Sloan School of Management
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SOLUTIONS TO HOMEWORK SET #3

1.

a. FALSE

Durable goods are more elastic in the *short run* rather than in the long run (i.e., more *inelastic* in the long run). Goods that are durable include things like automobiles, televisions, refrigerators, or capital equipment purchased by businesses. Relative to annual production, the stock of these goods is relatively large and so even a small price change in the short run can result in a large change in the quantity demanded. For example, in the short run if the price of automobiles increases, consumers can defer purchases of new cars because cars are durable (demand is elastic). In the long run, however, durable goods eventually wear out and will likely be replaced (demand is inelastic). See page 36 of the textbook for graphs of the difference in short-run and long-run elasticities for durable goods.

b. FALSE

The $20 fee is charged for each snowmobile to be produced: it is a variable cost of production and is not a sunk cost.

c. FALSE

$\ln P = 7 + .35 \ln (D) + .3 \ln (B) - .25 \ln (W)$

As stated in the problem, we do not need to use logarithms to solve the problem. Although we could calculate the prices for each configuration using the information provided and compare them, we can instead use the fact that our hedonic price equation is a log-linear format, which allows us to read the elasticities directly off the equation. To see if we should modify the laptop, we can use the percentage changes in the battery life and weight and multiply those by the elasticities to see the effect on price.

First, we know the battery life increases to 3 hours, which is a 50% increase over the current 2 hours. The weight increases to 6 pounds, which is a 20% increase over the current 5 pounds. Therefore, the change in price for the increased battery life is 50%*.3=15% and the change in price for the increased weight is 20%*-.25= -5%. Overall then, we could charge a price that is 15% - 5% = 10%, or $200, higher than before. However the marginal cost of the new battery is $300, so we would be spending $300 to get $200. Therefore, we **should not** sell the laptop with the new battery and the statement is **FALSE**.

d. FALSE

If a firm with seller market power faces a positive network externality, it will set price and quantity in any period taking account of the effect of the marginal unit on future demand and marginal revenue. For instance, in a two period example, the firm will operate in period 1 such that

$$MR_1 = MC_1 - \frac{\partial R_2}{\partial Q_1}$$

where the marginal change in second-period revenue with respect to an additional unit of output in the first period is positive. If the marginal cost is very low (*e.g.*, in the provision of some internet services where it may be near zero), then firms may maximize profit by producing this period in a region where MR_1 is negative.

e. FALSE

The manufacturer will maximize profits by producing at each plant until the point where marginal cost equals marginal revenue, in this case $50. The problem states that at least for the first 100 units, both plants would have MC<MR and, therefore, both plants should be operating. Note that at the optimal level of production, the marginal cost in each plant will be the same (set to equal MR=$50). This is achieved by producing more in Boston than in Pittsburg—not by shutting the Pittsburg plant down.

2.

a. Given: ln UL = 8.0 – 0.9 ln (N) – 0.4 ln (S)

Where:	UL	= labor input per air conditioner in Nth lot
	N	= cumulative lot number
	S	= mean lot size

Our objective is to determine the approximate change in labor input per air conditioner after an increase in lot size by 5%. Given the log-linear specification of the learning curve, we can read off the size elasticity from the equation as –0.4. This means that for a 1% increase in the mean lot size, there is a corresponding 0.4% decrease in the unit labor input per air conditioner. Multiplying by 5, a 5% increase in mean lot size implies approximately a **2% decrease in unit labor input per air conditioner**.

b. The lot size is S = 100 and the lot we are asked about is the first, so the cumulative number of lots is N = 1. In order to calculate how much labor is required to produce the first lot, we just need to plug the values for S and N in the learning curve equation to obtain UL (the labor input per air conditioner), then multiply UL by S to get the total labor required.

