

# Final Review Session

*15.010/011 Economic Analysis for Business Decisions*

December 10, 2004

## Contents

1. Two-part tariffs, bundling, and pricing.....	3
2. Transfer pricing.....	11
3. Asymmetric information.....	23
4. Cartels and auctions .....	27
5. Externalities/Common Property.....	33
6. Game Theory Part 1 .....	36
7. Game Theory Part 2 .....	43

# **Two-part tariffs, bundling, and pricing**

## Two-Part Tariffs

Consumers pay a one-time access fee (T) for the right to buy a product, and a per-unit price (P) for each unit they consume.

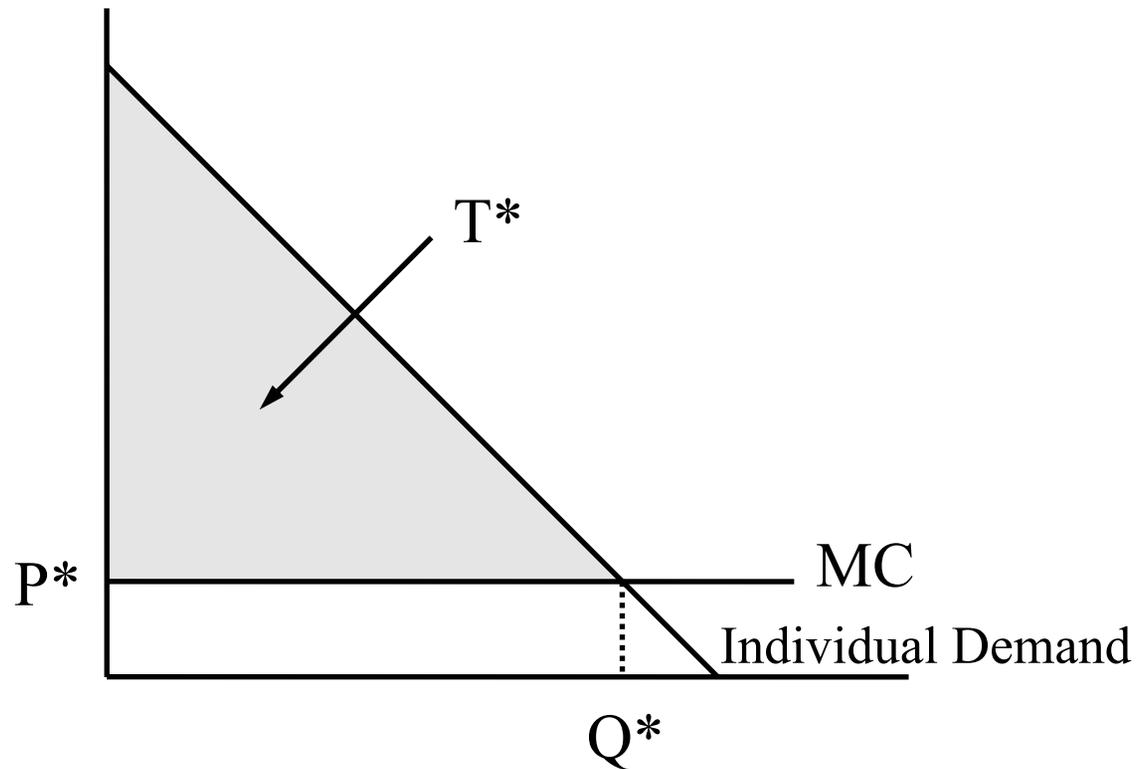
Examples: Amusement parks, Golf Clubs, T-passes, Dance Clubs

Necessary conditions for Two-Part Tariff implementation:

1. Firm must have **market power**
2. Firm must be able to **control access**
3. **Homogeneous consumer demand** (all the consumers within the same segment have the same demand curve)

Note: We are now dealing with individual demand curves (as opposed to market demand curves)

## Two-Part Tariffs - Single Consumer Group:

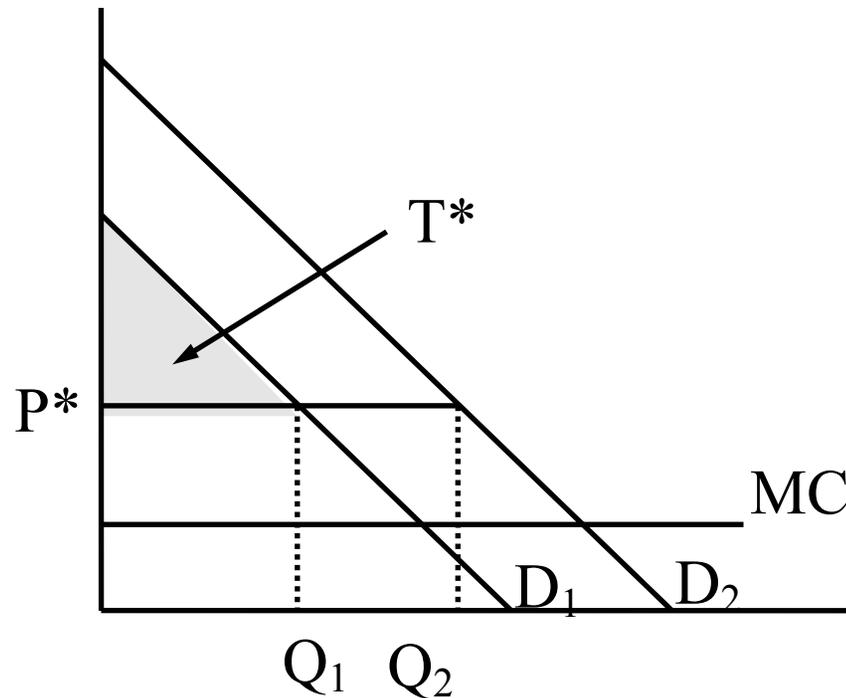


Optimal Pricing strategy:

Entry fee  $T^*$  equal to the *entire surplus of the consumer*

Usage fee  $P^*$  equal to *Marginal Cost*

## Two-Part Tariffs - Multiple Consumer Groups:



Optimal Pricing Strategy:

Entry fee  $T^*$  equal to *Surplus of the consumer with smaller demand*.

