8.851 Course 2006 (Lecture Notes, I. Stewart)

Soft - Collinear Effective Theory (SCET)

For this part we'll switch sign convention for g

 $= i g T^A \gamma^\mu$ to agree with literature

Outline

Class 1: Intro, Degrees of Freedom, Scales

Expansion of Spinors, Propagators,

Power Counting see ②, ③

Class 2: Construction of Currents, Lagrangian

Multipole Expansion, Labels, Grid in detail

see ②, ③, ⑩ (not in notes)

Class 3: SCET_I Lagrangian, Gauge Symmetry, ③, ④, ⑥

Reparameterization Invariance (RPI)

Class 4: More RPI, Ultrasoft - Collinear Fact.

Hard - Collinear Factorization, IR divs,

Matching, Running see ④, ①, ②, ③

Class 5: DIS see ⑧

Soft - Collinear Interactions ④

Class 6: SCET_{II} ④, ⑦, ⑩

Power Counting Formulae ⑤

eg. $\gamma^* \gamma \to \pi^0$ ⑧, eg $B \to D\pi$ ⑨

eg. $B \to X_s \gamma$. Define a Jet ④

(Jets in e^+e^-, see ⑪)

Refs I used

① hep-ph/0005275

② hep-ph/0011336

③ hep-ph/0107001

④ hep-ph/0109045
Gauge Inv.

Power Counting
⑤ hep-ph/0205289

⑥ hep-ph/0204229
RPI

Gauge Inv. at λ^2
⑦ hep-ph/0303156

⑧ hep-ph/0202088
Hard-Scattering

⑨ hep-ph/0107002
$B \to D\pi$

⑩ hep-ph/0605001
0-bin

⑪ hep-ph/0212255
hep-ph/0603066

Intro, Degrees of Freedom, Coordinates

Want an EFT for energetic hadrons, $E_H \simeq Q \gg \Lambda_{QCD}$

Why? • Many processes have large regions of phase space where the hadrons are energetic, $E_H \gg M_H$

eg B-decays $B \to \pi e \nu$, $B \to K^* \gamma$, $B \to \pi\pi$, $B \to X_u e \nu$
$B \to X_s \gamma$, $B \to D^* \pi$, ...

$M_B = 5.279$ GeV $\gg \Lambda_{QCD}$

eg. Hard Scattering

$e^- p \to e^- X$ (DIS), $p\bar{p} \to X \ell^+ \ell^-$ (Drell Yan),
$\gamma^* \gamma \to \pi^0$, $\gamma^* p \to \gamma^{(*)} p'$ (Deeply Virtual Compton Scattering)

• Need to separate perturbative, $\alpha_s(Q)$ & non-perturbative "$\alpha_s(\Lambda_{QCD})$" effects ➡ factorization

What are the low energy degrees of freedom?

eg 1 $B \to D\pi$

in B-rest frame $p_\pi^\mu = (2.310 \text{ GeV}, 0, 0, -2.306 \text{ GeV})$
$\simeq Q\, n^\mu$ to good aprox.

$Q \gg \Lambda$, $n^\mu \equiv (1,0,0,-1)$, $n^2 = 0$ light-like

in 0,1,2,3 basis

Basis vectors $n^\mu, \bar{n}^\mu$

Use Light-Cone coordinates : $n^2=0$, $\bar{n}^2=0$, $n\cdot\bar{n}=2$

vectors $$p^\mu = \frac{n^\mu}{2}\,\bar{n}\cdot p + \frac{\bar{n}^\mu}{2}\,n\cdot p + \underset{\updownarrow\text{ orthogonal } n^\mu,\bar{n}^\mu}{p_\perp^\mu}$$

metric $$g^{\mu\nu} = \frac{n^\mu\bar{n}^\nu}{2} + \frac{\bar{n}^\mu n^\nu}{2} + g_\perp^{\mu\nu}$$

epsilon $$\epsilon_\perp^{\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta}\,\frac{\bar{n}_\alpha n_\beta}{2}$$

Define

$p^+ \equiv n\cdot p$

$p^- \equiv \bar{n}\cdot p$

- since $n^2=0$ we needed to define complementary vector $\bar{n}$
- choice $n^\mu = (1,0,0,-1)$, $\bar{n}^\mu = (1,0,0,1)$ is possible, but other choices also work eg. $n^\mu = (1,0,0,-1)$
$\bar{n}^\mu = (3,2,2,1)$

(more on this later)

In $B\to D\pi$ the B, D are soft $E_H \sim M_H$
& we can use HQET for their constituents
ie quarks & gluons with $p^\mu \sim \Lambda$

But pion is "collinear", $E_H \gg M_H$

In rest frame $\textcircled{\pi}$ has quark & gluon constituents $p^\mu \sim (\overset{+,}{\Lambda}, \overset{-}{\Lambda}, \overset{\perp}{\Lambda})$

boosting for $B\to D\pi$ $\textcircled{\pi}\!\rightarrow$ has constituents $p^\mu \sim \left(\frac{\Lambda^2}{Q}, Q, \Lambda\right)$
$\equiv$ collinear fluctuations around $(0, Q, 0) = p_\pi^\mu$

Note : Boost in direction orthogonal to $\perp$ directions changes p^+, p^- multiplicatively

$p^+ \to a\,p^+$

$p^- \to \frac{1}{a}\,p^-$

Generically

$$(p^+, p^-, p^\perp) \sim Q\,(\lambda^2, 1, \lambda) \quad \text{is collinear}$$

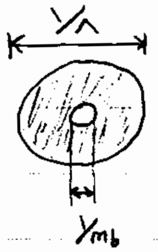
where $\lambda \ll 1$ is small parameter. (above eg. $\lambda = \frac{\Lambda}{Q}$)

What makes this EFT different?

- usually we separate scales $M_1 \gg m_2$ and have

$$\sum_{i=1}^{n} \underset{\substack{\uparrow \\ \text{short distance} \\ \text{Wilson Coeffs}}}{C_i(\mu, m_1)} \;\; \underset{\substack{\uparrow \\ \text{long distance} \\ \text{operators}}}{\mathcal{O}_i(\mu, M_2)}$$

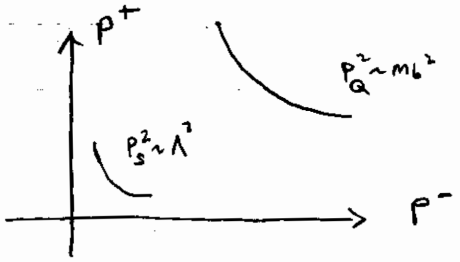
eg in HQET the B-meson



$m_b \gg \Lambda$

$p_Q^\mu \sim m_b$
$p_s^\mu \sim \Lambda$

picture momenta



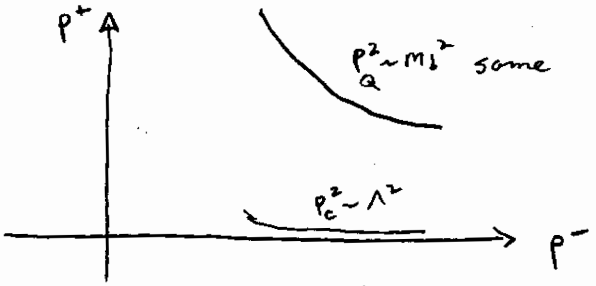
well separated in all components

- now we have overlap between perturbative & non-perturbative momenta in p^- component

for collinear pion

$E_\pi \sim m_b$

$p_c \sim (\frac{\Lambda^2}{m_b}, m_b, \Lambda)$



↑ overlap in p^-, but $p_c^2 \ll p_Q^2$ still

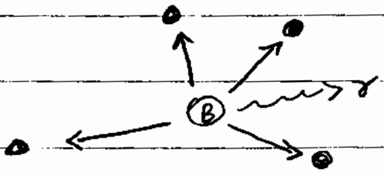
a. inclusive decay $B \to X_s \gamma$ from $b \to s\gamma$

$\uparrow \geq 1$ hadron, summed over

in general $E_\gamma = \dfrac{m_B^2 - m_{X_s}^2}{2m_B} \in \left[0 , \dfrac{m_B^2 - m_{K^*}^2}{2m_B} \right]$

for $m_X \in [m_B , m_{K^*}]$

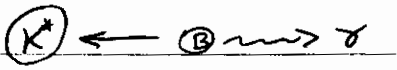
For $m_X^2 \sim m_B^2$



standard OPE just like we did for $B \to X_c e \bar{\nu}$

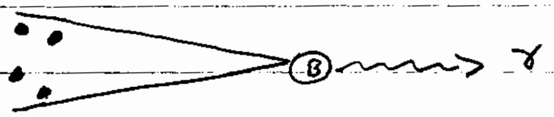
X has hadrons in all directions

For $m_X^2 \sim \Lambda^2$



exclusive decay (not inclusive)

For $m_X^2 \sim m_B \Lambda$



jet of hadrons in X

jet constituents $(p^+, p^-, p_\perp) \sim (\Lambda , Q , \sqrt{\Lambda Q}) \sim Q (\lambda^2, 1, \lambda)$

collinear again

this time $\lambda = \sqrt{\dfrac{\Lambda}{Q}} \ll 1$

Infrared Degrees of Freedom

have $p^2 \lesssim Q^2\lambda^2$

modes	$p^\mu = (+,-,\perp)$	p^2
collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$
ultrasoft (usoft)	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$

Off shell modes have $p^2 \gg Q^2\lambda^2$ and are integrated out into wilson coefficients $C(\mu)$

eg $p^\mu \sim Q(1,1,1)$

Two useful cases

				examples
$SCET_I$	$\lambda = \sqrt{\frac{\Lambda}{Q}}$	collinear usoft	$p_c^2 \sim Q\Lambda$ $p_u^2 \sim \Lambda^2$	$B \to X_s\gamma$, *DIS, ...
$SCET_{II}$	$\lambda = \frac{\Lambda}{Q}$	collinear soft	$p_c^2 \sim \Lambda^2$ $p_s^2 \sim \Lambda^2$	$B \to D\pi$, $\gamma^*\gamma \to \pi^0$, ...

The theory $SCET_{II}$ can be derived from $SCET_I$ so we'll study I first

Factorization : $\sum_i C_i O_i$ becomes continuous

$$\int d\xi \quad C(\xi) \quad O(\xi)$$

since p^- were same size

Collinear Spinors u_n : labelled by direction n

(recall HQET spinors u_v)

massless QCD spinors (Dirac Rep) $u(p) = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ \frac{\vec{\sigma}\cdot\vec{p}}{p^0}\, u \end{pmatrix}$, $v(p) = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\vec{p}\cdot\vec{\sigma}}{p^0}\, v \\ v \end{pmatrix}$

let $n^\mu = (1,0,0,1)$ and expand , $\bar{n}\cdot p = p^0 + p^3 = \frac{Q}{2} + \frac{Q}{2}$

$\bar{n}^\mu = (1,0,0,-1)$

$n\cdot p \ll Q$, $p_\perp \ll Q$

$\frac{\vec{\sigma}\cdot\vec{p}}{p^0} = \sigma^3$

$$u_n = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ \sigma^3 u \end{pmatrix} = \left\{ \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} , \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \right\} \quad \text{particles}$$

$$v_n = \frac{1}{\sqrt{2}}\begin{pmatrix} \sigma^3 v \\ v \end{pmatrix} = \left\{ \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} , \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} \right\} \quad \text{antipartic}$$

$$\not{n} = \begin{pmatrix} \mathbb{1} & -\sigma^3 \\ \sigma^3 & -\mathbb{1} \end{pmatrix} \qquad \not{n}\, u_n = \not{n}\, v_n = 0$$

$$\frac{\not{n}\bar{\not{n}}}{4} = \frac{1}{2}\begin{pmatrix} \mathbb{1} & \sigma^3 \\ \sigma^3 & \mathbb{1} \end{pmatrix} \qquad \frac{\not{n}\bar{\not{n}}}{4}\, u_n = u_n \ , \quad \frac{\not{n}\bar{\not{n}}}{4}\, v_n = v_n$$

$\nearrow$

Projection Operator , $\mathbb{1} = \frac{\not{n}\bar{\not{n}}}{4} + \frac{\bar{\not{n}}\not{n}}{4}$

field $\psi^{QCD} = \xi_n + \xi_{\bar{n}}$

we'll integrate out "small" component $\xi_{\bar{n}}$

<u>Collinear Propagators</u> $p^2+i\epsilon = \bar{n}\cdot p\, n\cdot p + p_\perp^2 + i\epsilon$

$\sim \lambda^0 * \lambda^2 + \lambda * \lambda$ same size

Fermions $$\frac{i\slashed{p}}{p^2+i\epsilon} = \frac{i\slashed{n}}{2}\frac{\bar{n}\cdot p}{p^2+i\epsilon} + \dots$$

$$= \frac{i\slashed{n}}{2}\frac{1}{n\cdot p + \frac{p_\perp^2}{\bar{n}\cdot p} + i\epsilon\,\text{sign}(\bar{n}\cdot p)} + \dots$$

↑ from $T\{\xi_n(x), \bar{\xi}_n(0)\}$

Gluons $\frac{-i g^{\mu\nu}}{p^2+i\epsilon}$ stays same as QCD $g^{\mu\nu}\sim\lambda^0$

(true in any gauge)

↑ (eg Feyn. Gauge)

Power counting for collinear fields

$$\mathcal{L} = \int d^4x \;\; \bar{\xi}_n \;\; \frac{\bar{\slashed{n}}}{2}\left[i n\cdot\partial + \dots\right] \;\; \xi_n$$

$$\lambda^{-4} \quad \lambda^{a} \quad \lambda^{2} \quad \lambda^{a} \quad = \lambda^{2a-2}$$

set $\mathcal{L}\sim\lambda^0$ ie normalize kinetic term so no λ's

then $\boxed{\xi_n \sim \lambda}$

For gluons find $A_n^\mu = (A_n^+, A_n^-, A_n^\perp) \sim (\lambda^2, 1, \lambda)$

just like collinear momenta

ie have $p^\mu + A^\mu = iD^\mu$ homogeneous covariant derivative class 1 ended here.