We know that	ln UL = 8.0 – 0.9 ln (N=1) – 0.4 ln (S=100) ⇔
Therefore:	ln UL = 8.0 – 0 – 1.84 = 6.16 ⇔
And:	UL = $e^{6.16}$ = 473.43 minutes per air conditioner

Total Labor required to make 100 air conditioners = 100 air conditioners x 473.43 hours per A/C = **47,343 minutes of labor, or equivalently 789 hours.**

c. We now have to compute the labor input required for 1,000 air conditioners produced in 4 equal lots of 250 and the average labor input per air conditioner. In order to do so we need to plug the numbers from the following table into the learning curve equation and calculate two more columns: UL and UL * S.

Cumulative Lot Number, N	**Lot Size, S**	***Ln* UL**	**UL**	**Labor Required per Lot (minutes)**
1	250	5.79	327.01	81,869
2	250	5.17	175.91	43,873
3	250	4.80	121.51	30,459
4	250	4.54	93.69	23,511
Total Labor Input to Produce		1000	A/Cs	**179,711**
Average Labor Input per A/C (over the four lots)				**179.71**

The total of the fourth column represents the total labor required to produce all 1000 air conditioners, while the average labor inputs per A/C are obtained just by dividing this total amount by the total size of production (1000 air conditioners).

d. We now have to determine the overall average labor input per A/C with different configuration of production. In order to solve this problem, all we need to do is recalculate the previous table for the different configurations.

One lot of 1000 units of output (N=1, S=1000).

Cumulative Lot Number, N	Lot Size, S	*Ln* UL	UL	Labor Required per Lot (minutes)
1	1000	5.24	188.67	188,670
Total Labor Input to Produce		1000	A/Cs	**188,670**
Average Labor Input per A/C				**188.67**

Two lots of 500 units of output each (N=1, S=500 and N=2, S=500).

Cumulative Lot Number, N	Lot Size, S	*Ln* UL	UL	Labor Required per Lot (minutes)
1	500	5.51	247.15	123,575
2	500	4.89	132.95	66,475
Total Labor Input to Produce		1000	A/Cs	**190,050**
Average Labor Input per A/C (over the two lots)				**190.05**

Assuming that wages are independent of lot size or number of lots run, then producing 1000 air conditioners in 4 separate lots of 250 has the lowest overall average labor input cost.

3. (a) Given:

MC = \$80
Price= \$100
Quantity=10,000
Fixed cost=\$185,000

Objective:
Find the Net Present Value and take on the project if it is positive.
Approach: sum up the costs and gains to the project being careful to discount future gains or costs.

Discount rate = 0.05:

$$NPV = -\$185{,}000 + \frac{(\$100-\$80)*10{,}000}{(1+0.05)} = -\$185{,}000 + \frac{\$200{,}000}{1.05} = \$5476.19 > 0$$

Discount rate = 0.10:

$$NPV = -\$185{,}000 + \frac{(\$100-\$80)*10{,}000}{(1+0.10)} = -\$185{,}000 + \frac{\$200{,}000}{1.1} = -\ \$3181.81 < 0$$

So, we would take on the project with a discount rate of 0.05 (NPV>0)
but would not with a discount rate of 0.10 (NPV<0)

(b)

Now we have 3 periods of production but a larger start-up cost.

Discount rate = 0.05:

$$NPV = -\$450{,}000 + \frac{\$200{,}000}{1.05} + \frac{\$200{,}000}{1.05^2} + \frac{\$200{,}000}{1.05^3} = \$94{,}649.61 > 0$$

Discount rate = 0.1:

$$NPV = -\$450{,}000 + \frac{\$200{,}000}{1.1} + \frac{\$200{,}000}{1.1^2} + \frac{\$200{,}000}{1.1^3} = \$47{,}370.40 > 0$$

We would produce with discount rates of 0.05 and 0.1.

c)

Objective: Find the expected present value (EPV).

Approach: The expected present value = prob(high price)* NPV at high price
+ prob(low price) * NPV at low price

Notice here that the price is revealed at the beginning of the next period. At the low price, the constant marginal cost of $80 is greater than the price of $70, so the optimal production is zero. At the high price, production will take place resulting in a NPV of $5,476 as found in part a.