Usage fee  $P^*$  to maximize  $\Pi = 2T^* + (P^* - MC)(Q_1 + Q_2)$ .

$P^*$  can be identified setting  $d\Pi/dP = 0$  and solving for  $P$ .

Note that  $T^*$  is a function of  $P^*$

## **Two-Part Tariffs - Example**

Some video stores offer customers two ways to rent films:

- (i) Pay an annual membership fee (e.g., \$40), and then pay a small fee for the daily rental of each film (e.g., \$2 per film per day) (Two part Tariff)
- (ii) Pay no membership fee, but pay a higher daily rental fee (e.g., \$4 per film per day) (Simple rental fee)

Why might it be more profitable to offer consumers a choice of two plans, rather than a single plan for all customers?

A classic price discrimination example. The store has created a menu of choices where each plan appeals to a different group of consumers that will self select into the option designed for them. The high demand consumer will probably choose the two-part tariff, while the casual consumer will prefer the simple rental fee.

Profits will be greater with price discrimination than with a single pricing scheme for all customers.

# Bundling

Bundling refers to selling more than one product at a single price.

When is bundling applicable:

- The firm has **market power**
- Price discrimination is **not** possible (inability to offer different prices to different customers or segments)
- Demand for two or more goods to be sold is **negatively correlated** (the more consumers demand one good, the less they will demand of the other good)

**Pure Bundling:** Consumers must buy both goods together; the choice of buying one good without buying the other is NOT given.

**Mixed Bundling:** Consumers have the choice of buying both goods or buying one good without the other.

## Pure Bundling Example

You have two consumers with known reservation prices for goods A and B. Should you price the goods separately or bundle? Assume  $MC=10$  for both goods.

	Product A	Product B
Consumer 1	\$ 120	\$ 30
Consumer 2	\$ 100	\$ 40

If you price separately:

$$\text{Price A} = 100 \quad \text{Profit A} = 2*(100-10) = 180$$

$$\text{Price B} = 30 \quad \text{Profit B} = 2*(30-10) = 40$$

$$\underline{\text{Total Profit} = \$220}$$

If you bundle the goods:

$$\text{Price Bundle} = 140 \quad \text{Profit} = 2*(140-20) = 240$$

$$\underline{\text{Total Profit} = \$240}$$

By bundling the goods you have increased profits.

## Overview of Pricing Tactics

<b>Pricing Method</b>	<b>When to Use</b>	<b>How to Use</b>
P=MC	Perfect Competition	Take P from horizontal demand curve. Set P=MC.
MR=MC	General Monopoly Power	Find MR, MC. Set MR=MC. Get P from Demand Curve
Learning Curve	Cost function of cumulative output.	Set MR=MC after learning, unless discount rate is high.
Perfect Price Discrimination	Excellent information about consumer preferences	Set P = reservation price of each customer
Customer Self-selection	Offer consumers a menu of choices with different prices	Built-in inconvenience in the “low” offering so that high-value consumers select the “high” option.
Observable market segments	Can distinguish segments and willingness to pay,	Set $MR_1 = MR_2 = MC$ across segments
Two Part Tariffs	Cannot price discriminate effectively, few sets of homogeneous demand, quantity varies for individuals	Price has 2 components: entry fee (T) and price per unit purchased (P).  Choose P to maximize profit function, which has entry fee component.  T = consumer surplus of low valuation consumer. If all consumers are identical, P=MC and T = consumer surplus at P.
Bundling (Pure)	Multiple products, heterogeneous demand, demand negatively correlated	Solve numerically for simple problems. Solve by trial and error if demand is unknown

# Transfer pricing

Internal price at which components from upstream division are sold to downstream division

Upstream Division: (component manufacturer)

- Sells components inside or outside firm

Downstream Division: (end product manufacturer)

- Buys components inside or outside firm

Net Marginal Revenue: extra revenue that an additional unit of upstream division's product brings after extra downstream production costs

- $NMR = MR - MC_d$
- NMR does NOT include MC of upstream (component) product
- Another way to think about NMR is 'Marginal Net Revenue' i.e. the marginal revenue that the downstream division gets net of its own marginal costs but not including the marginal cost of the upstream product.