Currents

eg. QCD $b \to u e \bar\nu$ $J = \bar u \Gamma b$ $\Gamma = \gamma^\mu(1-\gamma_5)$

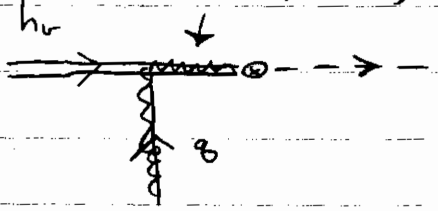
if u energetic match onto SCET (& HQET for b)

$$J^{eff} = \bar\xi_n \Gamma h_v$$

↑ dashed for collinear

for offshell, $k^\mu = m_b v^\mu + \frac{n^\mu}{2}\bar n\cdot q + \cdots$

Consider



$k^2 = m_b^2 + n\cdot v\, m_b\, \bar n\cdot q$

$k^2 - m_b^2 \sim m_b^2$

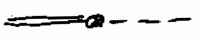
for $\bar n\cdot q \sim m_b$

$\bar n\cdot A_n \sim \lambda^0$ ← no power Suppression for these gluons

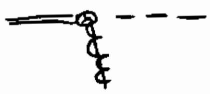
Find $$\bar\xi_n \Gamma \frac{i(\slashed{k}+m_b)}{k^2-m_b^2} i g T^A \gamma^\mu h_v = -g\, \bar\xi_n \Gamma \frac{\left(\cancel{m_b}(1+\slashed{v}) + \cancelto{0}{\frac{\slashed{n}}{2}\bar n\cdot q}\right)\frac{\slashed{n}}{2}\bar n^\mu T^A h_v}{n\cdot v\, \cancel{m_b}\, \bar n\cdot q}.$$

$$= \frac{-g\,\bar n^\mu}{\bar n\cdot q}\, \bar\xi_n \Gamma T^A \frac{\left(\cancelto{0}{-\frac{\slashed{n}}{2}(1-\slashed{v})} + \cancel{n\cdot v}\right)}{\cancel{n\cdot v}} h_v \qquad \slashed{v} h_v = h_v$$

ie



$= \Gamma$



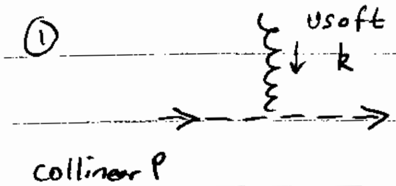
$= \frac{-g\,\bar n^\mu}{\bar n\cdot q}\Gamma T^A$

same order in λ

(add more gluons later)

Which fields can interact in a local way?

①

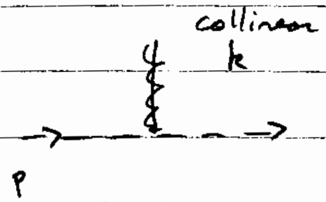


$$p+k = \frac{n^\mu}{2}\,\bar{n}\cdot p + \frac{\bar{n}^\mu}{2}\, n\cdot(p+k) + p_\perp + \ldots$$

still collinear

∴ local

②

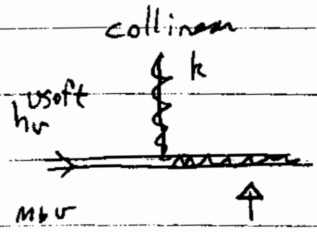


$$p+k = \frac{n^\mu}{2}\,\bar{n}\cdot(p+k) + \frac{\bar{n}^\mu}{2}\, n\cdot(p+k) + p_\perp + k_\perp$$

still collinear

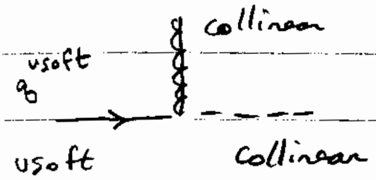
∴ local

③



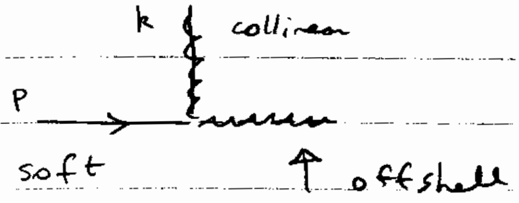
offshell integrate it out (prev. eg.)

④



OK → local

⑤



in $SCET_{II}$

$$p+k = \frac{n^\mu}{2}\,\bar{n}\cdot p + \frac{\bar{n}^\mu}{2}\, n\cdot k + \ldots$$

$$(p+k)^2 = \bar{n}\cdot p\; n\cdot k \sim \underset{\overset{\uparrow}{\text{UV}}}{Q^2\lambda} \gg \underset{\overset{\uparrow}{\text{IR d.o.f.}}}{Q^2\lambda^2}$$

Fields which mediate interactions in $SCET_{II}$ are offshell making it more complicated so we postpone further discussion to after developing $SCET_{I}$

skip to 201

More on Power Counting

Separate Q, $Q\lambda$, $Q\lambda^2$ momenta

Analogy

			label	residual	
b:	HQET	$P^\mu =$	$m_b v^\mu$	$+\ k^\mu$	$h_v(x)$
u:	SCET	$P^\mu =$	p^μ	$+\ k^\mu$	$\xi_{n,p}(x)$
			$\uparrow\ (1,\lambda)$ terms	$\uparrow\ \lambda^2$ terms	

Mode Expn

$$\psi(x) = \int d^4p\ \delta(p^2)\,\theta(p^0)\left[u(p)\,a(p)\,e^{-iP\cdot x} + v(p)\,b^\dagger(p)\,e^{iP\cdot x}\right]$$
$$= \psi^+ + \psi^-$$

expand & Write

$$\psi^+(x) = \sum_p e^{-ip\cdot x}\,\psi^+_{n,p}(x) \qquad \not{n}\,\psi^\pm_{n,p} = 0$$
$$\psi^-(x) = \sum_p e^{ip\cdot x}\,\psi^-_{n,p}(x)$$

$\uparrow$ both have $\theta(\bar{n}\cdot p)$

Now define $\xi_{n,p}(x) \equiv \psi^+_{n,p}(x) + \psi^-_{n,-p}(x)$

$\bar{n}\cdot p > 0$ particles $E = \frac{\bar{n}\cdot p}{2} > 0$

$\bar{n}\cdot p < 0$ antiparticles $E = -\frac{\bar{n}\cdot p}{2} > 0$

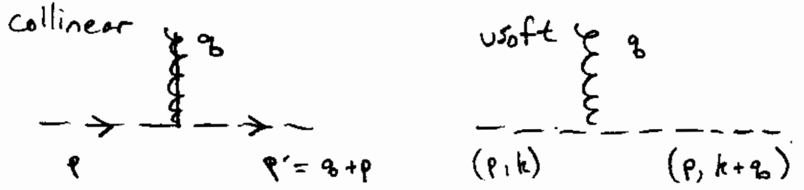
Similiar for Gluons $A^\mu_{n,q}$ destroy

$A^{\mu\,*}_{n,q} = A^\mu_{n,-q}$ create

In HQET label v^μ was conserved by gluons

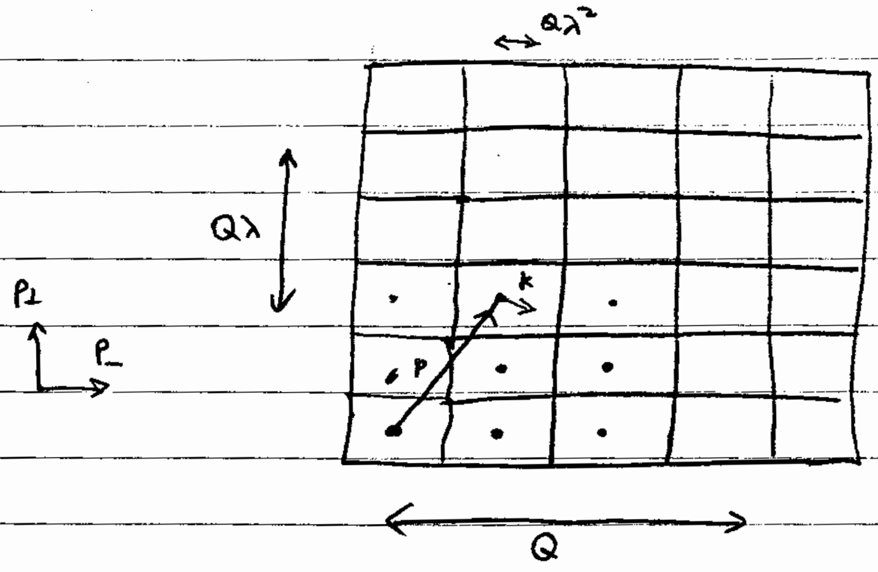
In SCET labels are changed by collinear gluons

″ are conserved by usoft gloons



(label, residual)

Page with SCET Grid

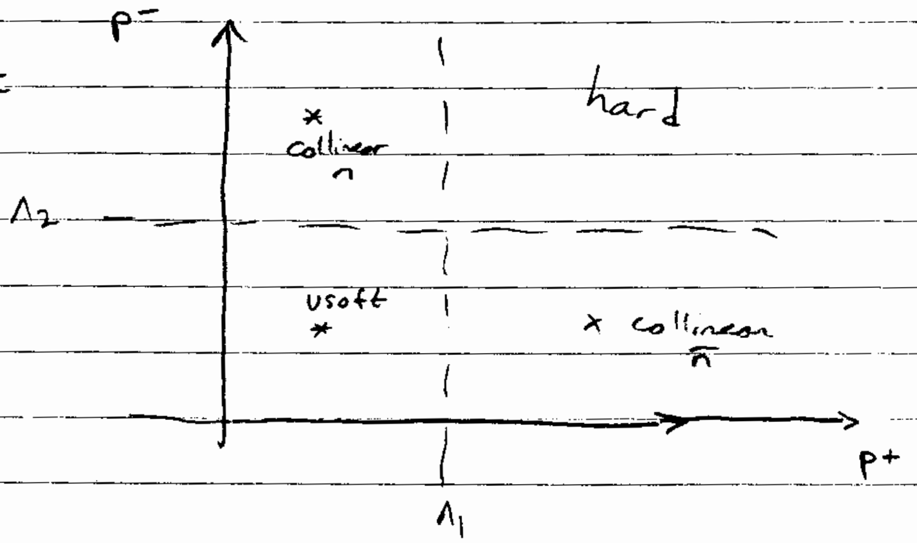


large momentum $P^- = p^- + k^-$

$\uparrow p^- \neq 0$

small momentum $P^- = k^-$, $p^- = 0$, zero-bin

SCET$_I$



See hep-ph/0605001 for information on the details that we discussed in lecture

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Introduce Label Operator for p^μ momenta

$$\mathcal{P}^\mu \left(\phi^\dagger_{q_1} \phi^\dagger_{q_2} \cdots \phi_{p_1} \phi_{p_2} \cdots \right) = \left(p_1^\mu + p_2^\mu + \cdots - q_1^\mu - q_2^\mu \right) \left(\phi^\dagger_{q_1} \cdots \phi_{p_1} \cdots \right)$$

eigenvalue eqtn

"derivative" for labels $\mathcal{P}^\mu$

derivative for residual $i\partial^\mu$

$$i\partial^\mu \sum_p e^{-ip\cdot x} \phi_{n,p}(x) = \sum_p e^{-ip\cdot x} \left(\mathcal{P}^\mu + i\partial^\mu \right) \phi_{n,p}(x)$$

$$= \sum_p e^{-ix\cdot\mathcal{P}} \left(\mathcal{P}^\mu + i\partial^\mu \right) \phi_{n,p}(x)$$

in products of fields this makes labels conserved

residual momentum conserved

5/13/06

Summary

Type	$(p^+, p^-, p^\perp)$	Fields		Field Scaling
collinear	$(\lambda^2, 1, \lambda)$	$\xi_{n,p}(x)$		λ
		$(A^+_{n,p}, A^-_{n,p}, A^\perp_{n,p})$		$(\lambda^2, 1, \lambda)$
soft	$(\lambda, \lambda, \lambda)$	$q_{s,p}$	essentially Fourier transform	$\lambda^{3/2}$
		$A^\mu_{s,p}$		λ
usoft	$(\lambda^2, \lambda^2, \lambda^2)$	q_{us}		λ^3
		A^μ_{us}		λ^2