So, EPV= 0.9 * $5476.19 + 0.1 * (- $185,000) = - $13,571.43 < 0

With the uncertainty, you would not take on the project (EPV<0).

d)

To calculate the willingness to pay, compare the expected profits with and without the information:

<u>If you buy the information</u>, you will take on the project if you find out the price is high, but you will not if you find out the price is low. Note that the probability that you find out the price is high is 0.9. So, your expected gain is:

EPV= 0.9 * $5476.19 + 0.1*0= 0.9 * 5476.19 = $4928.57

<u>If you do not buy the information</u>, you do not know the price. Part c showed that we do not take on the project in this case.

EPV=0

For the consulting service, you would be willing to pay the difference between the value of the project if you know the information and the value of the project when you do not know the information:

$4928.57 - $0= $4928.57.

4. (a) Given:

$MC_{US} = MC_{CAN}$ = \$25 in '000s per vehicle (call them cars)
$Q_{US} = 18{,}000 - 400\ P_{US}$ --> $P_{US} = 45 - 0.0025\ Q_{US}$
No fixed costs.

Objective:
1. Determine the optimal Q_{US} to produce
2. Determine the price P_{US} to charge
3. Determine profits

Approach: Profit-maximization
1. Specify the profit function Π_{US}
2. Maximize Π_{US} by choosing Q_{US}
3. Use the demand function to calculate P_{US}
4. Plug P_{US} and Q_{US} into Π_{US} to determine profits

Step 1: Specify the profit function Π_{US}

$$\begin{aligned}\Pi_{US} &= P_{US}Q_{US} - MC_{US}Q_{US} \\ &= (45 - 0.0025\ Q_{US})\ Q_{US} - 25\ Q_{US} \\ &= 45\ Q_{US} - 0.0025\ Q_{US}^2 - 25\ Q_{US}\end{aligned}$$

Step 2: Maximize Π_{US} by choosing Q_{US}

$$\text{Max}_{\{Q\}} \qquad 45\ Q_{US} - 0.0025\ Q_{US}^2 - 25\ Q_{US}$$

Taking the first order condition for a maximum,

$$d\Pi_{US} / dQ_{US} = 45 - 0.005\ Q_{US} - 25 = 0 \qquad \text{Equation 1}$$

Note that we can manipulate this to reveal that $MR_{US} = MC$ optimally,

$$45 - 0.005\ Q_{US} = 25$$

Solving, we find that

$Q_{US} = 20 / 0.005 = 4000$ vehicles

Step 3: Use demand to get P_{US}

Substituting the optimal Q_{US} into the demand function,

$$P_{US} = 45 - 0.0025\ Q_{US} = 45 - 0.0025(4000) = \$35$$

The price we should charge in the United States is \$35,000 per vehicle.

Step 4: Plug P_{US} and Q_{US} into Π_{US} to determine profits

$$\Pi_{US} = P_{US}Q_{US} - MC_{US}Q_{US}$$
$$= (\$35)(4000) - (\$25)(4000)$$

$$\Pi_{US} = \$40{,}000{,}000$$

(b) Given:

$Q_{CAN} = 8000 - 100\, P_{CAN}$ --> $P_{CAN} = 80 - 0.01\, Q_{CAN}$
MC_{CAN} = $25 in '000s per vehicle

Objective:

1. Determine the optimal Q_{CAN} to produce
2. Determine the price P_{CAN} to charge
3. Determine profits

Approach: Profit-maximization

1. Specify the profit function Π_{CAN}
2. Maximize Π_{CAN} by choosing Q_{CAN}
3. Use the demand function to calculate P_{CAN}
4. Plug P_{CAN} and Q_{CAN} into Π_{CAN} to determine profits

Step 1: Specify the profit function Π_{CAN}

$$\Pi_{CAN} = P_{CAN}Q_{CAN} - MC_{CAN}Q_{CAN}$$
$$= (80 - 0.01\, Q_{CAN})Q_{CAN} - 25\, Q_{CAN}$$
$$= 80\, Q_{CAN} - 0.01\, Q_{CAN}^2 - 25\, Q_{CAN}$$

