



SOLUTIONS TO HOMEWORK SET #6

1. In this problem, ACME has an upstream division that makes calculator displays, and a downstream division that assembles the calculator. We are given the demand for calculators, and the costs of the divisions:

$$P_{\text{calc}} = 135 - 2Q_{\text{calc}}$$

$$TC_D = 25 + 5Q_D + Q_D^2$$

$$\Leftrightarrow MC_D = 5 + 2Q_D$$

$$TC_{\text{calc}} = 10Q + TC_D = 10Q + 25 + 5Q + Q^2$$

$$\Leftrightarrow MC_{\text{calc}} = 15 + 2Q$$

- a. The optimal transfer price for displays is set to the marginal cost of displays:

$$P_D = MC_D$$

$$\text{Or, } P_D = 5 + 2Q$$

To calculate P_D , we need to find the optimal quantity of displays for Acme:

Profit = PQ – Processing Cost(Q) – Cost of Displays

$$= (135 - 2Q)Q - 10Q - (25 + 5Q + Q^2) \text{ (i.e. Net Revenue - Cost of Displays)}$$

Taking the derivative with respect to Q and setting it equal to zero:

$$135 - 4Q - 10 - 5 - 2Q = 0$$

$$\Leftrightarrow 125 - 4Q = 5 + 2Q \text{ (i.e. Net Marginal Revenue = Marginal Cost of Displays)}$$

$$\Leftrightarrow Q = 20.$$

With one display for each calculator, this means that $Q_D = 20$

And the optimal transfer price:

$$P_D = 5 + 2Q$$

$$P_D = \$45$$

$$\text{so, } Q_{\text{calc}} = Q_D = 20$$

$$P_{\text{calc}} = 135 - 2(20) = \$95 \text{ (read off the demand curve)}$$

$$\text{Profit}_{\text{calc}} = 95(20) - 10(20) - 45(20) = \$800$$

$$\text{Profit}_D = 45(20) - 25 - 5(20) - 20^2 = \$375$$

$$\text{Profit}_{\text{ACME}} = 800 + 375 = \$1175.$$

- b. Now the divisions set the prices instead. The upstream division will have monopoly power and this will result in a ‘double marginalization’—that is, the upstream division will have a markup ($P_D > MC_D$), as will the downstream division ($P_{calc} > MC_{calc}$). This leads to deadweight loss at both stages, with the deadweight loss resulting from the upstream markup coming straight out of ACME’s profits:

Upstream:

1st figure out the demand for displays from the downstream division:

The calculator producer will maximize:

$$\begin{aligned} \text{Profit} &= PQ - 10Q - P_D Q \\ &= (135 - 2Q) Q - 10Q - P_D Q \end{aligned}$$

Taking the derivative with respect to Q and setting it equal to zero:

$$\begin{aligned} 125 - 4Q - P_D &= 0, \text{ or} \\ P_D &= 125 - 4Q_D \end{aligned}$$

Note: The demand curve that the upstream firm faces is ACME’s Net Marginal Revenue

The upstream firm is then maximizing:

$$\text{Profit}_D = P_D Q_D - (25 + 5Q_D + Q_D^2)$$

Because the upstream firm can act like a monopoly, substitute in the demand curve for P_D , given above as $P_D = 125 - 4Q_D$, or:

$$\text{Profit}_D = (125 - 4Q_D) Q_D - (25 + 5Q_D + Q_D^2)$$

Taking the derivative with respect to Q_D and setting it equal to zero:

$$\begin{aligned} \Leftrightarrow 125 - 8Q_D &= 5 + 2Q_D \text{ (i.e. MR = MC)} \\ \Leftrightarrow Q_D &= 12 = Q_{Calc} \\ \Leftrightarrow P_D &= 125 - 4(12) = 77 \text{ (read off the downstream division’s demand)} \\ \Leftrightarrow \text{Profit}_D &= 77(12) - 25 - 5(12) - 12^2 = \$695 \\ P_{Calc} &= 135 - 2(12) = 111 \text{ (read off the market demand for calculators)} \\ \Leftrightarrow \text{Profit}_{Calc} &= 111(12) - 77(12) - 10(12) = \$288 \end{aligned}$$

Total profits to Acme: 695 + 288 = 983 < part a.