Last time

$$\begin{aligned} & && \underline{\text{label}} && \underline{\text{residual}} \\ P^- &= && p^- &&+ k^- \\ P_\perp &= && p_\perp &&+ k_\perp \end{aligned}$$

$\xi_{n,p}(x)$

$A^\mu_{n,p}(x)$

label operator $\mathcal{P}^\mu$

$$\mathcal{P}^\mu \, \xi_{n,p} = p^\mu \, \xi_{n,p}$$

$$\mathcal{P}^\mu \, \bar{\xi}_{n,p'} \Gamma \xi_{n,p} = (p^\mu - p'^\mu) \, \bar{\xi}_{n,p'} \Gamma \xi_{n,p}$$

$$i\partial^\mu \sum_{p\neq 0} e^{-ip\cdot x}\, \xi_{n,p}(x) = \underbrace{\underset{\substack{\uparrow\\ \text{labels}\\ \text{conserved}}}{e^{-ix\cdot\mathcal{P}}} \sum_{p\neq 0}}_{\substack{\text{often suppress}\\ \text{this}}} (\mathcal{P}^\mu + i\partial^\mu)\, \underset{\substack{\uparrow\\ \text{residual}\\ \text{momentum}\\ \text{conserved}}}{\xi_{n,p}(x)}$$

Summary

Type	$(p^+, p^-, p^\perp)$	Fields	Field Scaling
collinear	$(\lambda^2, 1, \lambda)$	$\xi_{n,p}(x)$ $(A^+_{n,p}, A^-_{n,p}, A^\perp_{n,p})$	λ $(\lambda^2, 1, \lambda)$
usoft	$(\lambda^2, \lambda^2, \lambda^2)$	$q_{us}(x)$ $A^\mu_{us}(x)$	λ^3 λ^2
soft (later)	$(\lambda, \lambda, \lambda)$	$q_{s,p}$ $A^\mu_{s,p}$	$\lambda^{3/2}$ λ

Collinear Lagrangian

Write $\psi = \xi_n + \xi_{\bar n}$, $\xi_n = P_n \psi$, $\xi_{\bar n} = P_{\bar n}\psi$

$P_n = \frac{\not n \not{\bar n}}{4}$ $\qquad P_{\bar n} = \frac{\not{\bar n} \not n}{4}$

$$\mathcal{L} = \bar\psi\, i\not D\, \psi = (\bar\xi_{\bar n} + \bar\xi_n)\left(i\frac{\not n}{2}\,\bar n\cdot D + i\frac{\not{\bar n}}{2}\, n\cdot D + i\not D_\perp \right)(\xi_n + \xi_{\bar n})$$

$$= \bar\xi_n \frac{\not{\bar n}}{2}\, i n\cdot D\, \xi_n + \underbrace{\bar\xi_{\bar n}\frac{\not n}{2}\, i\bar n\cdot D\, \xi_{\bar n}}_{} + \bar\xi_n\, i\not D_\perp\, \xi_{\bar n} + \underbrace{\bar\xi_{\bar n}\, i\not D_\perp\, \xi_n}_{}$$

So far we've done nothing, just written QCD in diff. vars.
Only ξ_n components are big, so lets take only external ξ_n's [do not couple current to $\xi_{\bar n}$ in path int.]

Integrate out $\xi_{\bar n}$

$\delta / \delta\bar\xi_{\bar n}$:

$$\frac{\not n}{2}\, i\bar n\cdot D\, \xi_{\bar n} + i\not D_\perp\, \xi_n = 0$$

$$i\bar n\cdot D\, \xi_{\bar n} + \frac{\not{\bar n}}{2}\, i\not D_\perp\, \xi_n = 0$$

$$\xi_{\bar n} = \frac{1}{i\bar n\cdot D}\, i\not D_\perp \frac{\not{\bar n}}{2}\, \xi_n$$

Think of $\frac{1}{i\bar n\cdot\partial} f(x) = \int đ^4p\, \frac{e^{-ip\cdot x}}{\bar n\cdot p} f(p)$ for inv. deriv.

Now

$$\mathcal{L} = \bar\xi_n \left(i n\cdot D + i\not D_\perp \frac{1}{i\bar n\cdot D}\, i\not D_\perp \right)\frac{\not{\bar n}}{2}\, \xi_n$$

Next: introduce collinear & usoft gluon fields & phases $e^{-ip\cdot x}$

- recall A^μ_{us} has $p^2 \sim Q^2\lambda^4 << p_c^2 \sim Q^2\lambda^2$
  ie long wavelength, its like a classical background field as far as A_n^μ & ξ_n are concerned

write $A^\mu = A_n^\mu + A^\mu_{us}$ $\qquad$ [not quite right, but suffices here]

- Phase Redefinition $\quad i\partial^\mu \to P^\mu + i\partial^\mu$
  get $e^{-ix\cdot P}$ out front irrespective of number of fields we have ($\frac{1}{i\bar{n}\cdot D}$ means we have Feyn rules with 0, 1, 2, 3, -- gluons)

$$\left.\begin{aligned} \xi_n &= \xi_{n,p} \\ in\cdot D &= \underset{\lambda^2}{in\cdot\partial} + \underset{\lambda^2}{g\, n\cdot A_{n,q}} + \underset{\lambda^2}{g\, n\cdot A_{us}} \end{aligned}\right\} \text{suppress } \sum_p,\ \sum_q$$

$$iD_\perp = \underbrace{(P_\perp + gA^\perp_{n,q})}_{iD_\perp^c \sim \lambda} + \underbrace{(i\partial^\perp + gA^\perp_{us})}_{\lambda^2 \text{ drop it}}$$

$$i\bar{n}\cdot D = \underbrace{(\bar{P} + g\bar{n}\cdot A_{n,q})}_{i\bar{n}\cdot D^c \sim \lambda^0} + \underbrace{(i\bar{n}\cdot\partial + g\bar{n}\cdot A_{us})}_{\lambda^2 \quad \text{drop it}}$$

Leading Order Action is $\mathcal{O}(\lambda^4)$ $\quad$ [$\ast$ λ^{-4} from measure]

$$\mathcal{L}^{(0)}_{qq} = \underset{\uparrow}{e^{-ix\cdot P}}\ \bar{\xi}_{n,p'} \left[in\cdot D + i\not{D}_\perp^c \frac{1}{i\bar{n}\cdot D_c} i\not{D}_\perp^c \right] \frac{\not{\bar{n}}}{2}\, \xi_{n,p}$$

- drop this if we remember to impose label conservation
- all fields are at x, derivatives $i\partial^\mu \sim \lambda^2$
  - action explicitly local at $Q\lambda^2$ scale
  - action local at $Q\lambda$ too ($D_\perp$ in numerator, mom. space version of locality)
  - only non-local at $\sim Q$ scale
- terms are same size in power counting

Repeat for Gluons

$$\mathcal{L} = -\tfrac{1}{4} G^A_{\mu\nu} G^{\mu\nu A} = -\tfrac{1}{2}\, \mathrm{tr}\left[G_{\mu\nu}G^{\mu\nu}\right], \quad G^{\mu\nu} = \frac{i}{g}\left[D^\mu, D^\nu\right]$$

...

$$\mathcal{L}_{cg}^{(0)} = \frac{1}{2g^2}\, \mathrm{tr}\left\{\left(\left[i\hat{\mathcal{D}}^{\mu} + gA^{\mu}_{n,q},\ i\hat{\mathcal{D}}^{\nu} + gA^{\nu}_{n,q'}\right]\right)^2\right\} \quad + \text{ gauge fixing}$$

$$i\hat{\mathcal{D}}^{\mu} = \frac{i\bar{n}^{\mu}}{2}\, n\cdot D + \mathcal{P}_{\perp}^{\mu} + \frac{n^{\mu}}{2}\,\bar{\mathcal{P}}$$

$\uparrow$ see hep-ph/0109045

- terms dropped in constructing $\mathcal{L}_{\xi\xi}^{(0)}$, $\mathcal{L}_{cg}^{(0)}$ give $\mathcal{L}_{\xi\xi}^{(1)}$, $\mathcal{L}_{cg}^{(1)}$, ...

Arguement so far was <u>tree level</u>. To go further we need symmetries (& power counting)

① Gauge Symmetry ⎤
② Reparameterization Invariance ⎦ v. Useful
③ Spin Symmetry ?

ω: Easiest in two-component form (rather than 4-components ξ_n with $\frac{\not{n}\not{\bar{n}}}{4}\xi_n = \xi_n$)

$$\xi_n = \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_n \\ \sigma^3\varphi_n\end{pmatrix}$$

$$\mathcal{L} = \varphi^{\dagger}_{n,p'}\left\{ i n\cdot D + iD_{\perp}^{c\mu}\frac{1}{i\bar{n}\cdot D_c}\, iD_{\perp}^{c\nu}\left(g^{\perp}_{\mu\nu} + i\epsilon^{\perp}_{\mu\nu}\sigma_3\right)\right\}\varphi_{n,p}$$

not SU(2)

just U(1): helicity $h = \frac{i\epsilon_{\perp}^{\mu\nu}}{4}[\gamma_{\mu}, \gamma_{\nu}]$ generator

$h \sim \sigma_3$, spin along direction of motion

Broken by masses

Broken by non-pert effects

Useful in pert. theory

① Gauge Symmetry $\qquad U(x) = \exp\left[i \alpha^A(x) T^A \right]$

Need to consider U's which leave us within EFT

eg. $i\partial^\mu \alpha^A \sim Q \alpha^A$ then $\xi_n' = U(x)\, \xi_n$ would no longer have $p^2 \lesssim Q^2\lambda^2$

collinear $U(x)$ $\qquad i\partial^\mu U_c(x) \sim Q(\lambda^2, 1, \lambda)\, U_c(x) \leftrightarrow A^\mu_{n,q}$

usoft $U(x)$ $\qquad i\partial^\mu U_u(x) \sim Q(\lambda^2, \lambda^2, \lambda^2)\, U_u(x) \leftrightarrow A^\mu_{us}$

- two classes of gauge trasfm for two gauge fields

- in momentum space we have convolutions for U_c

$$\xi_{n,p} \to \sum_q (U_c)_{p-q}\, \xi_{n,q}$$

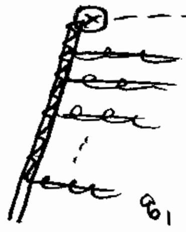
we'll write shorthand $\quad \xi_n \to U_c\, \xi_n$

Now $\quad q_{us} \overset{U_c}{\rightarrow} q_{us}$ $\quad$ since otherwise we give large mom. to an usoft field

<u>Aside</u> recall our heavy-to-light current

$\bar\xi_n \Gamma h_v^{us} \to \bar\xi_n U_c^\dagger \Gamma h_v^{us}$ is <u>not</u> gauge invariant

