Problem Set 4 Solution

17.881/882

October 26, 2004

1 Gibbons 2.1 (p.130)

This is a dynamic game of perfect information, we will use backward induction to solve.

We start at the final stage. The parent's objective is

$$\max_B V(I_p(A) - B) + kU(I_c(A) + B)$$

The first-order condition is:

$$-V'(I_p(A) - B) + kU'(I_c(A) + B) = 0 \quad (1)$$

(I'll omit discussion of the second-order condition).

This equation is defining an implicit relation $B(A)$.

In the first stage, the child anticipates his choice of A to affect B according to 1. The child's problem is

$$\max_A U(I_c(A) + B(A))$$

The first-order condition is:

$$U'(I_c(A) + B)[I_c'(A) + B'(A)] = 0 \quad (2)$$

(I'll omit discussion of the second-order condition).

Since $U' > 0$, the only way for 2 to hold is to have

$$I_c'(A) = -B'(A) \quad (3)$$

To find $B'(A)$, let us use the implicit function theorem on 1.

$$\frac{dB}{dA} = -\frac{-V''I_p'' + kU''I_c'}{V'' + kU''}$$

Using 3, and solving for I_c', we find

$$V''[I_c'(A) + I_p'(A)] = 0$$

Since V is strictly concave, this can only hold if $I_c'(A) + I_p'(A) = 0$, which is exactly the first-order condition of the joint-income maximization problem:

$$\max_A I_c(A) + I_p(A)$$