# Transfer Price Example

Upstream Division                    – electronic components  
Downstream Division                – radios

$$TC_{\text{radio}} = 30 + 2Q_r$$

$$TC_{\text{component}} = 70 + 6Q_c + Q_c^2$$

$Q_{\text{component}} = Q_{\text{radio}}$  i.e. there is a one to one relationship between a radio and a component

The demand for radios is :  $P_{\text{radio}} = 108 - Q_{\text{radio}}$

$$MR_{\text{radio}} = 108 - 2Q_{\text{radio}}$$

$$MC_{\text{radio}} = 2 \quad (\text{not including component costs})$$

$$NMR = MR_{\text{radio}} - MC_{\text{radio}} (\text{not including component costs})$$

$$NMR = 108 - 2Q_{\text{radio}} - 2$$

$$NMR = 106 - 2Q$$

$$MC_{\text{component}} = 6 + 2Q$$

## Transfer Price: No Centrally Set Transfer Price

Case 1: Let's say that there is no Outside Market (Yet), and each division maximizes its *OWN* profits

For the Upstream Division

- Demand curve facing upstream division = NMR
- Calculate  $MR_{\text{component}}$  by  $d\{NMR*Q\}/dQ$
- Set  $MR_{\text{component}} = MC$  and solve for  $Q_{\text{component}}$
- Get  $P_{\text{component}}$  from component demand curve (NMR) – and  $P_{\text{component}}$  will be the transfer price that the Upstream division will set

For the Downstream Division

- $Q_{\text{radio}} = Q_{\text{component}}$
- $P_{\text{radio}}$  from downstream demand curve

This situation is like having two separate companies, each with market power and hence creates Double Marginalization

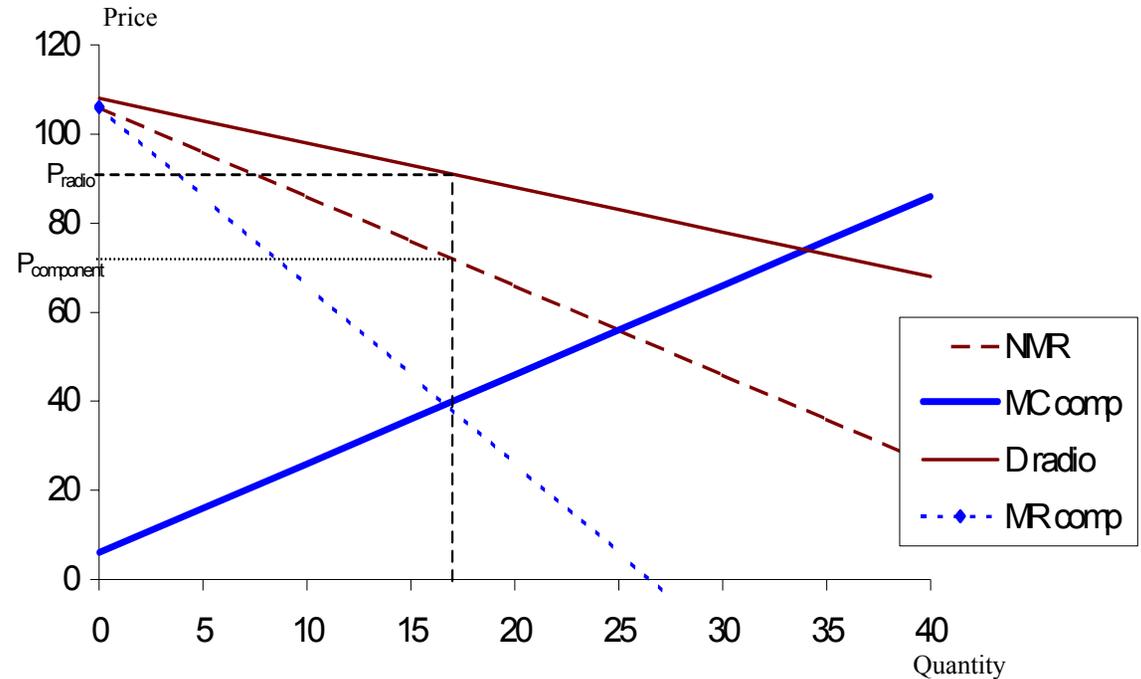
# Transfer Price: No Centrally Set Transfer Price

Upstream Division:

- Demand curve:  $P_{\text{component}} = 106 - 2Q$  (NMR of downstream division)
- $MR_{\text{component}} = 106 - 4Q$  (Recall that  $MC_{\text{component}} = 6 + 2Q$ )
- $MR=MC$ :
  - $106 - 4Q = 6 + 2Q$
  - $Q = 16.7$  components
- $P_{\text{component}} = 106 - 2(16.7) = \$72.7$

Downstream Division

- $Q_{\text{radio}} = 16.7$  radios
- $P_{\text{radio}} = 108 - 16.7 = \$91.3$



# Transfer Price: Central Transfer Pricing, No Outside Market

Case 2: Now, the firm wants to maximize its *OVERALL* profit

- Additional benefit of last unit of component = additional cost of last unit of component
- $NMR = MC_{\text{component}}$ 
  - Additional benefit of the last unit of component stems from the sale on an additional radio

For the Upstream Division:

- Set  $P_{TP}$  so upstream division produces  $Q_{\text{component}}^*$
- $P_{TP} = MC_{\text{component}} = NMR$  (use  $MC_{\text{component}} = NMR$  to get  $Q_{\text{component}}^*$ )
- Substitute  $Q_{\text{component}}^*$  in either  $MC_{\text{component}}$  or  $NMR$  to get  $P_{TP}$

For the Downstream Division:

- $P_{\text{radio}}$  from downstream (radio) demand curve at  $Q_{\text{radio}}^*$

## Transfer Price: Central Transfer Pricing, No Outside Market

Upstream Division

- $NMR = MC_{\text{component}}$  (Recall that  $MC_{\text{component}} = 6 + 2Q$  and  $NMR = 106 - 2Q$ )
  - $106 - 2Q = 6 + 2Q$
  - $Q^* = 25$  components (& radios)
- $P_T = MC_{\text{component}} = 6 + 2*(25) = \$56$

Downstream Division:

