Problem Set 3 Solution

17.881/882

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1 Morrow 4.11 (pp.107-8)

Note that the ideal point of the median voter is y_n, and that the mid-point between x_1 and x_2 is $(x_1 + x_2)/2$

a) Partition the set of ideal points and call $n_l = |\{i|y_i < (x_1 + x_2)/2\}|, n_r = |\{i|y_i > (x_1 + x_2)/2\}|, n_c = |\{i|y_i = (x_1 + x_2)/2\}|$

Voters at the midpoint vote for candidate 1 with probability 1/2, and for candidate 2 with probability 1/2.

Let v_j be the expected number of votes for party j; $v_1 = n_l + \overline{\frac{n_c}{2}}; v_1 = n_r + \overline{\frac{n_c}{2}}$

Let u_j be the utility of party j. Then $u_1 = v_1 - v_2 = n_l - n_r; u_2 = -u_1 = n_r - n_l$

b) If $i < n$, candidate 1 should choose x_1 such that $x_2 < x_1 < 2y_{i+1} - x_2$

If $i \geq n$ and $x_2 > y_i$, candidate 1 should choose x_1 such that $2y_i - x_2 < x_1 < x_2$

If $i > n$ and $x_2 = y_i$, candidate 1 should choose x_1 such that $2y_{i-1} - x_2 < x_1 < x_2$

If $i = n$ and $x_2 = y_i$, candidate 1 should choose $x_1 = x_2$

c) Both candidates choose the ideal point of the median voter, in which case both get utility of 0. If any candidate chooses a position marginally to the left of the right of the median voter, then that candidate would lose and get negative utility. So we get convergence at the median.