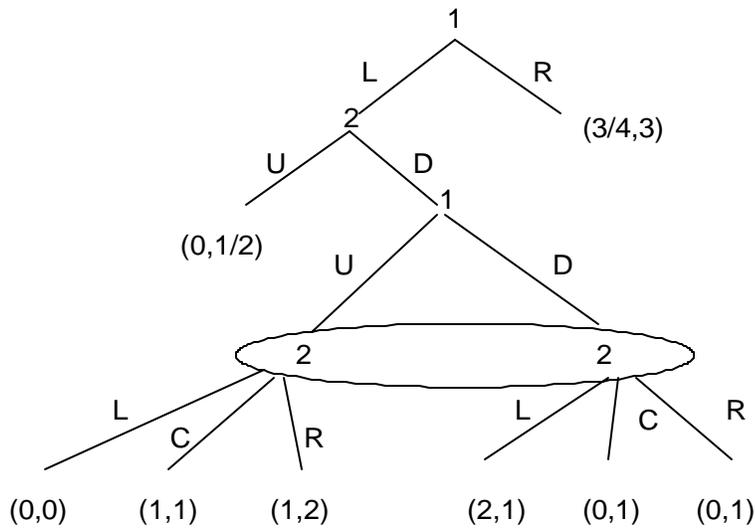


17.881/17.882 Final Exam

1. Consider the game below, and answer the following questions:

- (a) Find all the Nash Equilibria for the subgame starting with player 1's second move.
- (b) Can player 2 play U as part of a subgame perfect Nash Equilibrium? Can player 2 play U as part of a Nash Equilibrium?
- (c) Can player 2 achieve a payoff of 3 as part of a subgame perfect Nash Equilibrium? Can player 2 achieve a payoff of 3 as part of a Nash Equilibrium?
- (d) Find a pure strategy Nash Equilibrium for the whole game that is not subgame perfect.



2. Consider the following simplified version of “poker” with 2 players. Each player i is given a number t_i . The numbers are drawn independently and uniformly from the interval $[0, 1]$. Each player must decide whether to Bid or Fold. If both players Fold then both players receive -1. If one player Bids and the other player Folds then the player that Bids receives +1 and the player that Folds receives -1. If both players Bid then the player with the higher number receives $+k$ and the other player receives $-k$. (Note, the probability of a tie is zero and can be ignored.)

(a) Set $k = 2$. Suppose the players play simultaneously. Find a Bayesian Nash equilibrium. (Hint: look for an equilibrium in which both players use cutoff strategies of the form “Fold if $t_i \leq \alpha_i$, and Bid if $t_i > \alpha_i$.”) Is this equilibrium unique? Be sure to describe the equilibrium *completely* and *rigorously*.

(b) Suppose k increases. What happens to the cutoff levels? In words, what is the intuition for the change?

(c) Again set $k = 2$. What if the players bid sequentially? That is, suppose player 1 must decide whether to Bid or Fold first, and player 2 observes his choice before deciding whether to Bid or Fold. Do the players have the same cutoff levels, or are they different? If different, which player has the higher cutoff level? In words, what is the intuition for this result?

3. MegaByteTechAssociates (MBTA) has a potential investment project. This project would develop an application for MacroStuft a prestigious cwidget manufacturer. The game proceeds in 3 time periods. In period 1, MBTA must make an irreversible decision about how much to invest in development. An investment level of I costs $I^2/2$, and creates a product that can be manufactured at cost cI and has value to MacroStuft of $V_M I$, where $V_M > c$ (the product is a software template so only one unit is needed by MacroStuft). In period 2, MacroStuft observes I , chooses a price $p \geq 0$, and makes a take-it-or-leave-it offer to buy the product at price p . In period 3 MBTA accepts or rejects the offer. If MBTA accepts the offer, then it receives a payoff of $p - cI - I^2/2$ and MacroStuft receives a payoff of $V_M I - p$. If MBTA rejects the offer, then they receive a payoff of $-I^2/2$ and MacroStuft receives a payoff of 0.

(a) Write out an extensive form representation of the game. Find the pure strategy, subgame-perfect Nash equilibrium of the game. What is the optimal investment level (the level that maximizes the total surplus generated by the product)? How does it compare to the equilibrium investment level? Intuitively, why does MBTA choose the level of investment it does, even though the project creates more value than it costs?

Now modify the game in the following way. Suppose there are 5 time periods. The first three are as given above. Now, however, if MBTA refuses the MacroStuft offer, then it can offer the product to another company, Solar. Specifically, if MBTA refuses to sell to MacroStuft, then the game moves to period 4. In period 4, MBTA chooses a price q , and makes a take-it-or-leave-it offer to Solar to buy the product at price q . Solar values the product at $V_S I$, and the cost of production is the same as above. In period 5, Solar either rejects or accepts the offer and the game ends. If Solar rejects, then both it and MacroStuft receive payoffs of 0, and MBTA receives a payoff of $-I^2/2$. If Solar accepts the offer, then it receives a payoff $V_S I - q$, MBTA receives $q - cI - I^2/2$, and MacroStuft receives $-r$ (MacroStuft is a jealous company, and $-r$ represents its distress from seeing a competitor obtain the project).

(b) Find the Pure strategy subgame perfect Nash equilibrium of this game. Does your answer depend on the relative magnitudes of V_M and V_S ? If equilibrium investment by MBTA is different than in part (a), give an intuitive explanation for the difference.

Return to the three-period game of part (a), but now allow MacroStuft to make the following offer. In period 0 (before MBTA makes its investment decision), MacroStuft offers a legally enforceable and unbreakable option contract, that commits it to buy the project for V_M per unit of I at period 3 in return for a fixed payment of F now. MBTA may then accept or reject the option contract, and in either case play proceeds as before. Now of course, if MBTA buys the option the payoffs are different.

(c) Write out an extensive form for this game paying special attention to the payoffs on the branch where MBTA accepts the option contract. Find the pure strategy subgame perfect nash equilibrium in this game. What fee F will be charged for the option? What does this example tell us about the value of enforceable contracts?