- $P_{\text{radio}} = 108 - Q = 108 - 25 = \$83$

# Transfer Price: Competitive Outside Market

Case 3: A perfectly competitive outside market exists for the upstream product

- If  $P_T < P_{\text{competitive}}$  → lose opportunity cost of sale outside
- If  $P_T > P_{\text{competitive}}$  → lose on purchase of upstream product
- $P_T = P_{\text{competitive}}$

Upstream Division:

- Marginal revenue is the competitive market price
- Optimize at  $MR_{\text{component}} = P_{\text{competitive}} = MC_{\text{component}}$  and solve for  $Q_{\text{component}}$

Downstream Division:

- Marginal cost of upstream product is the competitive market price
- Optimize at  $MC = P_{\text{competitive}} = NMR$  and solve for  $Q_{\text{radio}}$
- $P_{\text{radio}}$  from downstream (radio) demand curve at  $Q_{\text{radio}}$

- If  $Q_{\text{component}} > Q_{\text{radio}}$  → sell components on outside market
- If  $Q_{\text{component}} < Q_{\text{radio}}$  → buy components from outside market

## Transfer Price: Competitive Outside Market

$$P_{\text{competitive}} = \$40$$

Upstream Division:

- $MR = P_{\text{competitive}} = 40$
- Optimize at  $MR = MC_{\text{component}}$ :
  - $40 = 6 + 2Q$
  - $Q_{\text{component}} = 17$  components

Downstream Division:

- $MC_{\text{component}} = 40$
- Optimize at  $MC = NMR$ :
  - $40 = 106 - 2Q$
  - $Q_{\text{radio}} = 33$  radios
- $P_{\text{radio}} = 108 - Q_{\text{radio}} = 108 - 33 = \$75$

$$Q_{\text{component}} < Q_{\text{radio}}$$

→ BUY  $(33 - 17) = 16$  components from competitive market

## Transfer Price: Monopoly Outside Market

Case 4: An outside market exists for the upstream product and the upstream division is a monopolist

- 2 potential sources of marginal revenue for the upstream division:
  1. MR from component sale on outside
  2. NMR from use of component internally→ Total MR curve for component is horizontal sum of 2 marginal revenue curves
- $MR_{\text{component}} = MC$   
→  $MR_{\text{component}} = NMR_{\text{inside}} = MR_{\text{outside}}$  (Use these relationships to get the quantities)  
$$\begin{array}{ccc} & \downarrow & \downarrow \\ & Q_{\text{inside}} & Q_{\text{outside}} \end{array}$$
  
→  $Q_{\text{components}} = Q_{\text{inside}} + Q_{\text{outside}}$

For the Upstream Division:

- $P_{\text{outside}}$  from external component demand curve at  $Q_{\text{outside}}$

For the Downstream Division:

- Transfer price is marginal cost
- $NMR = P_T$
- $P_{\text{radio}}$  from radio demand curve using  $Q_{\text{inside}}$

# Transfer Price Example: Monopoly Outside Market

$$P_{\text{outside}} = 72 - 1.5Q_{\text{outside}}$$

(External demand curve for components)

- $MR_{\text{outside}} = 72 - 3Q_{\text{outside}} \rightarrow Q_{\text{outside}} = 24 - MR_{\text{outside}}/3 = 24 - MR_{\text{component}}/3$
- $NMR_{\text{inside}} = 106 - 2Q_{\text{inside}} \rightarrow Q_{\text{inside}} = 53 - NMR_{\text{inside}}/2 = 53 - MR_{\text{component}}/2$
- $Q_{\text{total}} = Q_{\text{inside}} + Q_{\text{outside}} = 77 - 5/6 MR_{\text{component}}$
- $MR_{\text{component}} = 462/5 - 6/5Q_{\text{total}}$  (Note: that this relationship is true only for  $Q > 17$ . For  $Q < 17$   $NMR_{\text{inside}}$  is always greater than  $MR_{\text{outside}}$ . So the upstream division will only sell inside the firm.)
- $MR_{\text{component}} = MC$
- $462/5 - 6/5 * Q_{\text{total}} = 6 + 2Q_{\text{total}}$
- $Q_{\text{total}} = 27$  components
- $MR_{\text{component}} = 462/5 - 6 * 27/5 = 60 = NMR_{\text{inside}} = MR_{\text{outside}}$
- $Q_{\text{inside}} = (106 - 60)/2 = 23$  components (= number of radios)
- $Q_{\text{outside}} = 27 - 23 = 4$  components

Upstream Division:

- $P_{\text{outside}} = 72 - 1.5Q_{\text{outside}} = 72 - 1.5 * 4 = \$66$

Downstream Division:

- $P_T = NMR = \$60$
- $P_{\text{radio}} = 108 - Q_{\text{radio}} = 108 - 23 = \$85$

**Example: True, False or Uncertain:**

EKAR Corporation manufactures and sells electric cars. The Electronic Division of EKAR supplies the engine for these cars. This engine can also be bought or sold in an outside market for \$10,000. A recent technology innovation at EKAR has reduced their marginal cost of production for the engine. As a result of this cost reduction, EKAR should increase the number of electric cars it produces.

Answer: The engines can be bought and sold in a competitive market. The optimal transfer price is \$10,000 and is unchanged by the innovation at EKAR. The electric car divisions' production will remain unchanged, as they will still optimize so that the net marginal revenue from the engine is equal to the transfer price (\$10,000). The answer is FALSE.

(The engine division will optimize so that the transfer price (\$10,000) is equal to their new marginal cost. Consequently, the upstream division will increase their production of engines.)

