Sloan School of Management
Massachusetts Institute of Technology 15.010/15.011

## SOLUTIONS TO HOMEWORK SET #4

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1.

a. If the markets are open to free trade, the monopolist cannot keep the markets separated. Hence, arbitrage opportunities will mean that $P = P_1 = P_2$. Total market demand in this case is the sum of the demands from Market 1 and Market 2.

$Q = Q_1 + Q_2$
$= 25 - 1/2P_1 + 50 - P_2$
$\mathbf{Q = 75 - 3/2\ P}$

Which, after rearranging, will be:

$\mathbf{P = 50 - 2/3\ Q}$
$\mathbf{REV = 50Q - 2/3\ Q^2}$
$\mathbf{MR = 50 - 4/3\ Q}$

*Time-saving tip*: Note that slope of MR is twice that of demand. This is always true with linear demand. (We'll use this short-cut for the rest of the solutions.)

Now, the monopolist maximizes profits by selling the quantity that equates MR = MC. MC is the derivative of total cost with respect to $Q = Q_1 + Q_2$:

$TC = 10(Q_1 + Q_2)$
$= 10Q$
$\mathbf{MC = 10}$

Thus,

$MR = MC$
$50 - 4/3Q = 10$
**Q = <u>30 units</u>**

Substituting into demand equation we then find P:

$P = 50 - 2/3(30)$
**P = <u>$30</u>**

Total profits then will be:

$\Pi = TR - TC$
$= PQ - 10(Q)$ or alternatively $= Q\,(P\text{-}AC)$
$= (30)(30) - (10)(30)$ $= 30\,(30\text{-}10)$
**Π = <u>$600</u>**

**b.** If the markets are geographically separated, it is possible for the monopolist to price discriminate through market segmentation.

**Market 1**

$Q_1 = 25 - 1/2\, P_1$ , so

$\mathbf{P_1 = 50 - 2\, Q_1}$ and by the MR shortcut

$\mathbf{MR_1 = 50 - 4Q_1}$

$\mathbf{MC_1 = 10}$

Our monopolist will produce $Q_1$ such that $\mathbf{MR_1 = MC_1}$

$50 - 4Q_1 = 10$

$\mathbf{Q_1 = 10}$ **units**

By substituting into the market 1 demand equation,

$P_1 = 50 - 2(10)$

$\mathbf{P_1 = \$30}$

**Market 2**

Repeating this process:

$Q_2 = 50 - P_2$

$\mathbf{P_2 = 50 - Q_2}$

$\mathbf{MR_2} = \mathbf{50 - 2Q_2}$

$\mathbf{MC_2 = 10}$

Again, our monopolist produces $Q_2$ such that $\mathbf{MR_2 = MC_2}$

$50 - 2Q_2 = 10$

$\mathbf{Q_2 = 20}$ **units**

$\mathbf{P_2 = \$30} = 50 - 20$

In summary,

$\mathbf{P_1 =}$ **<u>$30</u>**, $\mathbf{Q_1 =}$ **<u>10 units</u>**

$\mathbf{P_2 =}$ **<u>$30</u>**, $\mathbf{Q_2 =}$ **<u>20 units</u>**

Total profits across the two markets will be:

$\Pi = TR - TC$

$= P_1Q_1 + P_2Q_2 - 10(Q_1 + Q_2)$

$= (30)(10) + (30)(20) - (10)(20+10)$

$\mathbf{\Pi = \underline{\$600}}$

2.

a. In the case where the firm sells the goods separately, the calculations are summarized in the following table:

| | Revenue | Cost | Profit |
|---|---|---|---|
| **MP3s** | **1 @ 96** | **15** | **81** |
| | 2 @ 30 | 30 | 30 |
| **Walkmans** | **1 @ 90** | **15** | **75** |
| | 2 @ 30 | 30 | 30 |

The optimal solution would be represented by:

**MP3s @ $96** and **Walkmans @ $90**

And total Profits would be**: Π =** $81 + $75 = **$156**

b. In the case where the firm decides to set up a pure bundling pricing scheme, the calculations are summarized in the following table:

| | Revenue | Cost | Profit |
|---|---|---|---|
| **Bundle** | 1 @ 126 | 30 | 96 |
| | **2 @ 120** | **60** | **180** |

The optimal solution is to bundle a MP3 and a Walkman at **$120**. This way, the firm will sell 2 bundles and will make a profit of **$180**.