BuT we had to integrate out offshell propagators



+ perms of assigning $q_1, -, q_m$ ie. crossed graphs

$$= \Gamma \sum_{m=0}^{\infty} \sum_{\text{perms}} \frac{(-g)^m\, \bar n \cdot \epsilon^{a_1}_{n,q_1} \cdots \bar n \cdot \epsilon^{a_m}_{n,q_m}}{\bar n \cdot q_1\, \bar n \cdot (q_1+q_2) \cdots \bar n \cdot (\Sigma q_i)} * T^{a_m} \cdots T^{a_1}$$

$$= \Gamma\, W$$

$\bar n \cdot A^{a_i}_{n,q_i} \to \bar n \cdot \epsilon^{a_i}_{n,q_i}$

we had first term previously $-\frac{g\,\bar n^\mu}{\bar n\cdot q}\,\Gamma T^a$

Here W is a Wilson Line

Short form $W = \Big[\sum_{\text{perms}} \exp\Big(\frac{-g}{\bar{\mathcal P}}\,\bar n\cdot A_{n,q}(x)\Big)\Big]$

If we set residual coordinat $x=0$ then Fourier transform $W = W(y,-\infty) = P\exp\Big(ig\int_{-\infty}^{y} ds\,\bar n\cdot A(s\bar n)\Big)$

ie like $\bar\xi_n(y)\; W(y,-\infty)\; h_v(-\infty)$

$\uparrow$ short dist. y $\uparrow$ usoft field at "long" dist. & doesn't see short dist. interactions

Now $W \to U_c W$ & $\bar\xi_n W \Gamma h_v$ is invariant

End Aside

Gauge Transformations

		U_c	U_{us}	U_{global}
collinear	$\xi_{n,p}$	$U_c\,\xi_{n,p}$	$U_{us}\,\xi_{n,p}$	easy
	$A_{n,p}$	$U_c A_{n,p} U_c^\dagger + \frac{i}{g} U_c[i\hat{\mathcal D}^\mu, U_c^\dagger]$	$U_{us} A_{n,p} U_{us}^\dagger$	---
	W	$U_c W$	$U_{us} W U_{us}^\dagger$	--
usoft	q_{us}	q_{us}	$U_{us}\, q_{us}$	--
	A_{us}	A_{us}	$U_{us}\big(A_{us}^\mu + \frac{i\partial^\mu}{g}\big)U_{us}^\dagger$	---
	Y	Y	$U_{us}\, Y$	--

- homogeneous in λ, recall $i\hat{\mathcal D}^\mu$ has $in\cdot D$ in it
  $U_{us} A_{n,p} U_{us}^\dagger$ is like background field trnsfm of quantum field $A_{n,p}$

Gauge Symmetry ties together

$$i n\cdot D = i n\cdot\partial + g n\cdot A_n + g n\cdot A_{us}$$
$$i D_\perp^c$$
$$i \bar{n}\cdot D^c$$

Mass Dimension & P.C. means either $i n\cdot D \sim \lambda^2$

or $\frac{1}{\bar{P}} (i D_\perp)^2 \sim \lambda^2$ (no other λ^2 ops)

What about coeff. between $i n\cdot D$ & $i \not{D}_\perp \frac{1}{i\bar{n}\cdot D} i\not{D}_\perp$?

What about other operators like

$$\bar{\xi}_n \; i D_{\perp c}^\mu \frac{1}{i\bar{n}\cdot D} i D_{\perp,c}^\mu \frac{\not{\bar{n}}}{2} \xi_n \qquad ?$$

(vi) <u>Reparameterization Invariance (RPI)</u>

$n, \bar{n}$ break Lorentz Inv. $n^\mu M_{\mu\nu}$, $\bar{n}^\mu M_{\mu\nu}$

(only $\epsilon_\perp^{\mu\nu} M_{\mu\nu}$ preserved)

rotations about 3-axis

3 types of RPI which keep $n^2 = \bar{n}^2 = 0$, $n\cdot\bar{n} = 2$

I $\quad n \to n + \Delta_\perp$	II $\quad n \to n$	III $\quad n \to e^\alpha n$
$\bar{n} \to \bar{n}$	$\bar{n} \to \bar{n} + \epsilon_\perp$	$\bar{n} \to e^{-\alpha} \bar{n}$

type III is simple ! implies for any operator with an n^μ

we have corresponding n in denominator

or a corresponding $\bar{n}$ in numerator

eg. $\mathcal{L}_{qq}^{(0)}$ had $\not{\bar{n}} \frac{1}{i\bar{n}\cdot D_c}$ ✓ , $\not{\bar{n}}\, n\cdot D$ ✓

can't have $\not{\bar{n}}\, \bar{n}\cdot D$

Power Counting $\Delta_\perp \sim \lambda$
$\epsilon_\perp \sim \lambda^0$, $\alpha \sim \lambda^0$ } max power that leaves scaling of collinear momenta intact

ie we only care about restoring Lorentz Inv. for the set of fluctuations described by SCET

stopped here ↓

Find

under I

$$n\cdot D \to n\cdot D + \Delta^\perp\cdot D^\perp$$
$$D^\perp_\mu \to D^\perp_\mu - \frac{\Delta^\perp_\mu}{2}\,\bar n\cdot D - \frac{\bar n_\mu}{2}\,\Delta^\perp\cdot D^\perp$$
$$\bar n\cdot D \to \bar n\cdot D$$
$$\xi_n \to \left(1 + \frac{\not\Delta_\perp \not{\bar n}}{4}\right)\xi_n$$
$$W \to W$$

under II

$$n\cdot D \to n\cdot D$$
$$D^\perp_\mu \to D^\perp_\mu - \frac{\epsilon^\perp_\mu}{2}\,n\cdot D - \frac{n_\mu}{2}\,\epsilon^\perp\cdot D^\perp$$
$$\bar n\cdot D \to \bar n\cdot D + \epsilon^\perp\cdot D^\perp$$
$$\xi_n \to \left(1 + \frac{\not\epsilon_\perp}{2}\,\frac{1}{i\bar n\cdot D}\,i\not D_\perp\right)\xi_n$$
$$W \to \left[\left(1 - \frac{1}{i\bar n\cdot D}\,i\epsilon^\perp\cdot D_\perp\right)W\right]$$

$$V^\mu = \frac{n\cdot V}{2}\,\bar n^\mu + \frac{\bar n\cdot V}{2}\,n^\mu + V_\perp^\mu$$ invariant under I, II, III

Last Time

RPI

$$p^\mu = \frac{n^\mu}{2}\,\bar{n}\cdot(p+k) + \frac{\bar{n}^\mu}{2}\,n\cdot k + (p_\perp^\mu + k_\perp^\mu)$$

- Any choice of basis vectors , $n^2 = 0 = \bar{n}^2$, $n\cdot\bar{n} = 2$ equally good

I $\quad n \to n + \Delta_\perp$ $\qquad$ II $\quad n \to n$ $\qquad$ III $\quad n \to e^{\alpha} n$

$\quad\;\; \bar{n} \to \bar{n}$ $\qquad\qquad\quad\;\; \bar{n} \to \bar{n} + \epsilon_\perp$ $\qquad \bar{n} \to e^{-\alpha}\bar{n}$

- Freedom in the component decomposition

$\bar{n}\cdot(p+k)$, $\quad p_\perp^\mu + k_\perp^\mu$

$\mathcal{P}_\mu \to \mathcal{P}_\mu + \beta_\mu$, $\quad i\partial_\mu \to i\partial_\mu - \beta_\mu$ $\qquad n\cdot\beta = 0$

$\xi_{n,p}(x) \to e^{i\beta\cdot x}\,\xi_{n,p+\beta}(x)$

Connects : $\quad \mathcal{P}^\mu + i\partial^\mu$

Gauge this

$$\left.\begin{array}{l} iD_\perp^{c\mu} + W\, iD_\perp^{us\,\mu}\, W^\dagger \\ i\bar{n}\cdot D^c + W\, i\bar{n}\cdot D_{us}\, W^\dagger \end{array}\right\} \text{nice properties under gauge symmetry}$$

Modifies earlier attempt : - due to W's this is not $A_n^\mu + A_{us}^\mu$

- doesn't effect $n\cdot D$ in LO $\mathcal{L}$.

I, II, III leave $\quad V^\mu = \frac{n^\mu}{2}\,\bar{n}\cdot V + \frac{\bar{n}^\mu}{2}\,n\cdot V + V_\perp^\mu \quad$ invariant

III last time

Under I

$$n\cdot D \to n\cdot D + \Delta_\perp\cdot D_\perp$$
$$D^\perp_\mu \to D^\perp_\mu - \frac{\Delta^\perp_\mu}{2}\,\bar n\cdot D - \frac{\bar n_\mu}{2}\,\Delta^\perp\cdot D^\perp$$
$$\bar n\cdot D \to \bar n\cdot D$$
$$\xi_n \to \left(1 + \frac{\not\Delta_\perp \bar{\not n}}{4}\right)\xi_n$$
$$W \to W$$

Under II

$$n\cdot D \to n\cdot D$$
$$D^\perp_\mu \to D^\perp_\mu - \frac{\epsilon^\perp_\mu}{2}\,n\cdot D - \frac{n_\mu}{2}\,\epsilon^\perp\cdot D^\perp$$
$$\bar n\cdot D \to \bar n\cdot D + \epsilon_\perp\cdot D_\perp$$
$$\xi_n \to \left(1 + \frac{\not\epsilon_\perp}{2}\,\frac{1}{i\bar n\cdot D}\,i\not D_\perp\right)\xi_n$$
$$W \to \left[\left(1 - \frac{1}{i\bar n\cdot D}\,i\epsilon^\perp\cdot D_\perp\right)W\right]$$

Power Counting: max power that leaves scaling for collin momentum

$$\epsilon_\perp \sim \lambda^0 \quad , \quad \alpha \sim \lambda^0$$
$$\Delta_\perp \sim \lambda \qquad [\text{else } n\cdot D \not\sim \lambda^2]$$

eg.

$$\delta^{(I)}\left(\bar\xi_n\, i\not D^c_\perp \frac{1}{i\bar n\cdot D}\, i\not D^c_\perp \frac{\bar{\not n}}{2}\xi_n\right) = -\bar\xi_n\, i\Delta^\perp\cdot D^\perp \frac{\bar{\not n}}{2}\xi_n$$
$$\delta^{(I)}\left(\bar\xi_n\, i n\cdot D \frac{\bar{\not n}}{2}\xi_n\right) = \underbrace{\bar\xi_n\, i\Delta^\perp\cdot D^\perp \frac{\bar{\not n}}{2}\xi_n}_{\text{connected}}$$

type-II rules out $\bar\xi_n D_\perp^\mu \frac{1}{i\bar n\cdot D} D_\perp^\mu \frac{\bar{\not n}}{2}\xi_n$ operator in $\mathcal{L}^{(0)}_{qq}$

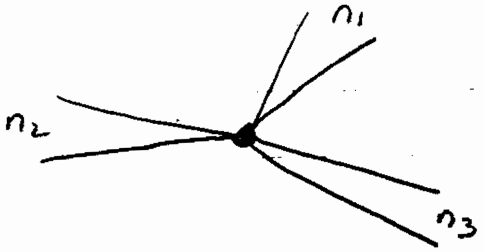
So

$$\mathcal{L}^{(0)}_{qq} = \bar\xi_n\left[i n\cdot D + i\not D^c_\perp \frac{1}{i\bar n\cdot D^c}\, i\not D^c_\perp\right]\frac{\bar{\not n}}{2}\xi_n$$

Unique by p.c., gauge inv, & RPI

More collinear fields: for >1 energetic hadron
or >1 " jet
need more n's (& $\bar{n}$'s)

Generalize to $\sum_n \mathcal{L}^{(0)}_{\xi\xi}$



For $n_1, n_2, n_3, \ldots$ the modes are distinct only if
$n_i \cdot n_j \gg \lambda^2$ $\quad i \neq j$

eg. $P_2 = Q n_2$
$n_1 \cdot P_2 = Q\, n \cdot n_2 \sim Q\lambda^2$ then P_2 is n_1-collinear

Discrete Symmetries $\quad n = (1,0,0,1)$, $\bar{n} = (1,0,0,-1)$

$$C^{-1}\, \xi_{n,p}\, C = -\left[\,\bar{\xi}_{n,-p}\, \mathcal{C}\,\right]^T$$
$$P^{-1}\, \xi_{n,p}(x)\, P = \gamma_0\, \xi_{\bar{n},\tilde{p}}\,(x_P)$$
$$T^{-1}\, \xi_{n,p}(x)\, T = T\, \xi_{\bar{n},\tilde{p}}\,(x_T)$$

$p = (p^+, p^-, p^\perp)$
$\tilde{p} = (p^-, p^+, -p^\perp)$
$x_P = (x^-, x^+, -x_\perp)$
$x_T = (-x^-, -x^+, x^\perp)$

Study $\mathcal{L}_{\xi\xi}^{(0)}$

① Propagator

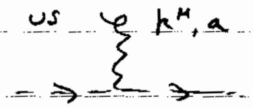
$$i\frac{\bar{n}\!\!\!/}{2}\,\frac{\theta(\bar n\cdot p)}{n\cdot p+\frac{p_\perp^2}{\bar n\cdot p}+i\epsilon} + i\frac{\bar{n}\!\!\!/}{2}\,\frac{\theta(-\bar n\cdot p)}{+n\cdot p+\frac{p_\perp^2}{\bar n\cdot p}-i\epsilon} = i\frac{\bar{n}\!\!\!/}{2}\,\frac{\bar n\cdot p}{n\cdot p\,\bar n\cdot p+p_\perp^2+i\epsilon}$$

✓ expn. of QCD

particles $\bar n\cdot p>0$ anti $\bar n\cdot p<0$

② Interactions

- only $n\cdot A_{us}$ gluons at LO



$$= i g T^a n^\mu \frac{\bar{n}\!\!\!/}{2}$$

& only sees $n\cdot k$ usoft momentum (multipole expn.)