# **Asymmetric information**

## Asymmetric Information

Can exist if sellers of a product have better information about its quality than the buyers.

- Nothing wrong with having both high quality and low quality goods in the market since there can be a demand for both types - as long as consumers know what quality product they are buying!
  - EX Five Star Restaurant vs. Joe's Bar & Grill
- The problem is that Asymmetric Info can lead to market failure
  - If prices are driven down because consumers do not know the quality level of the product, owners of high quality goods will not sell their products and thus lead to a condition where only low quality goods exist.
    - EX. Autos that are just a few months old (only lemons sold)
    - Producers of high quality goods can send a credible signal as to their quality level to avoid market failure. (Money back guarantees, Warrantees, customer service reputation)

## Signaling

High quality producers can send a *credible and informative* signal that their product is high quality

- Benefit of signal exceeds costs for high quality producer
- Cost exceeds benefit for low quality producer
- For the signal to work, high quality producer sends signal but low quality producer doesn't
  - EX1: Harry / Lew car dealers (Prob#9, P&R Chap 17, P. 620)
  - EX2: B-School Degrees

## Adverse Selection

Can exist if buyer knows more about the actual cost of the service she is buying – only customers who will use a disproportionately high amount of the service will pay, driving up total cost.

- EX1: All you can eat restaurant – buyer knows how much he will eat, seller does not → Big eaters more likely to buy all you can eat, “eating” into seller’s profits
- EX2: Insurance. Buyer knows if she is not feeling well and can then run and buy extra insurance

## **Principal—Agent Problem**

Agent is the person making the decision, Principal is the party whom the decision affects

- EX1: Doctors (Agent) making decisions regarding operations, medications; patients (Principal) are the ones affected.
- EX2: Management (Agent) / Owners and Debt Holders (Principal)
- EX3: Real Estate Agent (Agent) / Homeowner (Principal)

## **Moral Hazard**

Customers' modify behavior *after* entering agreement

- EX1: Insurance – buyer can start skydiving or race driving *because* insurance company will pay (for part) of downside
- EX2: Savings and Loan: were able to get large sums of government insured capital. Investors asked little questions since money was insured by Feds – but S&L's invested aggressively and ended up in crisis

# **Cartels and auctions**

## Cartels

- Producers in a cartel explicitly agree to cooperate in setting prices and output levels
- Requirements for cartel success:
  - A stable cartel organization must be formed whose members can agree on price and production levels and then adhere to that agreement
    - Agreement, Monitoring, Enforcement
  - There is the potential for market power.

## Things to Remember about Cartels

- Cartel's market power is affected by price elasticity of demand and price elasticity of competitive supply.
  - The more price-inelastic the demand, the greater the market power.
  - The more that competitive supply is price-elastic, the lower the market power of the cartel.
- Cartel must be able to overcome organizational issues such as monitoring, compliance, enforcement, etc.
- Optimizing profit in a cartel occurs when firms behave as if they were single monopolist.

## Cartel Sample Question

A cartel's *potential* monopoly power is its ability to raise price above competitive levels, assuming that the cartel members can agree on and adhere to production cutbacks. A cartel's actual monopoly power depends, in addition, on the willingness of the members to agree on and adhere to those cutbacks.

List the specific factors that affect a cartel's potential monopoly power. Explain each briefly, using an illustrative example such as OPEC.

## Auction structure

### Bidding Structure

- **Open outcry:** The bids are openly declared by buyers
  - **English (Ascending):** seller solicits progressively higher bids. When no one bids at the next level, the last bidder wins at the price they bid
  - **Dutch (Descending):** Seller starts high and then reduces price by fixed amounts. First buyer accepting an offered price wins
- **Sealed-bid:** Buyers put bids into an envelope and submit them at the same time
  - **First-price:** highest bidder wins and pays their bid (like Dutch)
  - **Second-price:** highest bidder wins and pays the 2nd highest bid (like English)

## Auctions (continued)

### Valuation

- **Private:** Each bidder has private information about his/her own personal valuation of the auctioned good. Example: art
- **Common Value:** Each bidder has private information about the value of the object auctioned. At the end of the day, the object will be worth the same to all.

### Strategy

- **Private:**
  - **2<sup>nd</sup> price sealed:** bid reservation price. **English:** bid in small increments until you hit your reservation price. *Risk averse* do not bid any differently
  - **Dutch, 1<sup>st</sup> price sealed:** Shade your bids (lower). *Risk averse* shade less for fear of losing item
- **Common Value:** Shade values to avoid *winners curse*. Amount of shading depends on accuracy of estimates and your risk aversion.

## Auctions Example

There are 5 potential buyers in a 2<sup>nd</sup> price sealed-bid auction for an object. Each bidder has a private value and there is no possibility of resale. The bidders and their valuations are:

Bidder	Value
A	\$420
B	\$760
C	\$550
D	\$430
E	\$600

**Question:** How much should each person bid? If they all bid optimally, who will win the auction and how much will that buyer pay?

**Answer:** Each person should bid his/her true private value. Bidder B will bid \$760 and win the auction, but only pay \$600, the second-highest bid.

# **Externalities and common property**

## Common Property Resources

- Common property resources are those to which anyone has free access. As a result, negative externalities can arise and the resources are likely to be overutilized. Examples are fishing waters (depletion) or oil fields (pressure reduction).
- The simplest solution to the common property resource problem is to let a single owner manage the resource. The owner will set a fee for the use of the resource that is equal to the marginal external cost of exploitation, so that utilization will be limited to the optimal level. Unfortunately, most common property resources are vast, and single ownership may not be practical. Thus, government ownership or direct government regulation may be needed.