3.

a. If Sloan charges one fixed tuition fee (or "tariff") with a price per hour ("per-unit price") of $0, the normal students (N) will consume 100 course hours and each obtain a consumer surplus of 0.5*400*100 = $20,000. The workaholics (W) will consume 200 course hours and each obtain a consumer surplus of 0.5*400*200 = $40,000. (Note that the price intercept at Q=0 is $400 for both the normal and the workaholic students.)

Sloan basically has two options in charging one fixed fee, either set the tariff = $20,000 and serve both populations of students, or set the tariff = $40,000 and only serve the Ws.

**Profits (T = $20,000)** = 360*20,000 – 100*180*(100+200) – 2,000,000
= **-$200,000**

**Profits (T = $40,000)** = 180*40,000 – 100*180*200 – 2,000,000
**= $1,600,000**

Thus, Sloan will set the tariff (tuition fee) equal to $40,000. Only the 180 W students will choose to enroll.

*Aside:* An even simpler way to solve this problem would have been to observe that total revenues are the same regardless of whether Sloan attracts both types of students or just the workaholics. Since there is some cost associated with providing additional course-hours, then, it is obvious that Sloan will prefer to attract only the workaholics. The solution approach provided above, of course, can also be applied in problems where the answer is not so obvious.

b. In this situation, again, we must consider two cases: 1) only the Ws enroll or 2) both types of students enroll.

**Only WORKAHOLICS**

When only serving one type of customer with a two-part pricing scheme, we know from class that the monopolist maximizes profit by setting per-unit price equal to MC, here **p = <u>$100</u>**, and the tariff so as to extract all consumer surplus at that price. At p=100, workaholics would consume $Q_W$ = 200-0.5(100) = 150 course hours. So the consumer surplus would be .5(400-100)(150) = $22,500 per workaholic student. Thus, Sloan would charge tariff **T = <u>$22,500</u>** and profits are

$$\Pi = 180[22{,}500 + 100(150) – 100(150)] – 2{,}000{,}000$$
$$= 180*22{,}500 – 2{,}000{,}000 = \underline{\mathbf{\$2{,}050{,}000}}$$

*Reality check* (not required): In part b, Sloan has the option of charging p = 0, so profits here should be no less than in part a. Since $2,050,000 > $1,600,000, our solution passes the reality check.

**Both TYPES OF STUDENTS**

When a monopolist serves more than one type of customer, it does not set per-unit price equal to MC in a two-part pricing scheme. However, as we learned in class, the tariff will always be set so as to extract all consumer surplus from the lower type (the N students). In this situation, both types will enroll. Thus,

$$T = 0.5*(400 - p)*(100 - 0.25p)$$

We may therefore express profits solely as a function of per-unit price p:

| | |
|---|---|
| $\Pi = 360*0.5*(400-p)*(100-0.25p)$ | Tariff revenues |
| $+ 180*(p-100)*(100-0.25p)$ | Variable Profit from per-unit sales to N's |
| $+ 180*(p-100)*(200-0.5p)$ | Variable Profit from per-unit sales to W's |
| $– 2{,}000{,}000$ | Fixed Costs |

Maximize profits by taking a derivative with respect to p and setting it equal to zero:

$0 = 360*0.5*(-100 - 100 + 0.5p) + 180*(100+25-.5p) +$
$+ 180*(200+50-p)$
$0 = 175 – p$ (dividing all by 180 and gathering terms)
**p = $\underline{\$175.00}$**

The optimal tariff and Sloan profits are then:

**T =** 0.5*(400-175)*(100-0.25*175) = **$\underline{\$6328.13}$**

**Π** = 360*6,328.13 + 180*(175-100)*(100-.25*175) +
+ 180*(175-100)*(200-.5*175) – 2,000,000
= **$\underline{\$2{,}556{,}250}$**

Sloan makes more profit from attracting both types of students.

c. If Sloan can use a different two-part pricing scheme for each of the two student types, then we are back to the setting in which it can extract all surplus. The profit-maximizing per-unit price, then, equals MC: **p = $\underline{\$100}$ for both types of students.**

The optimal tariffs for W students and N students extract all consumer surplus at this price:

$\mathbf{T_W}$ = 0.5*(400-100)*(200-.5*100) = **$\underline{\$22{,}500}$**
$\mathbf{T_N}$ = 0.5*(400-100)*(100-.25*100) = **$\underline{\$11{,}250}$**

Total Sloan profits therefore amount to:

**Π** = 180*22,500 + 180*11,250 + 0 – 2,000,000 = **$\underline{\$4{,}075{,}000}$**
(Note that per-unit profits are zero since P=MC.)

*Reality check* (not required): In part c, Sloan is able to extract all surplus from both types of students, so profits here should be no less than in part b. Since $4,075,000 > $2,556,250, our solution passes the reality check.

**4. (a)** Given:

| | |
|---|---|
| Demand | $P = (1/6)(340 – 5Q_1 – 5Q_2)$ |
| Total costs | $TC_1 = 30Q_1 + 0.5Q_1^2$ |
| | $TC_2 = 30Q_2 + 0.5Q_2^2$ |

**Approach Overview:**

1. Calculate $MC_1(Q_1)$, $MC_2(Q_2)$, and $MC_{TOT}(Q_1+Q_2)$
2. Calculate $MR(Q_1+Q_2)$
3. Get $Q_{TOT}$ by setting $MC_{TOT}(Q_1+Q_2) = MR(Q_1+Q_2)$
4. Get $Q_1,Q_2$ by setting $MC_{TOT}(Q_{TOT}) = MC_1(Q_1) = MC_2(Q_2)$

**Step 1:** Calculate MC

$MC_1(Q_1) \quad = 30 + Q_1$
$MC_2(Q_2) \quad = 30 + Q_2$

Both plants have the same initial marginal cost ($MC_1(0) = MC_2(0) = 30$), so both plants will be used no matter what total quantity is produced.

To find the total marginal cost curve, $MC_{TOT}(Q_1+Q_2)$ from $MC_1(Q_1)$ and $MC_2(Q_2)$, we follow the basic steps: (a) "Invert" each plant's MC curve.

$Q_1 = -30 + MC_1$
$Q_2 = -30 + MC_2$

(b) "Add" these inverted marginal cost relationships (under presumption that $MC_1 = MC_2 = MC_{TOT}$):

$Q_{TOT} = -60 + 2MC_{TOT}$

(c) "Invert back" to get total marginal cost:

$MC_{TOT} = (1/2)(Q_{TOT} + 60)$

**Step 2:** Calculate MR

$TR = P\,Q_{TOT} = (1/6)(340 – 5\,Q_{TOT})Q_{TOT} = (1/6)(340\,Q_{TOT} – 5Q_{TOT}^2)$
$MR = d\,TR/\,d\,Q_{TOT} = (1/6)(340 – 10\,Q_{TOT})$
(Note: A faster approach would simply "double the slope" in the inverse demand relationship $P = (1/6)(340 – 5\,Q_{TOT})$.)

**Step 3:** $MR = MC_{TOT}$ and solve for $Q_{TOT}$

$(1/6)(340 – 10\,Q_{TOT}) = (1/2)(Q_{TOT} + 60)$

Solving,

$Q_{TOT} \quad = 160/13 =$ **12.31 million dolls**
$P \quad = (1/6)(340 – 5*160/13) =$ **<u>$46.41</u>**

**Step 4:** $MC_1(Q_1) = MC_2(Q_2) = MC_{TOT}(Q_{TOT}) = (1/2)(160/13+60) = 30 + 80/13$. Thus,

$Q_1 = Q_2 = 80/13 =$ **<u>6.15 million dolls</u>** at each plant.