Assuming that the managers are maximizing their own profits implies that the manager of the upstream division will behave monopolistically, restricting output to the point at which $MC_D = MR_D$. Note that this is *not* the point at which $MC_D = \text{Net Marginal Revenue}$, i.e. the point at which *total* firm profits are maximized. In effect, we now have two monopolies and the “double marginalization” will restrict output more than is optimal for the firm as a whole. (This double marginalization can be seen from the slopes of the divisions’ marginal revenue curves: the slope of MR_{Calc} doubles the slope of the market demand from 2 to 4, and the slope of the MR_D doubles the slope again, from 4 to 8).

In contrast, part (a) had ACME setting the transfer price to the marginal cost of displays—eliminating the deadweight loss from the upstream production decision and maximizing total firm profits.

2.

- a. If Harry offers a one-year warranty, the question asks whether Lew will match it or not match it—revealing the quality of Lew’s cars. Because we assume that Lew is a rational actor, he will decide to match the one-year warranty only if this allows him to earn higher profits than by not matching it. We therefore need to study the payoffs.

If Lew does not match Harry’s warranty, the warranty becomes a credible signal of quality, and consumers will be able to discern which car is good quality and which isn’t. Consequently, they will pay \$10,000 for Harry’s cars and only \$7,000 for Lew’s.

If on the other hand Lew matches the warranty, this gives no information to the consumer about the quality of the car, so each dealer will still receive \$8,500 for his cars.

Knowing this, we can calculate Lew’s payoffs as:

One-Year Warranty

	If Lew does not offer warranty		If Lew matches warranty	
	Harry’s	Lew’s	Harry’s	Lew’s
Offers Warranty	Yes	No	Yes	Yes
Cost of buying + servicing car	\$ 8,000	\$ 5,000	\$ 8,000	\$ 5,000
Warranty cost	\$ 500	\$ -	\$ 500	\$ 1,000
Price obtained	\$10,000	\$ 7,000	\$ 8,500	\$ 8,500
Profit	\$ 1,500	<u>\$ 2,000</u>	\$ -	<u>\$ 2,500</u>

Thus, Lew will match the warranty, because by doing so he obtains \$500 more than by not matching it. The one-year warranty will not be a credible signal of quality, since both car dealers will offer it.

- b. Using the same logic as before we can construct the payoffs for Lew, which now are:

Two-Year Warranty

	If Lew does not offer warranty		If Lew matches warranty	
	Harry's	Lew's	Harry's	Lew's
Offers Warranty	Yes	No	Yes	Yes
Cost of buying + servicing car	\$ 8,000	\$ 5,000	\$ 8,000	\$ 5,000
Warranty cost	\$ 1,000	\$ -	\$ 1,000	\$ 2,000
Price obtained	\$10,000	\$ 7,000	\$ 8,500	\$ 8,500
Profit	\$ 1,000	<u>\$ 2,000</u>	\$ (500)	<u>\$ 1,500</u>

A two year warranty will be a credible signal of quality, because Lew will not match it (he gets only \$1,500 per car as opposed to \$2,000 if he doesn't).

Three-Year Warranty

	If Lew does not offer warranty		If Lew matches warranty	
	Harry's	Lew's	Harry's	Lew's
Offers Warranty	Yes	No	Yes	Yes
Cost of buying + servicing car	\$ 8,000	\$ 5,000	\$ 8,000	\$ 5,000
Warranty cost	\$ 1,500	\$ -	\$ 1,500	\$ 3,000
Price obtained	\$10,000	\$ 7,000	\$ 8,500	\$ 8,500
Profit	\$ 500	<u>\$ 2,000</u>	\$(1,000)	<u>\$ 500</u>

Again, a three-year warranty will also be a credible signal of quality. Now Lew gets only \$500 for each car if he offers a warranty. He is clearly better off not offering a warranty and getting \$2,000 per car. However, Harry is now getting \$500, which is as much as he made when he offered no warranty at all.