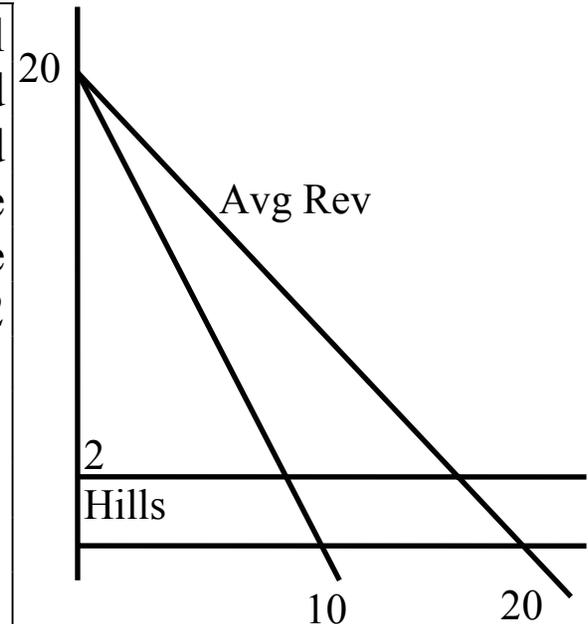
*True or false?*

By “unitizing” oil fields in Texas, fewer production wells are drilled, but each well is more profitable than it would be if landowners operated independently. Hence unitizing oil fields leads to monopoly power and monopoly profits.

*Answer:*

FALSE. The size and competitiveness of the oil market is such that it is impossible to increase monopoly power by unitizing oil fields. The reason why unitized oil fields are more profitable is the existence of negative externalities in oil extraction. Negative externalities exist because when an additional oil well is drilled on the same oil field, the pressure in the underground deposit is reduced, affecting the output of all the other wells on the same field.

Farmers have free access to a common land. The farmers sell milk from their cows at \$1/gallon. Because cows trample and eat the grass, the amount of milk each cow produces (and therefore, the revenue) depends on the number of cows on the land so that  $AR = 20 - C$  (where  $C$  is the # of cows). If the farmers leave their cows in the hills, the cows will produce 2 gal/wk.



1. How many cows (total) will the farmers want to bring onto the common land?
2. What number of cows would maximize the benefits to the community as a whole?

3. What mechanism could you use to get the farmers to only bring the optimum number of cows onto the land?
4. Given that the number of cows is  $N$  ( $N > 18$ ), what would be the net welfare/wk. in each case (1-3).

**Solution:**

1. Farmers will bring cows down until there are 18 cows on the common land.
2. The community wants farmers to bring cows down until Marginal Milk Revenue = Opportunity Cost in the hills:  $2 = 20 - 2C$  or # Cows = 9.  
(alternatively, maximize  $\pi = (20 - C) \times C + (N - C) \times 2$  with respect to  $C$ )
3. Charge a usage fee of  $AR(9) - \text{Opportunity Cost} = \$11 - \$2 = \$9$  per cow.
4.
  1.  $\pi_{\text{common land}} + \pi_{\text{hill}} = 18 \times (20 - 18) + (N - 18) \times 2 = 2N$  \$/wk.
  2.  $\pi_{\text{common land}} + \pi_{\text{hill}} = 9 \times (20 - 9) + (N - 9) \times 2 = 81 + 2N$  \$/wk.
  3.  $\pi_{\text{common land}} + \pi_{\text{fee}} + \pi_{\text{hill}} = 9 \times [(20 - 9) - 9] + 9 \times 9 + (N - 9) \times 2 = 81 + 2N$  \$/wk.

# Game theory, part 1

# Game Theory I - Overview

**Cournot models**

**Stackelberg variations on Cournot model**

**Bertrand model with undifferentiated products**

**When Cournot? When Bertrand?**

**COURNOT MODEL:** Two duopolists producing *undifferentiated* product (price will be same for both) and making *quantity* decisions *at the same time*. Solve for the reaction curves showing optimal Q for one firm for any given Q made by the other firm.

**Model premises:**

$$\begin{aligned} \mathbf{TC}_1 &= \mathbf{10Q_1} \\ \mathbf{TC}_2 &= \mathbf{10Q_2} \\ \mathbf{P} &= \mathbf{100 - Q_1 - Q_2} \end{aligned}$$

$\mathbf{MC}_1$	=	$\mathbf{10}$	Derivative of Firm 1's total cost curve
$Q_T$	=	$100 - P$	Set up demand in "P=" format
	=	$100 - Q_1 - Q_2$	There are only two firms, so $Q_T = Q_1 + Q_2$
$TR_1$	=	$(100 - Q_1 - Q_2) * Q_1$	Firm 1 revenue is price * Firm 1 quantity
	=	$100Q_1 - Q_1^2 - Q_1Q_2$	
$\mathbf{MR}_1$	=	$\mathbf{100 - 2Q_1 - Q_2}$	
$10$	=	$100 - 2Q_1 - Q_2$	Set MR = MC
$2Q_1$	=	$90 - Q_2$	
$\mathbf{Q_1}$	=	$\mathbf{45 - Q_2/2}$	Firm 1's reaction curve
$\mathbf{Q_2}$	=	$\mathbf{45 - Q_1/2}$	Firm 2's reaction curve, by symmetry

## COURNOT MODEL (Cont.):

### TO SOLVE FOR COURNOT EQUILIBRIUM:

Substitute one reaction curve into the other, finding the point where the two curves meet (Cournot-Nash equilibrium).

