



**15.010 / 15.011 Economic Analysis For Business Decisions**

**HOMEWORK SET #5**

**(Due Tuesday, November 16, 2004)**

**Name:** \_\_\_\_\_

**Section:** \_\_\_\_\_

**Please staple this sheet to the front of your answer set,  
Do NOT write your answers on this sheet.**



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**(Due Tuesday, November 16, 2004)**

**DIRECTIONS:** Answer all questions and show your work. Your completed homework is due in class or may be placed in the assignment box for your section before 4:30 p.m. *Late homework sets will not be corrected.* Please read the statement on 15.010/15.011 homework policy contained in the course overview.

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1. In your course packet you will find the case “Lesser Antilles Lines – The Island of San Huberto”. This case is useful for reviewing basic concepts of game theory and considering the implications of alternative decisions. Read the case carefully, then answer the following questions. Note: The contribution matrix in Exhibit 4 of the case is denominated in thousands of dollars.
    - a. With reference to the game situation depicted in the matrix at the end of the case:
      - i. What is a Nash Equilibrium? Are there any Nash equilibria in this game?
      - ii. What is a Dominant Strategy? Is there a dominant strategy for LAL? Is there a dominant strategy for KL?
      - iii. What is a maximin strategy? What would the maximin strategy for LAL be? And for KL?
      - iv. What is the ‘best’ outcome for the firms?  
[HINT : To answer this question, think about what LAL and KL would agree on if they could write a binding contract. Consider both the cases that the contract can dictate prices and that the contract can dictate prices as well as a transfer from one firm to the other.]
      - v. If LAL could somehow (irreversibly) commit to a price, would it do so? And if so, to what price would it commit? If LAL could somehow commit that it would always charge the same as KL, would it do so?
      - vi. What would you recommend that LAL do? Why?
    - b. What happens if – due to an increase of world trade following a sudden deregulation wave - the market size suddenly doubles? Would that change the Nash equilibrium described above? [HINT: You do not need to recalculate the payoff matrix.]
  2.
    - a. Suppose the airline industry consisted of only two firms : American and Texas Air Corp. Let the two firms have identical cost functions,  $C(q)=40q$ . Assume the demand curve for the industry is given by  $P=100-Q$  and that each firm expects the other to behave as a Cournot competitor.
      - i. Calculate the Cournot-Nash equilibrium for each firm, assuming that each chooses the output level that maximizes its profits when taking its rival’s output as given. What are the profits of each firm?
      - ii. What would be the equilibrium if Texas Air had constant marginal and average costs of 25 and American had constant marginal and average costs of 40?
      - iii. Suppose that Texas Air’s cost advantage in ii) is based on a patented business practice. What is the minimum price that Texas air is willing to accept to license the patent to American. What is the maximum price that American wants to pay? Discuss.

- b. Determine analytically the Nash equilibrium of the LAL case by going through the following steps :
- Write the equations for gross profits (i.e. not considering fixed costs) for LAL and for KL.
  - Using the first-order condition (i.e. taking the derivative and setting it equal to zero), determine the optimal  $p_{LAL}$  for a given  $p_{KL}$ . Determine analogously the optimal  $p_{KL}$  for a given  $p_{LAL}$ .
3. Magna Company must decide whether to buy all or part of its aluminum from the Steele Corporation. At the same time, Steele must decide whether or not to provide prompt delivery, which is more costly. If Steele Corporation provides prompt delivery of the aluminum it sells Magna, then Magna will make \$2 million if it buys all its aluminum from Steele and \$1 million if it buys only part of its aluminum from Steele. But if Steele provides slow delivery, then Magna will lose \$5 million if it buys all its aluminum from Steele and lose \$1 million if it buys only part from Steele. If it receives an order for all of Magna's aluminum requirements, then Steele will make \$3 million if it provides prompt delivery and \$4 million if it is slow. If it receives an order for part of Magna's aluminum requirements, then Steele will make \$1 million if it provides prompt delivery and \$2 million if it is slow.
- Magna must decide whether to buy all or part of its aluminum from Steele, and Steele must decide whether or not to provide prompt delivery. What is the payoff matrix for this game?
  - Does any player have a dominant strategy? If so, what is it? Explain.
  - Does this interaction have a Nash equilibrium? If so, what is it? Explain.
4. Consider two firms: Chip and Dale. Each of them is considering whether to expand its capacity. The decision depends on the payoffs, which depend on what the other player does. Assume, for sake of simplicity, that each faces three possibilities: Do Nothing (DN), a Small expansion, or a Large expansion. The payoff matrix is given by (Dale's Payoff, Chip's Payoff):

Dale	Chip		
	DN	Small	Large
DN	\$36m, \$36m	\$30m, \$40m	\$18m, \$36m
Small	\$40m, \$30m	\$32m, \$32m	\$16m, \$24m
Large	\$36m, \$18m	\$24m, \$16m	\$0m, \$0m

- Because of the long lead times, assume that the two players' actions are simultaneous. What is the equilibrium strategy in this game? Explain.
- If one player, say Dale, preempts the other by moving first, plot the resulting game tree. What is the equilibrium strategy now? Explain.
- Does the equilibrium strategy change if Chip instead moves first? Explain how.