- c. Harry should offer the cheapest warranty that creates a credible signal of quality. The recommendation would therefore be to offer a two-year warranty as opposed to a three-year warranty.

To be more precise we can calculate the cheapest warranty that will create a credible signal of quality. The warranty will be credible if Lew does not match it, but Lew will match it if his profits are higher by doing so, i.e. if:

$$\begin{aligned}
 & \$8,500 - \$5,000 - \$1,000 * \text{Years} > \$7,000 - \$5,000, \text{ or if} \\
 & \text{Years} < 1.5 \text{ years (18 months)}
 \end{aligned}$$

So, any warranty covering 18 or more months will not be matched and will be credible. Thus, Harry should offer an 18-month warranty.

3.
 - a. **FALSE.** In method (2), there is limited incentive for OEM's to buy anyone else's software for their machines, since they have already paid for every one of their units of output. Thus method (2) prevents penetration by competing systems. If Microsoft uses low prices for the "per-processor" contract to switch OEM's from method (1) to method (2), as was charged in the anti-trust suit, then the overall scheme is anti-competitive.
 - b. **FALSE.** In descending price auctions, a participant will not think of bidding until the bid price equals her maximum willingness to pay. The participant will then consider the benefits and costs of waiting. The benefit is a lower price, while the cost is a smaller probability of winning the auction. Agents who are more risk averse will bid quicker than less risk-averse agents, but all agents have an incentive to "shade down" the bid from the maximum willingness to pay. (Note that knowing everyone else is shading down, will cause you to shade down even more).
 - c. **FALSE.** Because of moral hazard, students may take less care to prevent theft resulting in more thefts. Sloan would break even only if the number of thefts remained the same following the introduction of the insurance.

4.

a. The utility of an independent farmer is

$$u_i = 80h_i - \frac{h_i^2}{2}$$

while that of a cooperative farmer is

$$u_i = \frac{90(h_1 + h_2)}{2} - \frac{h_i^2}{2} = 45(h_1 + h_2) - \frac{h_i^2}{2}$$

b. Taking the derivative of the independent farmer's utility with respect to hours of work and setting it equal to zero yields:

$$80 - h_i = 0$$

or $h_i=80$. The utility of an independent farmer is then $u_i = 3200$.

c. Consider farmer 1, with utility: $u_1 = 45(h_1 + h_2) - \frac{h_1^2}{2}$. Taking the derivative of his utility with respect to his hours of work and setting it equal to zero yields:

$$45 - h_1 = 0 \Leftrightarrow h_1 = 45 \quad \text{and} \quad u_1 = 45(45 + 45) - 2025/2 = 3037.5.$$

(Note: the same results are true for farmer 2 as well.)

d. The problem of the cooperative farm is that the output is common property, so farmers get only part of the output they produce. The incentives to work change from $MR=MC$ to a fraction of $MR=MC$. Compare, in particular, independent and cooperative farmers who consider the gain from putting in one more hour after they have already worked 45 hours. The cost to each of working the 46'th hour is $\frac{46^2}{2} - \frac{45^2}{2} = 45.5$. The gain for an independent worker is 80, which outweighs the cost of 45.5. For a cooperative worker, however, the gain is only 45 (since half of his output goes to the other farmer), so working the extra hour is not worth it. Note that farmer 1 putting in more hours does not change how many hours farmer 2 works. This problem is magnified with more farmers, as the share of the marginal bundle kept by the farmer ($1/\#$ of farmers) decreases with the number of farmers.

If the number of hours that someone works is observable, the farmers could write a contract that exploits the gains from specialization and maximizes utility. (For those who do the calculation, the optimal number of hours for each farmer is 90 per week in the cooperative and farmers would get a utility of 4050.) If effort is not observable, we would have to explore incentive schemes (have farmers keep 100% of the bushels after a certain number are contributed to the cooperative, back-loaded pay, tournaments, etc.) or screening mechanisms (allowing only people who love to farm into the cooperative) to align incentives.