$$Q_2 = 45 - Q_1/2$$

$$Q_1 = 45 - (45 - Q_1/2)/2$$

$$= 45 - 22.5 + Q_1/4$$

$$Q_1 = 22.5 * 4/3$$

$$Q_1 = 30$$

Since firms have same costs,  $Q_2 = 30$  also.

$$\Pi_1 = (100 - 30 - 30) * 30 - [10 * 30]$$

$$= \$900$$

### Cournot-Nash solution:

$$Q_1 = 30, \quad \Pi = \$900$$

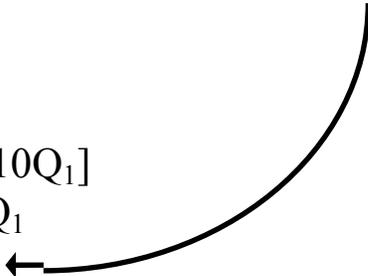
$$Q_2 = 30, \quad \Pi = \$900$$

**STACKLEBERG VARIATION:** Same as Cournot, *except* one firm (say Firm 1) goes first. Solve for the reaction curve of Firm 2 showing optimal Q for any given Q made by Firm 1. Substitute this reaction curve into the profit equation of Firm 1.

**TO SOLVE FOR STACKLEBERG EQUILIBRIUM:** Substitute reaction curve of firm going *second* into profit equation of firm going *first* - solve for  $\Pi$ -maximizing point for the firm going first.

We already know reaction curve for Firm 2:  $Q_2 = 45 - Q_1/2$

Profit equation for Firm 1:

$$\begin{aligned}
 \Pi &= TR - TC \\
 &= (100 - Q_1 - Q_2) * Q_1 - [10Q_1] \\
 &= 100Q_1 - Q_1^2 - Q_1Q_2 - 10Q_1 \\
 &= 90Q_1 - Q_1^2 - Q_1Q_2 \\
 &= 90Q_1 - Q_1^2 - Q_1*(45 - Q_1/2) \\
 &= 90Q_1 - Q_1^2 - 45Q_1 + Q_1^2/2 \\
 &= 45Q_1 - Q_1^2/2
 \end{aligned}$$


## STACKLEBERG VARIATION (Cont.):

$$\begin{aligned}\delta\Pi/\delta Q_1 &= 45 - Q_1 = 0 \\ Q_1 &= 45 && \text{Substitute optimal } Q_1 \text{ back into profit equation} \\ \Pi_1 &= 45(45) - (45)^2/2 = \mathbf{\$1,012.5} \\ Q_2 &= 45 - (45)/2 \\ &= \mathbf{22.5} \\ \Pi_2 &= (100 - 45 - 22.5) * 22.5 - 10 * (22.5) = \mathbf{\$506}\end{aligned}$$

### Stackleberg equilibrium:

#### First mover:

$$Q = 45, \quad \Pi = \$1,012.5$$

#### Second mover:

$$Q = 22.5, \quad \Pi = \$506$$

### Value of being a first mover vs. simultaneous mover:

$$\$1,012.5 - \$900 = \mathbf{\$112.5}$$

### Value of being a first mover, not a second mover:

$$\$1,012.5 - \$506 = \mathbf{\$506.5}$$

**BERTRAND WITH UNDIFFERENTIATED PRODUCTS:** Two firms producing *undifferentiated* products (price will be same) each selecting price at the same time. Assumption: each firm has enough capacity to supply entire market demand.

In any Bertrand situation, price will equal marginal cost and result is perfect competition. Why?

- ➔ Each firm can gain the entire market but just slightly undercutting its competitor's price and lose the entire market by holding price just above its competitor's price.
- ➔ Game theory dynamics drive price down to MC.

Is this a Nash equilibrium?

- ➔ Yes, each firm is doing the best it can, given what its competitor is doing

**In price competition, what conditions might lead to a Nash Equilibrium with a price above Marginal Cost?**

- Some differentiation
- Firms don't maintain enough capacity to take the market
- Protective Mechanisms (for example, price guarantees)

# **Game theory, part 2**

# Important Concepts

- Nash Equilibrium: A solution at which each player is doing its best given what competitors are doing. No one has an incentive to deviate
- Dominant Strategy: A strategy that is optimal for a player regardless of opponents' actions
- Maximin Strategy: A strategy that tries to maximize the payout of the worst possible outcome
- Backwards Induction (Unraveling): In a finite game, looking at what happens in the last stage and working backwards to the beginning

# Know the Game

## Types of Games

- Cooperative vs. non-cooperative
  - Are negotiations and binding contracts possible?
- One time vs. repeated games
- Finite vs. infinitely repeated games
  - Unraveling possible for finite game when ending point is known
- Simultaneous vs. sequential games

# Know the Game (cont.)

## Structure of Games

- What are the rules?
- Who goes first?
- How does the game end?
- What is the objective of the game?
  
- Always assume perfect information, rational behavior from opponents, and objective maximizing behavior unless told otherwise

# Example 1: Nash Equilibrium

Fred and Wilma are selling beer on the beach. The beer is identical and equally priced. Customers are evenly distributed across the beach and purchase from the closest vendor.

True, False, Explain:

Fred opens his business at point F in the diagram below and Wilma opens her business at point W. At these points, each have 50% of the market. Therefore, the current situation is a Nash Equilibrium.