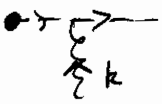
$$\frac{\bar n\cdot p}{\bar n\cdot p\; n\cdot(p+k)+p_\perp^2+i\epsilon} = \frac{\bar n\cdot p}{\bar n\cdot p\; n\cdot k+p^2+i\epsilon} \underset{\text{on-shell}}{=} \frac{\bar n\cdot p}{\bar n\cdot p\; n\cdot k+i\epsilon}$$

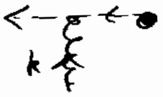
(Compare Collinear Gluon

$$\frac{\bar n\cdot(p+q)}{(p+q)^2+i\epsilon}$$)

Propagator reduces to eikonal approx when appropriate

$\bar n\cdot p>0$ $\bar n\cdot p<0$

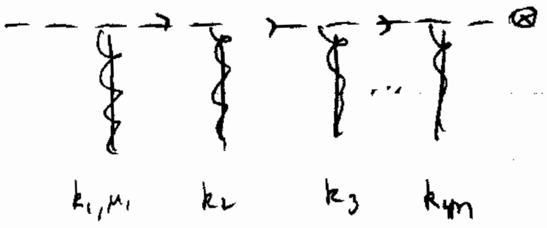




$$\frac{n^\mu}{n\cdot k+i\epsilon} \qquad \frac{n^\mu}{-n\cdot k+i\epsilon} \qquad \frac{n^\mu}{-n\cdot k-i\epsilon} \qquad \frac{n^\mu}{n\cdot k-i\epsilon}$$

Usoft - Collinear Factorization

Consider



$$= \Gamma \sum_{m} \sum_{\text{perms}} \frac{(-g)^m\, n\cdot A^{a_1} \cdots n\cdot A^{a_m}\, T^{a_m} \cdots T^{a_1}}{n\cdot k_1\; n\cdot(k_1+k_2) \cdots n\cdot\left(\Sigma k_i\right)} \times u_n$$

on-shell so $\frac{1}{n\cdot k + \frac{p^2}{\bar n \cdot p}} \to \frac{1}{n\cdot k}$

Motivates us to consider a field redefinition

$$\xi_{n,p}(x) = Y(x)\, \xi_{n,p}^{(0)}(x) \qquad A_{n,p} = Y A_{n,p}^{(0)} Y^{+}$$

↑ adjoint version

$$Y(x) = P \exp\left(ig \int_{-\infty}^{0} ds\; n\cdot A_{us}^{a}(x+ns)\, T^{a} \right)$$

$$n\cdot D\, Y = 0 \quad , \quad Y^{+} Y = 1 \qquad \text{find} \quad W = Y\, W^{(0)}\, Y^{+}$$

$$\begin{aligned}
\mathcal{L}_{\xi\xi}^{(0)} &= \bar\xi_{n,p'} \frac{\bar{n}\!\!\!/}{2} \left[in\cdot D + \cdots \right] \xi_{n,p} \\
&= \bar\xi_{n,p'}^{(0)} \frac{\bar{n}\!\!\!/}{2} \left[Y^{+} in\cdot D_{us} Y + Y^{+} (Y g \bar n \cdot A_n Y^{+}) Y + \cdots \right] \xi_{n,p} \\
&= \bar\xi_{n,p'}^{(0)} \frac{\bar{n}\!\!\!/}{2} \Big[\underbrace{in\cdot\partial + g\bar n\cdot A_n}_{in\cdot D_c} + \cdots \Big] \xi_{n,p}
\end{aligned}$$

↑ all $n\cdot A_{us}$'s disappear!

True for gluon action too

$$\mathcal{L}\left(\xi_{n,p},\, A_{n,q}^{\mu},\, n\cdot A_{us} \right) = \mathcal{L}\left(\xi_{n,p}^{(0)},\, A_{n,q}^{(0)},\, 0 \right)$$

Interactions don't disappear, but are moved out of L.O. $\mathcal{L}$ and into currents

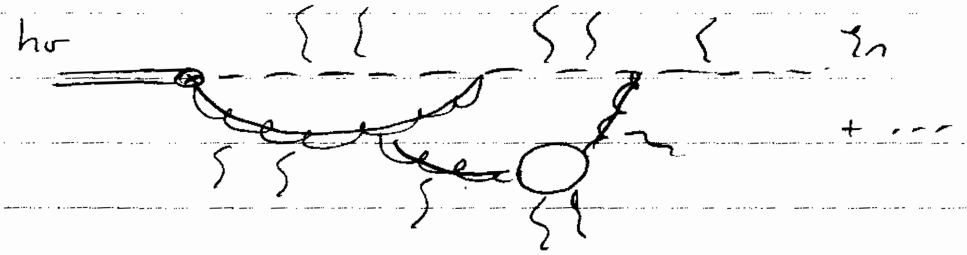
eg 1 $$J = \bar{\xi} W \Gamma h_v = \bar{\xi}_n^{(0)} Y^+ Y W^{(0)} Y^+ \Gamma h_v$$
$$= (\bar{\xi}_n^{(0)} W^{(0)}) \Gamma (Y^+ h_v)$$

If our current was a collinear color singlet

eg 2 $$J = (\bar{\xi}_n W) \Gamma (W^+ \xi_n) = \bar{\xi}_n^{(0)} W^{(0)} \cancel{Y}^+ \cancel{Y} \Gamma (W^{+(0)} \xi_n^{(0)})$$

Quite powerful, sums an ∞ class of diagrams

in eg 1



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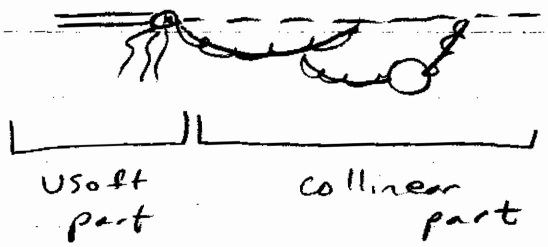


in eg 2 usoft gluons decouple at L.O. from any graph
This is color transparency

- usoft gluons decouple from energetic partons in color singlet state
- they just "see" overall color singlet due to multipole expansion

What about Wilson Coefficients?

have $C(\bar{P},\mu)$ ie depend on large momenta picked out by label operator $\bar{P} \sim \lambda^0$

eg. $$C(-\bar{P},\mu)\,(\bar{\xi}_n W)\,\Gamma h_v = (\bar{\xi}_n W)\,\Gamma h_v\, C(\bar{P}^+)$$

must act on product $(\bar{\xi} W)$ since only momentum of this combination is gauge invariant

Write $$(\bar{\xi} W)\,\Gamma h_v\, C(\bar{P}^+) = \int d\omega\, C(\omega,\mu)\,\left[(\bar{\xi} W)\,\delta(\omega-\bar{P}^+)\,\Gamma h_v\right]$$

$$= \int d\omega\, \underset{\uparrow}{C(\omega,\mu)}\;\; \underset{\uparrow}{O(\omega,\mu)}$$

convolution (as promised)

Hard-Collinear Factorization of "C" and collinear "O"

Recall defn of W, $i\bar{n}\cdot D_c W = 0$, $W^+ W = 1$

as operator
$$i\bar{n}\cdot D_c\, W = W\,\bar{P}$$
$$i\bar{n}\cdot D_c = W\,\bar{P}\,W^+$$
$$(i\bar{n}\cdot D_c)^k = W\,\bar{P}^k\,W^+$$

$$f(i\bar{n}\cdot D_c) = \underset{\uparrow}{W}\; \underset{\uparrow}{f(\bar{P})}\; \underset{\uparrow}{W^+}$$ trades $\bar{n}\cdot A \to W$

Part of collin op. $p^2 \sim \lambda^2 Q^2$

hard coefficient

$$= \int d\omega\; f(\omega)\;\; W\,\delta(\omega-\bar{P})\,W^+$$

In general define $\chi_n = (W^\dagger \xi_n)$

$$\chi_{n,\omega} = \delta(\omega - \bar{P})(W^\dagger \xi_n)$$

Operators $\int d\omega_1 \, d\omega_2 \; \bar{\chi}_{n,\omega_1} \Gamma \chi_{n,\omega_2}$ etc.

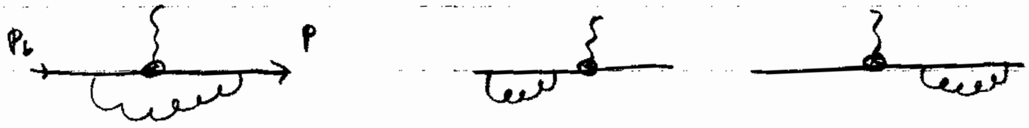
IR divergences, Matching, & Running

Consider heavy-to-light current for $b \to s\gamma$

$J^{QCD} = \bar{s}\Gamma b$ $\qquad \Gamma = \sigma^{\mu\nu} P_R F_{\mu\nu}$, $O_{7\gamma}$

$J^{SCET} = (\bar{\xi} W)\Gamma h_v \, C(\bar{P}^\dagger)$ $\qquad$ (pre Y- field redefn)

QCD graphs at one-loop, take $p^2 \neq 0$ to regulate IR of collin-quark



$$= -\bar{s}\Gamma b \, \frac{\alpha_s C_F}{4\pi}\left[\ln^2\left(\frac{-p^2}{m_b^2}\right) + 2\ln\left(\frac{-p^2}{m_b^2}\right) + \cdots \right]$$

$$Z_b = 1 - \frac{\alpha_s C_F}{4\pi}\left[\frac{1}{\epsilon_{UV}} + \frac{2}{\epsilon_{IR}} + 3\ln\frac{\mu^2}{m_b^2} + \cdots \right]$$ ← IR reg. by D.R. here

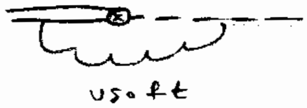
$$Z_s = 1 - \frac{\alpha_s C_F}{4\pi}\left[\frac{1}{\epsilon_{UV}} - \ln\frac{p^2}{\mu^2} \right]$$

← full Z's, not $\overline{MS}$ match - S-matrix

$$Z_{tensor} = 1 + \frac{\alpha_s C_F}{4\pi}\frac{1}{\epsilon}$$ tensor current not conserved

