

## 2000 Final Exam Answers: Prepared for TA Grading Purposes

We have tried to cover a wide range of possible answers in the following. To determine how many points are allocated to individual parts of questions, see the professors.

### 1) True, False, Uncertain

#### a) Uncertain

Recall that by the Dorfman-Steiner formula, the optimal advertising-sales ratio is

$$A/S = -\varepsilon_a / \varepsilon_p$$

This means that the optimal advertising to sales ratio can be calculated by dividing the effectiveness of advertising (Advertising elasticity) by the price elasticity.

If we assume that the statement is correct so that both the price elasticity and the advertising effectiveness have increased, the impact on the total ratio is **uncertain**.

There is also a wrinkle that we asked for advertising budgets, which should be  $(A/S) * S$ , so the answer is uncertain for yet another reason – we didn't say what happens to total sales. Anyone who makes this argument should be clear for credit and needs a discussion of  $A/S$  to get full credit.

**b) True.** This is the essence of the cartel problem. If industry profits are maximized, then any individual firm has an incentive to produce more, given the quantities produced by its rivals. The reason is that the firm's own marginal revenue is higher than the firm's marginal cost when industry profit is at a maximum.

Why? The firm's decision to maximize profits equalizes *firm* marginal revenue to *firm* marginal cost while the industry profit maximizing output levels will equalize *industry* marginal revenue to *industry* marginal cost. Recall marginal revenue consists of two components. First, the additional revenue you get from selling one more unit of the good and second, the reduction in revenue you suffer from having to sell all the earlier units at a lower price. The first part is the same whether it is firm or industry that sells the extra unit, but the second is very different: The firm only takes into account the reduction in their *own* revenue, while the industry takes into account the reduction in *everyone's* revenue that results from increasing output by one more unit and thereby reducing the price on every unit sold by all firms.

**Alternative Answers to 1b):**

**Math Answer: True** industry profit maximizing behavior yields the first order conditions (FOC) that industry marginal revenue equals industry marginal cost at the optimal industry output:  $QP'(Q) + P(Q) = c$ . (\*)

While firm profit maximization given the output of rival firms yields the FOC that:  $q_i P'(Q) + P(Q) = c$ .

Adding across the three firms yields the condition that total output satisfies the equation:

$$QP'(Q) + 3P(Q) = 3c \quad \text{or} \quad 1/3 Q P'(Q) + P(Q) = c \quad (**)$$

Now since  $P'(Q)$  is negative, the  $Q$  that solves (\*) is bigger than the  $Q$  that solves (\*\*). (Must also discuss intuition for full credit.)

**Monopoly Answer: Uncertain** Suppose fixed costs are big enough that only one firm produces in Cournot equilibrium. Then that firm maximizes profits and is the only active firm. Therefore it is possible that both firm and industry profits are at a maximum when one firm takes the other firms outputs (zero) as given and maximizes own profits. (Must also discuss stated answer for full credit.)

**Repeated Game Answer: Uncertain** Suppose these firms are involved in a repeated interaction that is expected to last for a long time. By maximizing profits in the sense of the net present value of profits across time, it is possible that the benefits to each firm from cooperating outweigh the benefits of cheating today and then suffering in the cournot equilibrium in future periods. Thus, the cartel outcome can be an equilibrium of the dynamic game. In that case, each firm can maximize own profits in a long run sense subject to the output levels of rivals *and* be maximizing industry profits. (Must also discuss stated answer for full credit.)

- c) **False.** The statement is false because of both adverse selection and moral hazard. The people who take the insurance will tend to be those who know they will be heavy users of services (and thus will impose costs above the overall average). Also, having the insurance will cause some cat owners to consume more pet medical services than they otherwise would. So, the expected "fair" rate of  $10 + (.05*600) = \$ 40$  is very unlikely to cover the actual health costs incurred by cats that are insured.

**2) Murcky and Pfizzier**

- a) Both companies have a dominant strategy to play Now.  
 b) The Nash equilibrium of the game is (Now,Now) for payoffs (5,5) since each player is playing their best response to the strategy of their rival.

c)  $X = 11$ , for following reasons:

If choose Now, then your payoff's go down by  $\$X$  million. Viz.,

		<u>Pfizzier</u>	
		Now	Later
<u>Murck</u>	Now	5-X, 5-X	20-X, 0
	Later	0, 20-X	10, 10

If  $X=1$ , then the equilibrium is still (Now, Now.)

		<u>Pfizzier</u>	
		Now	Later
<u>Murck</u>	Now	4, 4	19, 0
	Later	0, 19	10, 10

If  $X=5$ , then there are three equilibria is still (Now, Now), (Now, Later) and (Later, Now) since if one player chooses Now, the other is indifferent between Now and Later.

		<u>Pfizzier</u>	
		Now	Later
<u>Murck</u>	Now	0, 0	15, 0
	Later	0, 15	10, 10

If  $X = 10$ , then there are still three equilibria: (Now, Now), (Now, Later) and (Later, Now) since if one player chooses Later, the other is indifferent between Now and Later.