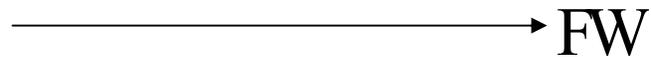
---

F

W

# Answer 1:

False. In a Nash Equilibrium each player is doing the best it can do, given what the other is doing. In the current situation, both players have an incentive to change their behavior. For example, Fred can move just to the left of Wilma and capture roughly  $2/3$  of the market.



# Example 2: Matrix Game

Two firms are each about to introduce a new variety of cookie. They are considering what type of cookie to make: oatmeal, chocolate, or peanut butter. Each will have to choose its type of cookie simultaneously and without communicating with the other. Each firm wants to maximize profits. The possible pay-offs are given below:

		Firm 2		
		Oatmeal	Chocolate	Peanut
Firm 1	Oatmeal	-25,-25	20,40	50,15
	Chocolate	50,30	-10,-10	60,10
	Peanut	15,60	10,45	-50,-50

# Questions:

1. Does either firm have a dominant strategy? If so what are they?
2. Does the game have any Nash Equilibria in pure strategies? If so what are they?
3. If both companies try to attain a Nash Equilibrium, what outcome or outcomes will result?
4. If both sides use a maximin strategy, what outcome or outcomes will result?
5. What outcome results if firm 1 goes first [sequential game]? If firm 2 goes first?

# Answers to Matrix Game

1. There are no dominant strategies for either player. For example, look at the strategy for firm 1 of always making chocolate cookies. This is the best strategy if firm 2 makes oatmeal or peanut butter, but not if firm 2 also makes chocolate (oatmeal is now a better choice).
2. There are two Nash Equilibria (Choc, Oatmeal) and (Oatmeal, Choc).
3. The results are not clear. Either side could reasonably play chocolate or oatmeal, so (C,O), (O,C), (C,C), and (O,O) are all possible outcomes.
4. Following a maximin strategy, both sides make chocolate cookies.
5. The firm that goes first plays chocolate and the other firm responds by playing oatmeal.

# Example 3: Bargaining Game

Professor Miron and Professor Stoker are attending a game theory conference in Hawaii. While sitting on the beach discussing how to create a HW Set 5 for next year, someone drops a container next to them with **32 ounces** of Ben & Jerry's ice cream. The person leaves before they can stop him so Miron and Stoker decide to split the ice cream.

Miron and Stoker will decide how to split the ice cream as follows:

1. Each round, one professor makes a proposal on how to split the ice cream. The smallest unit of division is 1 ounce.
2. If the other agrees they split the ice cream accordingly.
3. If the other disagrees, they argue for 10 minutes about theories of equality and equilibrium, and then other professor makes a new proposal.
4. They continue alternating proposals and arguing for 10 minutes when one is rejected until a decision is made.

It is a very hot day on the beach so each round they argue, **8 ounces** of the ice cream melts. Professor Stoker goes first. What should he propose?

# Answer to Bargaining Game

Write out the structure of the game:

Round	Proposal Maker	Ice Cream Left
1	Stoker	32 ounces
2	Miron	24
3	Stoker	16
4	Miron	8

Solve the game by starting at the end and working back

**In the last round, Miron can propose a split of 7 for him and 1 for Stoker. Stoker must accept because the rest of the ice cream will melt in round 5 and he will get 0.**

Thus in round 3 Stoker could propose a split of 8 for Miron and 8 for Stoker. Miron accepts because he can only get 7 by waiting.

By similar logic, in round 2 Miron proposes a split of 15 for Miron and 9 for Stoker.

Thus in round 1, Stoker should propose a split of 16 and 16 which Miron should accept.

**FIRST-MOVER ADVANTAGE:** By moving first, you force your opponent to accept whatever Nash equilibrium you find optimal. Classic examples:

**Two differentiated products**, one with higher profits than the other. First mover can commit to producing the more profitable product, forcing competitor to produce the less attractive product. Crispy vs. sweet cereals:

		FIRM 2	
		Crispy	Sweet
FIRM 1	Crispy	-5, -5	10, 20
	Sweet	20, 10	-5, -5

If Firm 1 can go first, where will it go? Where will Firm 2 go?

**Stackelberg:** commitment to a large production capacity, forcing opponent to a smaller production capacity.

Note: you have a **first mover advantage** when reaction functions are *downward sloping* - that is, if the more you do, the less your competitor will optimally do (e.g. capacity commitments tend to be negatively correlated). You have **second mover advantage** if reaction functions are *upward-sloping* - that is, the more you do, the more your competitor will do (e.g. pricing decisions).

**ENTRY-DETERRENCE:** Incumbent firm must convince any potential entrant that entry into industry will be unprofitable. But must do it *credibly*.

**Limit pricing** - incumbent charges low price before entry occurs. Potential entrant infers that post-entry price will be that or lower and entry is unprofitable. - Credible when there are large sunk costs. Incumbent can ignore them (remember why?) but entrant cannot (because they are not yet sunk).

**Excess capacity:** you can “Bertrandify” your industry in a jiffy. Without excess capacity, incumbent should accommodate entrant in this game:

		Potential entrant	
		Enter	Stay out
FIRM 1	High price (accomodation)	50, 10	100, 0
	Low price (warfare!)	30, -10	40, 0

With excess capacity (here assumed \$30 million to cost and maintain), incumbent commits itself to low price is entry occurs:

		Potential entrant	
		Enter	Stay out
FIRM 1	High price (accomodation)	20, 10	70, 0
	Low price (warfare!)	30, -10	40, 0

**Reputation effects may make short-term irrational strategy rational, provided game is sequential. A reputation for irrationality makes a threat to make an irrational move credible.**