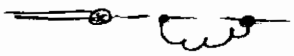
$$\text{Sum} = \bar{s}\Gamma b \left[1 - \frac{\alpha_s C_F}{4\pi} \Big(\ln^2\left(\frac{-p^2}{m_b^2}\right) + \frac{3}{2}\ln\left(\frac{-p^2}{m_b^2}\right) + \frac{1}{\epsilon_{IR}} + \cdots \right]$$

usoft



$$\int \frac{\not{d}^d k \quad n\cdot v}{(v\cdot k + i\epsilon)\,(k^2+i\epsilon)\,(n\cdot k + p^2/\bar{n}\cdot p + i\epsilon)}$$

$$= -\bar{\xi}\,\Gamma h_v \;\frac{\alpha_s C_F}{4\pi}\left[\frac{1}{\epsilon^2} + \frac{2}{\epsilon}\ln\left(\frac{\mu\,\bar{n}\cdot p}{-p^2 - i\epsilon}\right) + 2\ln^2\left(\frac{\mu\,\bar{n}\cdot p}{-p^2}\right) + \frac{3\pi^2}{4}\right]$$



$\propto \quad n^\mu n_\mu = 0 \qquad$ Feyn. Gauge

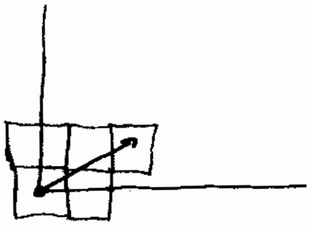
$$Z_{HQET} = 1 + \frac{\alpha_s C_F}{4\pi}\left[\frac{2}{\epsilon_{UV}} - \frac{2}{\epsilon_{IR}}\right]$$

Collinear Graphs

$$= \sum_{\substack{k\neq 0 \\ k\neq -p}} \int \not{d}^d k_r \; \frac{n\cdot\bar{n} \quad \bar{n}\cdot(p+k)}{\bar{n}\cdot k \quad k^2 \quad (k+p)^2}$$

↑ each has label & residual (k, k_r)

recall grid



Grid is like Wilsonian EFT

To make it Continuum

if $k=0$, gluon is usoft

$$\sum_{k\neq 0}\int \not{d}^d k_r \; F(k,p,k_r) = \int \not{d}^d k\left[F(k,p) - F^{subt}(k,p)\right]$$

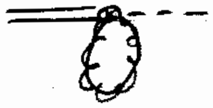
$k=-p$ usoft quark (harmless)

k scales towards usoft

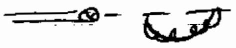
$$\frac{n\cdot\bar{n} \quad \bar{n}\cdot p}{\bar{n}\cdot k \;\; k^2 \; (n\cdot k\,\bar{n}\cdot p + p^2)}$$

$\propto \frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}}$

$$= -\bar{\xi}\,\Gamma h_v \;\frac{\alpha_s C_F}{4\pi}\left[-\frac{2}{\epsilon^2} - \frac{2}{\epsilon} - \frac{2}{\epsilon}\ln\left(\frac{\mu^2}{-p^2}\right) - \ln^2\left(\frac{\mu^2}{-p^2}\right) - 2\ln\left(\mu^2/-p^2\right) - 4 + \pi^2/6\right]$$



$\propto \; n\cdot\bar n = 0$



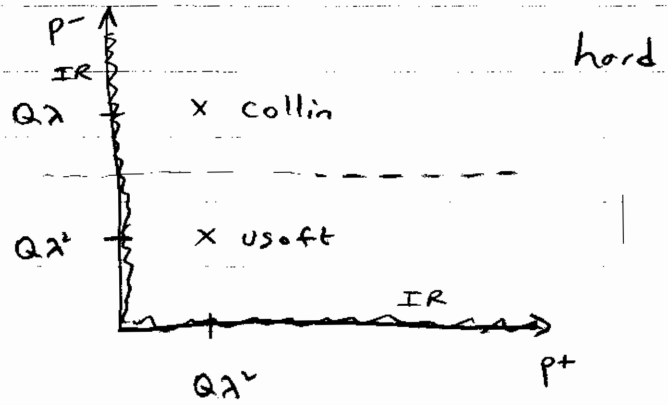
$$Z = 1 - \frac{\alpha_s C_F}{4\pi}\left[\frac{1}{\epsilon_{UV}} + \ln\frac{\mu^2}{p^2}\right]$$

IR matches $\ln^2(p^2)$ QCD = SCET

$\ln(p^2)$ ''

$1/\epsilon_{IR}$ ''

If we had neglected collinear graphs this would not be true [historically LEET ...]



degrees of freedom tile momentum space while maintaining p.c.

UV divergences in SCET need a c.t.

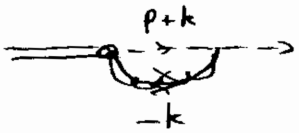
$$Z = 1 + \frac{\alpha_s C_F}{4\pi}\left[\frac{1}{\epsilon^2} + \frac{2}{\epsilon}\underset{\uparrow_{LL}}{\ln}\left(\frac{\mu}{\bar n\cdot p}\right) + \underset{\uparrow_{\text{part of NLL}}}{\frac{5}{2\epsilon}}\right]$$

Running

In general we must be careful with coeffs since they act like operators $C(\mu, \bar{P})$

In our eg. $\bar{P} \to \bar{n}\cdot p$ of external field always

non-trivial case



$$C(\mu, \bar{n}\cdot(p+k) + \bar{n}\cdot(-k)) = C(\mu, \bar{n}\cdot p)$$

$$\mu \, {}^{d}/_{d\mu} \; C(\mu) = - \frac{\alpha_s(\mu) \, C_F}{\pi} \ln\left(\frac{\mu}{\bar{P}}\right) C(\mu)$$ LO anom dim

Soln. QED α_s = fixed, $C_F = 1$

$$C(\mu) = \exp\left[-\frac{\alpha}{2\pi} \ln^2\left(\frac{\mu}{\bar{P}}\right)\right]$$ Sudakov Suppression

QCD $$C(\mu) = \exp\left[\frac{-4\pi C_F}{\beta_0^2 \, \alpha_s(m_b)} \left(\frac{1}{z} - 1 + \ln z\right)\right]$$

$z = \frac{\alpha_s(\mu)}{\alpha_s(m_b)}$

here m_b = matching scale

In more complicated cases $C(\bar{P}, \bar{P}^+)$ will be sensitive to $\bar{n}\cdot k$ loop momentum and we'll get

$$\mu \frac{\partial}{\partial\mu} C(\mu, \omega) = \int d\omega' \; \gamma(\omega, \omega') \; C(\mu, \omega')$$

examples	DIS	Alterelli-Parisi evolution
	$\gamma^* \pi^0 \to \pi^0$	Brodsky-Lepage "
	$\gamma^* p \to \gamma p'$	Deeply-Virtual Compton Scatt'g

these are actually all the evolution of a single SCET operator

$$(\bar{\xi}_n W)\, C(\bar{P}, \bar{P}^+)\, (W^\dagger \xi_n)$$

Note: series in $\ln C(\mu)$

		one-loop	two-loop	3-loop
LL	$\alpha_s^n \ln^{n+1}$	$1/\epsilon^2$	—	—
NLL	$\alpha_s^n \ln^n$	$1/\epsilon$	$1/\epsilon^2$	—
NNLL	$\alpha_s^n \ln^{n-1}$	matching	$1/\epsilon$	$1/\epsilon^2$

$1/\epsilon^2 \to 1/\epsilon \ln(\mu)$ term

Differs from single log case somewhat

At LHC, Sudakov effects are important in

Parton showers [Prob. to evolve without branching]

Jets

ast Time

SCET$_I$:
- usoft - collinear factorization
- hard - collinear factorization

[$\mathcal{L}_{qq}^{(0)}$, RPI, IR div., Running]

hard $p^\mu \sim (Q, Q, Q)$ $C = H$

collin $\sim (Q\lambda^2, Q, Q\lambda)$

usoft $\sim (Q\lambda^2, Q\lambda^2, Q\lambda^2)$

SCET$_{II}$: still to come,
- soft - collinear factorization
- Wilson coeffs

hard-collin p^μ $(Q\eta, Q, \sqrt{Q\eta})$ $C = J$ jet function

collin $p^\mu \sim (Q\eta^2, Q, Q\eta)$

soft $p^\mu \sim (Q\eta, Q\eta, Q\eta)$

Note: identification of d.o.f. is frame dependant, but relationships between d.o.f. are frame indep.

eg. boost can swap collin ↔ soft

Results for observable which tie d.o.f together are "Factorization Theorems"

eg $\int [d \cdots] \, H(q^-) \, J(q^-, p^-, k^+) \, \phi(p^-) \, \phi(k^+)$

Processes

- $\gamma^*\gamma \to \pi^0$ π-γ form factor at $Q^2 \gg \Lambda^2$ for γ^*

  Breit frame $q^\mu = \frac{Q}{2}(n^\mu - \bar{n}^\mu)$, $p_\gamma^\mu = E\,\bar{n}^\mu$

  $$p_\pi^\mu = \frac{Q}{2} n^\mu + \underbrace{\left(E - \frac{Q}{2}\right)}_{m_\pi^2/2Q} \bar{n}^\mu$$

  pion = collinear in n-direction (SCET_{II})

- $\gamma^* M \to M'$ M-M' (meson) form factor $Q^2 \gg \Lambda^2$ for γ^*

  M = collinear in n

  M' = " " $\bar{n}$ (say) (SCET_{I})

- $B \to D\pi$ Matrix Elt. of 4-quark operators

  $Q = \{m_b, m_c, E_\pi\} \gg \Lambda$

  B, D are soft $p^2 \sim \Lambda^2$, π-collinear (SCET_{II})

- DIS Structure Functions at $Q^2 \gg \Lambda^2$

  $e^- p \to e^- X$ and $1 - x \gg \Lambda/Q$ (ie not near endpts in Bjorken x)

  Breit frame: proton n-collinear , X - hard (SCET_{I})

- Drell-Yan $\frac{d\sigma}{dQ^2}$ Q^2 = inv. mass of $\ell^+\ell^- \gg \Lambda^2$

  $p\bar{p} \to \ell^+\ell^- X$

  p - n-collin, $\bar{p}$ - $\bar{n}$-collin, X - hard

- $e^+e^- \to$ jets

  $\bar{p}$ → jets • depends on observable we formulate

  pp → jets as two jets n-collin jet

  $\bar{n}$-collin jet

etc.

$\boxed{\text{DIS}}$ A rich subject, only aspects related to QCD factorization are covered here using SCET

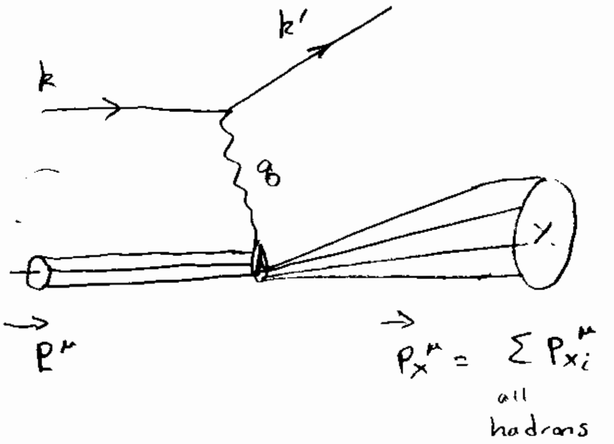
Refs: § 1.8 of text

Aneesh M.'s review: hep-ph/9204208

Bob J.'s review: hep-ph/9602236

paper: hep-ph/0202088 (for material below)

$e^- p \to e^- X$



$$Q^2 \gg \Lambda^2$$

$$q^2 = -Q^2 \,, \qquad x = \frac{Q^2}{2P\cdot q}$$

$$P_X^\mu = P^\mu + q^\mu$$

$$P_X^2 = \frac{Q^2}{x}(1-x) + M_p^2$$

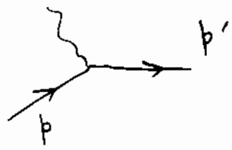
regions

P_X^2	$\left(\frac{1}{x}-1\right)$	
$\sim Q^2$	~ 1	inclusive OPE
$\sim Q\Lambda$	$\sim \Lambda/Q$	endpt. region
$\sim \Lambda^2$	$\sim \Lambda^2/Q^2$	resonance region

$e^- p \to e^- p'$

$\uparrow$ eg. excited state

Parton Variables



struck quark carries some fraction ξ of proton momentum

$$\bar{n}\cdot p = \xi\, \bar{n}\cdot P \qquad \leftarrow \text{we'll see how to formulate } \xi \text{ in QCD}$$

$$p'^2 \simeq Q^2\left(\frac{\xi}{x} - 1\right)$$

Frames

Breit Frame

$$q^\mu = \frac{Q}{2}\left(\bar{n}^\mu - n^\mu\right)$$

$$P^\mu = \frac{n^\mu}{2}\,\bar{n}\cdot P + \frac{\bar{n}^\mu m_p^2}{2\,\bar{n}\cdot P} = \frac{n^\mu}{2}\,\frac{Q}{x} + \cdots \qquad \text{collinear}$$

$$p_X^\mu = \frac{n^\mu}{2}\,Q + \frac{\bar{n}^\mu}{2}\,\frac{Q(1-x)}{x} + \cdots \qquad \text{hard}$$