		<u>Pfizzier</u>	
		Now	Later
<u>Murck</u>	Now	-5, -5	10, 0
	Later	0, 10	10, 10

If  $X = 11$ , then (Later, Later) is the only equilibrium since if one player chooses Later, the other is indifferent between Now and Later.

		<u>Pfizzier</u>	
		Now	Later
<u>Murck</u>	Now	-6, -6	9, 0
	Later	0, 9	10, 10

So,  $X = 11$  is the minimum amount that would cause (Later, Later) to be the only equilibrium.

3) **Black Velvet Paintings.** There are two groups, 1,000 BVCs with demand  $Q = 20 - 2P$ , and 10,000 BVPs, with demand  $Q = 10 - P$ .

a) One admission price: To set the admission price, you needed to derive the total demand and then solve for the optimal price given that demand.

i) Total demand:

$$Q = 1,000 * Q_{BVC} + 10,000 * Q_{BVP} \rightarrow$$

$$Q = 1,000 * (20 - 2P) + 10,000 * (10 - P) \rightarrow$$

$$Q = 120,000 - 12,000P$$

ii) Given total demand, maximize profit, which is revenue – cost(0)

In terms of Q, we have

$$\text{Profit} = Q * (10 - Q/12,000) = 10Q - Q^2/12,000$$

$$\partial \Pi / \partial Q = 0 \rightarrow 10 - 2Q/12,000 = 0 \rightarrow Q = 60,000.$$

At  $Q = 60,000$ , from the aggregate demand function,

For  $P = \$5$  and  $Q = 60,000$ , **Profit is \$300,000.**

b) Maximize profits given that attract only BVCs.

Since BVCs have individual demand  $Q = 20 - 2P$ , their consumer surplus given zero admission-price is  $\frac{1}{2} * 10 * 20 = \$100$ .

**Thus, the membership fee, which will be paid only by the BVCs, is \$100.**

This extracts all surplus from BVCs. Furthermore, BVPs don't want to pay for the membership since their consumer surplus given zero admission-price is  $\frac{1}{2} * 10 * 10 = \$50$ .

Total profit under this scheme is membership fees from BVCs =  $1,000 * \$100 \rightarrow \Pi = \$10,000$

c) Membership for all. There are two options: setting the membership fee so that only the BVCs join (\$100 as above) or set the fee so that BVPs and BVCs join.

If the fee is \$100 and only the BVCs join, the revenue will be \$100,000.

The fee that will induce the BVPs to join is their consumer surplus at  $P=10$ , or  $\frac{1}{2} * 10 * 10 = \$50$ .

**At \$50, both groups will join and revenue will be  $11,000 * \$50 = \$550,000$ .**

#### 4) Trash Sorting Systems

a) Net revenue from producing sorting systems from servos

$$NR = PQ - \text{Assembly Costs} = 1120Q - Q^2 - 900Q = 220Q - Q^2$$

$$\text{Therefore } NMR = 220 - 2Q.$$

- b) Perfect outside market for servos, so we set transfer price  $p = 100$ . For production amounts, we set  $p = MC$  for each facility. That is

$$100 = MC_1 = 20Q_1 \rightarrow Q_1 = 5$$

$$100 = MC_2 = 10Q_2 \rightarrow Q_2 = 10$$

$$100 = MC_3 = 5Q_3 \rightarrow Q_3 = 20$$

so total quantity produced in all three facilities is **35** (thousand). Total needs for servos for sorting systems is found by setting  $p = NMR$ , or

$$100 = NMR = 220 - 2Q \rightarrow Q = 60$$

So,  $Q = 60$  (thousand) systems are produced, and  $60 - 35 = 25$  (thousand) servos are purchased in the outside market.

- c) We seek a solution where servos are evaluated inside at the same level as any servos either bought or sold in the outside market. Here the buying price is  $100 + 20 = 120$ , and the selling price is 110, and these are the two most natural candidates for a solution. By trying  $p = 120$ , we see that it provides the optimal transfer price. Namely, suppose we set the transfer price  $p = 120$ . For production amounts, we set  $p = MC$  for each facility. That is

$$120 = MC_1 = 20Q_1 \rightarrow Q_1 = 6$$

$$120 = MC_2 = 10Q_2 \rightarrow Q_2 = 12$$

$$120 = MC_3 = 5Q_3 \rightarrow Q_3 = 24$$

so total quantity produced in all three facilities is **42** (thousand). Total needs for servos for sorting systems is found by setting  $p = NMR$ , or

$$120 = NMR = 220 - 2Q \rightarrow Q = 50$$

So,  $Q = 50$  (thousand) systems are produced, and  $50 - 42 = 8$  (thousand) servos are purchased in the outside market at price  $p = 120$ .

Notice, if  $p = 110$  were set as the transfer price,  $Q = 55$  (thousand) systems are produced, total in-house production is 38.5 (thousand) and  $55 - 38.5 = 16.5$  (thousand) servos would be needed to be purchased in the outside market.

Unfortunately, they cannot be purchased at  $p = 110$ , but rather only at  $p = 120$ , so  $p = 110$  cannot be the optimal transfer price.