Note: Another approach would have been to express total profits as a function of $Q_1$ and $Q_2$ and then to maximize profits by taking multiple derivatives, etc… This approach is much lengthier and lacking in educational value, so we omit it.

**4. (b)** Given:

$P = (1/6)(340 - 5(Q_1+Q_2)) \rightarrow MR(Q_1+Q_2) = (1/6)(340 - 10(Q_1 +Q_2))$
$TC_1 = 30Q_1 + 0.5Q_1^2 \rightarrow MC_1(Q_1) = 30 + Q_1$
$TC_2 = 10Q_2 + (5/2)Q_2^2 \rightarrow MC_2(Q_2) = 10 + 5Q_2$

**Approach Overview**

1. Since $MC_1(0) > MC_2(0)$, produce only at plant 2 up to $Q^*$, where $MC_1(0) = MC_2(Q^*)$. So, as long as $Q < Q^*$, $MC_{TOT}(Q) = MC_2(Q)$.
2. Compare $MR(Q^*)$ with $MC_1(0)$. If $MR(Q^*) > MC_1(0)$ then firm will produce at both plants. If $MR(Q^*) < MC_1(0)$, then the firm will produce only using plant 2.
3. If the firm uses both plants, follow steps from 4(a) to compute $MC_{TOT}(Q)$ for $Q > Q^*$ as well as the optimal quantities $Q_1$, $Q_2$, $Q_{TOT}$ and price P.

Step 1: $Q^* = 4$ since $30+0 = 10+5*4$.

Step 2: $MR(4) = 50 > 30$, so will produce at both plants.

Step 3: To calculate the firm's marginal cost curve for quantities greater than Q = 4, we need to repeat the "Invert", "Add", and "Invert Back" steps from 4(a):

$MC_1(Q_1) = 30 + Q_1 \rightarrow Q_1 = -30 + MC_1$
$MC_2(Q_2) = 10 + 5Q_2 \rightarrow Q_2 = -2 + (1/5)MC_2$
$\rightarrow Q_{TOT} = -32 + (6/5)MC_{TOT}$
$\rightarrow MC_{TOT}(Q_{TOT}) = (80/3) + (5/6)Q_{TOT}$

Putting this all together,

If $Q_{TOT} < 4$, then $MC_{TOT} = 10 + 5Q_{TOT}$
If $Q_{TOT} > 4$, then $MC_{TOT} = (1/6)(160+5Q_{TOT})$

Solving $MC_{TOT} = MR$, we obtain $Q_{TOT} = 12$

At $Q_{TOT} = 12$, $MC_1 = MC_2 = MC_{TOT} = (1/6)(160+60) = 36\ 2/3$. Thus,

**$Q_1$ = <u>6.67 million dolls</u>**
**$Q_2$ = <u>5.33 million dolls</u>**

Lastly, we find $(1/6)(340 - 5(12))$ = **<u>P = $46.67</u>**

**4. (c)**

We need to compare total profit from parts (a) and (b). (Note that profit numbers may differ somewhat depending on when / how you rounded numbers.)

Part (a) profit $= \text{Revenue} - TC_1(Q_1) - TC_2(Q_2)$
$= (46.41)(12.31) - (30(6.15)+0.5(6.15)^2) - (30(6.15)+0.5(6.15)^2)$

$= \$164,484,600$

Part (b) profit $= \text{Revenue} - TC_1(Q_1) - TC_2(Q_2)$
$= (46.67)(12) - (30(6.66)+0.5(6.66)^2) - (10(5.33)+2.5(5.33)^2)$
$= \$213,333,333$

RPI's profits are higher with scheme (b). Thus, they **should** implement Mr. Warner's plan (given that there is no cost to the reorganization he proposes).