Proton is made of collinear quarks and gluons

Rest Frame

$$P^\mu = \frac{m_p}{2}\left(n^\mu + \bar{n}^\mu\right) \qquad \text{soft}$$

$$q^\mu = \frac{\bar{n}^\mu}{2}\,\frac{Q^2}{m_p x} - \frac{n^\mu}{2}\,m_p x + \cdots$$

$$p_X^\mu = \text{sum} \qquad \text{"collinear"}\ p_X^2 \sim Q^2$$

Like $B \to X_c e \nu$ we can write cross-section in terms of leptonic & hadronic tensors

$$d\sigma = \frac{d^3k'}{2|\vec{k}'|}\,\frac{e^4}{sQ^4}\,L^{\mu\nu}(k,k')\,W_{\mu\nu}(P,q)$$

we'll look at spin-avg. case

$$W_{\mu\nu} = \frac{1}{\pi}\,\mathrm{Im}\,T_{\mu\nu}$$

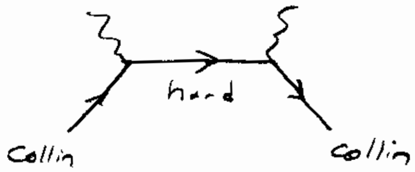
$$T_{\mu\nu} = \frac{1}{2}\sum_{\text{spin}} \langle p|\,\hat{T}_{\mu\nu}(q)\,|p\rangle$$

$$\hat{T}_{\mu\nu} = i\int d^4x\; e^{iq\cdot x}\; T\left[\underset{\substack{\uparrow\\ \text{e.m. currents}}}{J_\mu(x)\,J_\nu(0)}\right]$$

$$T_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) T_1(x,Q^2) + \left(P_\mu + \frac{q_\mu}{2x}\right)\left(P_\nu + \frac{q_\nu}{2x}\right) T_2(x,Q^2)$$

Satifies current conservation, P, C, T, etc.

Want imaginary part of forward scattering



First Match onto SCET Ops.
at L.O. :

$$\hat{T}^{\mu\nu} = \frac{g_\perp^{\mu\nu}}{Q}\left(O_1^{(i)} + \frac{O_1^g}{Q}\right) + \frac{(n^\mu+\bar{n}^\mu)(n^\nu+\bar{n}^\nu)}{Q}\left(O_2^{(i)} + \frac{O_2^g}{Q}\right)$$

O(λ^2) operators

$\downarrow$ flavor = u, d, ...

$$O_j^{(i)} = \bar{\xi}_{n,p'}^{(i)} W \frac{\bar{n}\!\!\!/}{2} C_j^{(i)}(\bar{P}_+, \bar{P}_-) W^\dagger \xi_{n,p}^{(i)}$$

$$O_j^{(g)} = \mathrm{tr}\left[W^\dagger B_\perp^\lambda W C_j^g(\bar{P}_+, \bar{P}_-) W^\dagger B_{\perp\lambda} W\right]$$

where $ig B_\perp^\lambda \equiv [i\bar{n}\cdot D_c, iD_{\perp c}^\lambda] \sim \lambda \sim \xi_n$

$\bar{P}_\pm = \bar{P}^\dagger \pm \bar{P}$

$O_j^{(i)}$ will lead to quark, anti-quark p.d.f.'s

O_j^g " " gluon p.d.f.'s

Quark contribution in detail :

$$O_j^{(i)} = \int d\omega_1 d\omega_2 \, C_j^{(i)}(\omega_+,\omega_-) \left[\underset{\substack{\uparrow \\ \delta(\omega_1 - \bar{P}^\dagger)}}{(\bar{\xi}_n W)_{\omega_1}} \frac{\bar{n}\!\!\!/}{2} \underset{\substack{\uparrow \\ \delta(\omega_2 - \bar{P})}}{(W^\dagger \xi_n)_{\omega_2}} \right]$$

$$\omega_\pm = \omega_1 \pm \omega_2$$

- coord space $f_{i/p}(z) = \int dy\ e^{-i2z\bar{n}\cdot p y} \langle p| \bar{\xi}(y) W(y,-y) \bar{n}\!\!\!/ \xi(y) |p\rangle$

parton distn for quark i in proton p

$$\bar{f}_{i/p}(z) = -f_{i/p}(-z) \quad \text{for anti-quark}$$

mom. space $\langle p_n| (\bar{\xi}_n W)_{\omega_1} \bar{n}\!\!\!/ (W^{\dagger}\xi_n)_{\omega_2} |p_n\rangle = 4\bar{n}\cdot p \int_0^1 d\xi\ \delta(\omega_-)$

$$* \left[\delta(\omega_+ - 2\xi\bar{n}\cdot p) f_{i/p}(\xi) - \delta(\omega_+ + 2\xi\bar{n}\cdot p)\bar{f}_{i/p}(\xi) \right]$$

recall

positive $\omega_1=\omega_2$ gives particles

negative $\omega_1=\omega_2$ gives anti-particles

- $\underbrace{(\bar{\xi}_n W)_\omega}_{\text{a parton}} \bar{n}\!\!\!/ (W^{\dagger}\xi_n)_\omega$ is a number operator for collinear quarks with momentum ω

[If we tried to couple usoft or soft gluons to this op. its a singlet so they decouple, more later]

Charge Conjugation

$$C_j^{(i)}(-\omega_+,\omega_-) = -C_j^{(i)}(\omega_+,\omega_-)$$

- relates Wilson-Coeff for quarks & anti-quarks at operator level
- Only need matching for quarks
- δ-functions set $\omega_- = 0$, $\omega_+ = 2\xi\bar{n}\cdot p = 2Q\frac{\xi}{x}$

Relate basis

$$\frac{1}{\pi}\,\mathrm{Im}\,T_1 = \int [d\omega]\ \frac{-1}{Q}\left(\frac{1}{\pi}\,\mathrm{Im}\,C_1(\omega)\right) \langle O^{(i)}(\omega)\rangle$$

$$\frac{1}{\pi}\,\mathrm{Im}\,T_2 = \int [d\omega]\ \left(\frac{4x}{Q}\right)^2 \frac{1}{Q}\,\frac{1}{\pi}\mathrm{Im}\left(C_2(\omega) - \frac{C_1(\omega)}{4}\right) \ \langle O^{(i)}(\omega)\rangle$$

Define $H_j(z) = \frac{\mathrm{Im}}{\pi}\ C_j\,(\underset{\omega_+}{2Qz},\ \underset{\omega_-}{0},\ Q^2,\ \mu^2)$ do $\omega_\pm$ with δ-functions

$$T_1(x,Q^2) = \frac{-1}{x}\int_0^1 d\xi\ \ H_1^{(i)}\left(\frac{\xi}{x}\right)\left[f_{i/p}(\xi) + \bar{f}_{i/p}(\xi)\right]$$

$$T_2(x,Q^2) = \frac{4x}{Q^2}\int_0^1 d\xi\ \left(4H_2^{(i)}\left(\frac{\xi}{x}\right) - H_1^{(i)}\left(\frac{\xi}{x}\right)\right)\left[f_{i/p}(\xi) + \bar{f}_{i/p}(\xi)\right]$$

- this is factorization for DIS (to all order in α_s) into
  computable coefficients H_i
  universal non-pert. functions $f_{i/p}$, $\bar{f}_{i/p}$
  (show up in many processes)
- Coefficients C_j were dimensionless and can only
  have $\alpha_s(\mu)\ \ln(\mu/Q)$ dependence on Q
  → Bjorken scaling

[Analysis valid to LO in $\frac{\Lambda^2}{Q^2}$]

- $H_i(\mu)\ f_{i/p}(\mu)$ traditionally this μ-dependence is called the "factorization-scale" $\mu = \mu_F$ & one also has "renorm. scale" $\alpha_s(\mu = \mu_R)$

In SCET the μ is just the ren. scale in SCET. We have new UV divergences associated with running of p.d.f., along with running for $\alpha_s(\mu)$.

- Tree Level Matching
  (upon which a lot of intuition is based)

find just $g_\perp^{\mu\nu}$ ie $C_2 = 0$

↳ Callan-Gross relation that $W_1/W_2 = Q^2/4x^2$

$$C_1(\omega_+) = 2e^2 \underset{\substack{\uparrow \\ \text{charges}}}{Q_i^2} \left[\frac{Q}{(\omega_+ - 2Q) + i\epsilon} - \frac{Q}{-(\omega_+ + 2Q) + i\epsilon} \right]$$

$$H_1 = -e^2 Q_i^2 \, \delta\left(\frac{\xi}{x} - 1\right)$$

gives parton-model interpretation $\xi = x$

Comments on DIS

- contrast ∞ set of ops in text
- power of $SCET_I / SCET_{II}$ not really needed, no soft (think of it as $SCET_I$ for example)

-do eg. first-

Soft-Collinear Interactions (SCET$_{II}$)

Recall $q = q_s + q_c \sim Q\,(\overset{+}{\lambda}, \overset{-}{1}, \overset{\perp}{\lambda})$

$q^2 = Q^2\lambda \gg (Q\lambda)^2$

offshell w.r.t s, c

On-shell modes $q^\mu \sim Q\,(\lambda, 1, \sqrt{\lambda})$ are hard-collinear

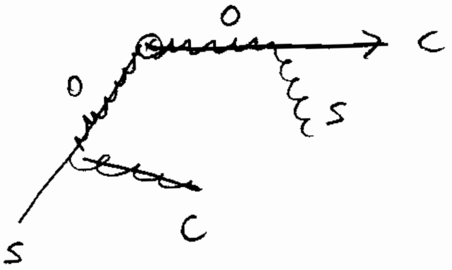
compared to collinear $q^\mu \sim Q(\lambda^2, 1, \lambda)$

Integrating out these fluctuations builds up a soft Wilson line S_n (analogous to $Y(n\cdot A_{us})$ but with soft fields)

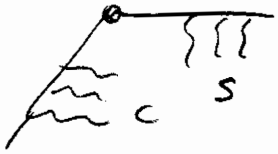
Toy eg. heavy-to-light soft-collin current $\bar\xi_n \Gamma h_v$

s = soft, c = collinear

o = offshell



adding more gives

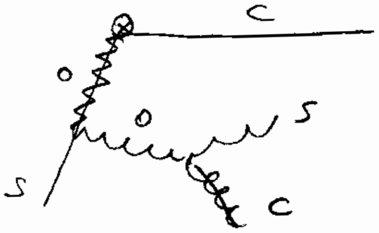


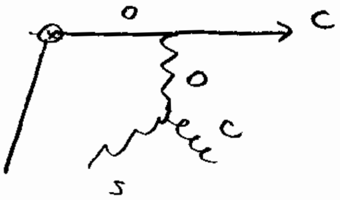
$\bar\xi_n S_n^+ \Gamma W h_v$

$S_n^+[n\cdot A_{us}]$

$W[\bar n\cdot A_c]$

In QCD need 3-gluon, 4-gluon vertices too: these flip order of S^+ & W





$$\underbrace{(\bar\xi_n W)}_{\text{collinear gauge invariant}}\ \Gamma\ \underbrace{(S_n^+ h_v)}_{\text{soft gauge invariant}}$$

[can be extended to all orders]

this is soft-collinear factorization

↓ stopped here

Another Method

- construct SCET$_{II}$ operators using SCET$_I$

i) Match QCD onto SCET$_I$ usoft $p_u^2 \sim \Lambda^2$

collinear $p_c^2 \sim Q\Lambda$

ii) Factorize usoft with field redefinition

iii) Match SCET$_I$ onto SCET$_{II}$ soft $p_s^2 \sim \Lambda^2$

collin $p_c^2 \sim \Lambda^2$

Notes • this gives us a simple procedure to construct SCET$_{II}$ ops. (even though they're non-local)