Step 2: Maximize Π_{CAN} by choosing Q_{CAN}

$$\text{Max}_{\{Q\}} \quad 80\, Q_{CAN} - 0.01\, Q_{CAN}^2 - 25\, Q_{CAN}$$

Taking the first order condition for a maximum,

$$d\Pi_{CAN} / dQ_{CAN} = 80 - 0.02\, Q_{CAN} - 25 = 0$$

Note that we can manipulate this to reveal that MR_{CAN} = MC optimally,

$$80 - 0.02\, Q_{CAN} = 25$$

Solving, we find that

$$Q_{CAN} = 55 / 0.02 = 2750 \text{ vehicles}$$

Step 3: Use demand to get P_{CAN}

Substituting the optimal Q_{CAN} into the demand function,

$$P_{CAN} = 80 - 0.01\ Q_{CAN} = 80 - 0.01(2750) = \$52.50$$

The price we should charge in Canada is $52,500 per vehicle.

Step 4: Plug P_{CAN} and Q_{CAN} into Π_{CAN} to determine profits

$$\Pi_{CAN} = P_{CAN}Q_{CAN} - MC_{CAN}Q_{CAN}$$
$$= (\$52.50)(2750) - (\$25)(2750)$$
$$\Pi_{CAN} = \$75{,}625{,}000$$

(c) Given:

Separated markets
Produce for both US and Canada, with their distinct demands

Objective:
1. Determine the optimal Q_{US}
2. Determine the optimal Q_{CAN}
3. Determine what total profits will be

Approach: Profit-maximization
1. Develop a total profit function Π_{TOT}
2. Maximize Π_{TOT} by choosing both Q_{CAN} and Q_{US}
3. Determine Π_{TOT} by obtaining P_{CAN} and P_{US}

Step 1: Develop a total profit function Π_{TOT}

$$\Pi_{TOT} = P_{US}Q_{US} + P_{CAN}Q_{CAN} - MC_{US}Q_{US} - MC_{CAN}Q_{CAN}$$

Substitution, as above, yields

$$\Pi_{TOT} = 45Q_{US} - 0.0025Q_{US}^2 + 80Q_{CAN} - 0.01Q_{CAN}^2 - 25Q_{US} - 25Q_{CAN}$$

Step 2: Maximize total profit Π_{TOT} by choosing Q_{US} and Q_{CAN}

$$\text{Max}_{\{Q\}} \quad 45Q_{US} - 0.0025Q_{US}^2 + 80Q_{CAN} - 0.01Q_{CAN}^2 - 25Q_{US} - 25Q_{CAN}$$

Taking the first order conditions for a maximum,

$$\partial\Pi_{TOT} / \partial Q_{US} = 45 - 0.005\ Q_{US} - 25 = 0$$

$$\partial\Pi_{TOT} / \partial Q_{CAN} = 80 - 0.02\ Q_{CAN} - 25 = 0$$

By inspection, we can see that these conditions are the same as in part (A) and part (B), so we can conclude that we will obtain the same levels of production, the same prices in each market and Π_{TOT} will be equal to the sum of $40,000,000 and $75,625,000.

Q_{US}	4000 vehicles
Q_{CAN}	2750 vehicles
P_{US}	$35,000 per vehicle
P_{CAN}	$52,500 per vehicle
Π_{TOT}	$115,625,000

(d) Given:

FC of $50,000,000

Objective: Determine what happens in each of case A, case B and case C

First, we note that none of the marginal conditions are affected. Therefore, we will produce with the already computed prices and quantities, **as long as profits in each case are positive after accounting for the fixed costs.**

Case A: We will not produce because profits ex ante without the fixed costs are less than the fixed costs.

Case B: We will produce with the same quantity (2750 vehicles) and price ($52,500 per vehicle) because profits ex ante without the fixed costs are greater than the fixed costs.

Case C: We will produce with the same quantities (4000 vehicles in the US and 2750 vehicles in Canada) and the same prices ($45,000 per vehicle in the US and $52,500 per vehicle in Canada) because profits ex ante without the fixed costs are greater than the fixed costs.