3) - 500 = 60p - p^2/3 - 500$. Maximize by setting d Profit/dp = 0, or $60 - 2p/3 = 0$, or $p = 90$.

This implies that $Q = 30$ and $r = 60$. Profits for the Tennis Club is 90*30 – 500 = $2,200. Profit for Racquets R' Us is 60*30 – 100 – (30*30)/2 = $1,250.

6c) Straightforward monopoly problem. Revenue is $PQ = (180 - Q)Q$ so MR = $180 - 2Q$. MC comes from racquet production, with MC = Q. So, MR = MC implies $180 - 2Q = Q$, or $Q = 60$, which in turn implies $P = 120$. Joint profits are 120*60 – 100 – (60*60)/2 – 500 = $ 4,800.

6d) By opening their own racquet store and selling the bundle, the Tennis Club forecloses demand for Racquets R' Us. In particular, the Tennis Club is providing both the membership and the racquet; there is no customer left that has a membership but needs a racquet. Therefore, Racquets R' Us will have no demand, and since they have a fixed cost, they would choose to not operate in this market.

7. Footspring

7a) Net revenue for the downstream division is $PQ - 100Q = (300 - Q)Q - 100\,Q = 200\,Q - Q^2$. Differentiate this to get NMR = $200 - 2Q$, as requested.

7b) Marginal cost of the operating system is 0, so it is optimal to set the transfer price = MC = 0. Determine output level by NMR = 0, giving $Q = 100$ thousand. There are no fixed costs, so profits are $PQ - 100Q = 200{*}100 - 100^2 = 100{*}100 = 10{,}000$ thousand, or $ 10,000,000.

7c) Footspring will now charge a monopoly price to Toehold, where Toehold has "demand" curve given by their net marginal revenue: $p = 200 - 2Q$. Therefore, Footspring sets MR from O/S to MC = 0 of O/S. In particular, MR = $200 - 4Q = 0$, which gives $Q = 50$ thousand. The transfer price p is obtained from NMR, namely $p = 100$.

7d) Profits to Footspring are $pQ = 100*50 = 5{,}000$ thousand, or $ 5,000,000. Profits to Toehold are $PQ - 100\ Q - pQ = 250*50 - 100*5 - 100*50 = 50*50 = 2{,}500$ thousand, or $2,500,000. Total profits are $7,500,000, which are much less than the total profits of $10,000,000 from **7b)**. The reason is that **7c)** involves *double marginalization*; output restriction by Footspring plus output restriction by Toehold results in $Q = 50$ instead of the optimal value of $Q = 100$.

8. Cartown

8a) The net benefit of the subway is 20. The option of driving will continue to attract people until its net benefit falls to 20. That is, until $20 = b_c - c_c = 50 - 10 - N_c$. This implies that $N_c = 20$ thousand people drive their cars, and that $N_s = 5$ thousand take the subway.

8b) Total Welfare is

$$N_s *(b_s - c_s) + N_c (b_c - c_c) = (25 - N_c)\, 20 + N_c (50 - 10 - N_c) = 500 + 20 N_c - N_c^2 .$$

Maximizing gives $2 N_c = 20$, or $N_c = 10$ thousand drivers. This is less than the answer to **8a)** because this is a common resource problem, with externalities to more drivers. Namely, in **8a)**, if you switch one driver to the subway, you improve the welfare of all the other drivers without affecting the subway rider's welfare; therefore the answer to **8a)** cannot be a welfare maximum.

8c) Methods for implementing this solution include requiring a driving permit or charging tolls per day. The government could implement this by issuing 10,000 permits only with some method for allocating them to drivers. Noting that with $N_c = 10$ thousand drivers the net benefits to driving are 30, the government could implement the solution by selling permits at 10 dollars/day or charging a daily driving toll of $10.

9. True, False, Uncertain

9a) FALSE**.** The Winner's Curse refers to the tendency in common-value auctions for the winner to be the one with the highest (over)estimate of the goods true value. In private value auctions, there is no Winner's Curse since every bidder knows his own true value of the object (willingness to pay).

9b) FALSE. With Positive Network Externalities, you increase future revenues by selling a greater number initially, so that you want to price such that current MR is less than current MC. This means you price lower initially, not higher.

9c) FALSE or UNCERTAIN with the right explanation. Through adverse selection, the Eyehumvee will be bought by less safe drivers, so there may be more accidents. Through moral hazard, drivers of the Eyehumvee may be more careless because they know their car is safer. Either effect works against the Eyehumvee having lower accidents per driver.