• usoft fields in I are renamed soft for II

eg. i) $J^I = (\bar\xi_n W)\,\Gamma\, h_v$

ii) $J^I = (\bar\xi_n^{(0)} W^{(0)})\,\Gamma\,(Y^\dagger h_v)$

iii) $J^{II} = (\bar\xi_n W)\,\Gamma\,(S^\dagger h_v)$ as before

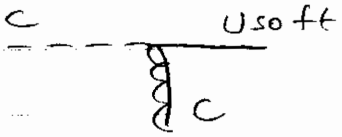
↑ here all T-products in SCET$_I$ & SCET$_{II}$ match up, so matching was trivial

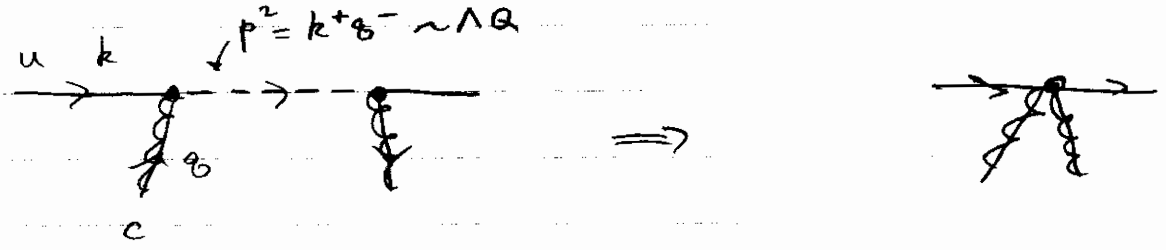
"Thm" • In cases where we have T-products in SCET$_I$ with ≥ 2 operators involving both collin & usoft fields, we can generate a non-trivial coefficient in SCET$_{II}$ (jet-function J)

T e.g. $\int dp_-\, dk_+\, J(p_-, k_+)\ \overbrace{(\bar\xi W)_{p_-}\ \Gamma\ (S^\dagger q_s)_{k^+}}^{p^2\sim\Lambda^2}$

↑ ↑

SCET$_I$ loops $p^2 \sim Q\Lambda$ ind's allow k^+ dependence eg. two operators





when we lower offshellness of ext. collin fields the intermediate line still has $p^2 \sim Q\Lambda$ and must really be integrated out

P.C. $T^{I} \sim \lambda^{2K} \Rightarrow O^{II} \sim \eta^{K+E}$

where $\lambda^2 = \eta = \frac{\Lambda}{Q}$,

factor $E > 0$ from changing the scaling of ext. fields

eg. $\xi_I \sim \lambda$

$\xi_{II} \sim \eta = \lambda^2$

$\Rightarrow$ No mixed soft-collin $\mathcal{L}$ at leading order

- after field redefn no mixed $\mathcal{L}_I$ ops at LO
- mixed $\mathcal{L}_I^{(1)}$ gives $T\{\mathcal{L}_I^{(1)}, \mathcal{L}_I^{(1)}\} \sim \lambda^2$ matches onto $O_{II} \sim \eta$ or higher

SCET$_I$ λ^δ

$$\delta = 4 + \underset{\uparrow\ u=1\text{ no c., else } u=0}{4u} + \overset{\swarrow\text{ rest}}{\sum_k (k-4)\, V_k^c} + \overset{\swarrow\text{ pure usoft}}{(k-8)\, V_k^u}$$

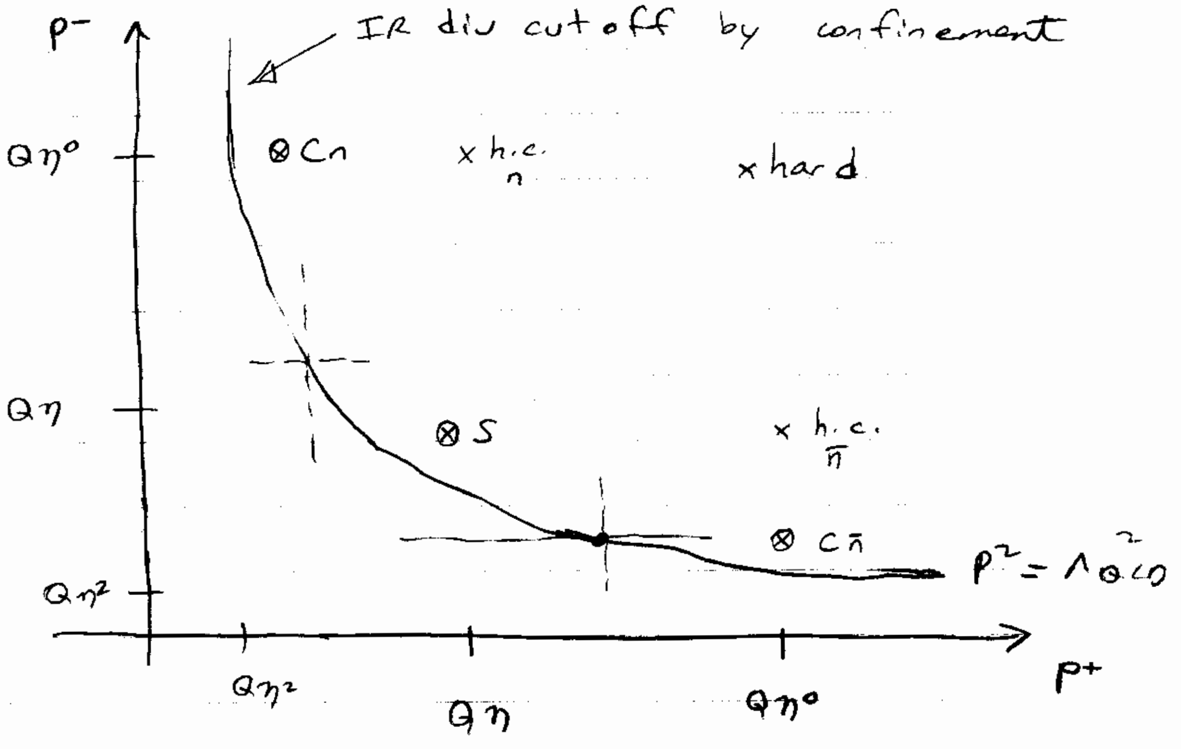
SCET$_{II}$

$$\delta = 4 + \sum_k (k-4)\big(\underset{\substack{\uparrow\\ \text{pure}\\ c}}{V_k^c} + \underset{\substack{\uparrow\\ \text{pure}\\ s}}{V_k^s} + \underset{\substack{\uparrow\\ \text{mixed}}}{V_k^{sc}}\big) + \underset{\substack{\uparrow\\ p\sim(\eta^2,\eta,\eta)\\ \text{loops}}}{L^{sc}}$$

$$\delta = 5 - \underbrace{N_c - N_s}_{\substack{\uparrow\ \uparrow\\ \#\text{ connected}\\ \text{soft, collin components}}} + \sum_k (k-4)\left(V_k^s + V_k^c\right) + (k-3)\, V_k^{sc}$$

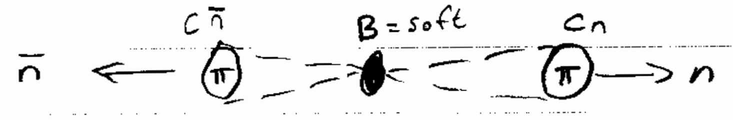
[in eg. SCET$_I$ $\lambda^3\lambda \frac{1}{\lambda^2} \lambda^3\lambda \sim \lambda^{6-4} \sim \lambda^2$ or $\lambda * \lambda \sim \lambda^2$ $\Rightarrow$ $(\eta^{3/2}\eta)^2 \frac{1}{\eta} = \eta^{4-3} = \eta$]

$$\mathcal{L}_{SCET^{II}} = \mathcal{L}^{(0)}_{soft}[q_s, A_s] + \mathcal{L}^{(0)}_{collin\text{-}n}[q_n, A_n] + \mathcal{L}^{(0)}_{collin\text{-}\bar n}[q_{\bar n}, A_{\bar n}]$$



Non-pert d.o.f in different sectors $B \to \pi\pi$

eg.



Exclusive

eg. $\gamma^* \gamma \to \pi^0$ hard - collin factorization

[Breit frame : soft modes have no active role so this does not really probe difference between $SCET_I$ & $SCET_{II}$]

QCD has

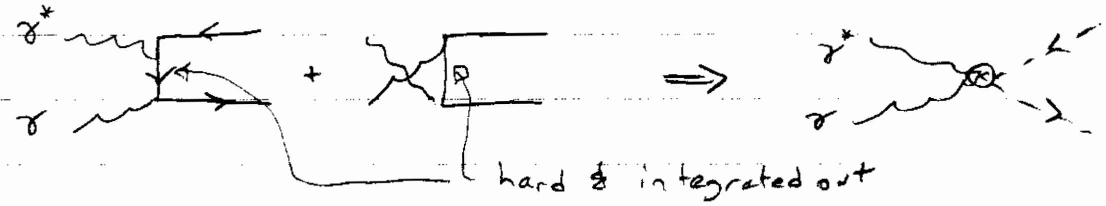
$$\langle \pi^0(p_\pi) | J_\mu(0) | \gamma(p_\gamma, \epsilon) \rangle = ie\, \epsilon^\nu \int d^4z\, e^{-ip_\gamma \cdot z} \langle \pi^0(p_\pi) | T\, J_\mu(0) J_\nu(z) | 0 \rangle$$
$$= -ie\, F_{\pi\gamma}(Q^2)\, \epsilon_{\mu\nu\rho\sigma}\, p_\pi^\nu \epsilon^\rho q^\sigma$$

e.m. current $J^\mu = \bar\Psi \hat{Q} \gamma^\mu \Psi$, $\hat{Q} = \frac{\tau_3}{2} + \frac{1}{6} = \begin{pmatrix} 2/3 & \\ & -1/3 \end{pmatrix}$

<u>For $Q^2 \gg \Lambda^2$</u> $F_{\pi\gamma}$ simplifies (a la Brodsky-Lepage)

frame $q^\mu = \frac{Q}{2}(n^\mu - \bar{n}^\mu)$, $p_\gamma^\mu = E\, \bar{n}^\mu$

$p_\pi^\mu = p + p_\gamma = \frac{Q}{2} n^\mu + (E - \frac{Q}{2})\bar{n}^\mu$



SCET Operator at Leading-order (for T-product) is

$$\mathcal{O} = \frac{i\, \epsilon^\perp_{\mu\nu}}{Q} [\bar\xi_{n,p} W]\, \Gamma\, C(\bar{P}, \bar{P}^+, \mu)\, [W^+ \xi_{n,p'}]$$

order λ^2 (" twist -2 ")

- obeys current conservation
- dim analysis fixes $\frac{1}{Q}$ pre-factor for C dimless

- Charge Conj : $T\{J, J\}$ even so $\mathcal{O}$ even

  so $C(\mu, \bar{P}, \bar{P}^+) = C(\mu, -\bar{P}^+, -\bar{P})$

- flavor & spin structure $\quad \Gamma = \underbrace{\bar{n}\!\!\!/\, \gamma_5}_{\text{for pion}} \; 3\sqrt{2} \; \underbrace{\hat{Q}^2}_{\text{2nd order e.m.}}$

- color singlet, purely collinear (again) so soft gluons decouple

$\swarrow$ SCET$_{II}$

equate $\quad \frac{Q^2}{2} F_{\pi\gamma} = \frac{i}{Q} \langle \pi^0 | (\bar\xi W) \,\Gamma\, C\, (W^\dagger \xi) | 0 \rangle$

write $\quad \bar{P}_\pm = \bar{P}^\dagger \pm \bar{P}$

now $\quad \bar{P}_-$ gives total mom of $(\xi W)\Gamma(W^\dagger \xi)$ operator

ie momentum of pion

$$\bar{P}_- = \bar{n}\cdot P_\pi = Q$$

$\longrightarrow$ total mom.

$$F_{\pi\gamma}(Q^2) = \frac{2i}{Q^2} \int d\omega\, C(\omega,\mu) \langle \pi^0 | (\bar\xi W)\, \Gamma\, \delta(\omega - \bar{P}_+) (W^\dagger \xi) | 0 \rangle$$

<u>Non-perturbative Matrix Elt</u> — finite Wilson line $\left(\text{Pexp } ig \int_x^y ds \ldots\right)$

Fourier Transform of $\bar{n}\cdot p$ label

position space

$$\langle \pi^0(p) | \bar\xi_n(y) \frac{\bar{n}\!\!\!/\,\gamma_5 \tau^3}{\sqrt{2}} W(y,x) \xi_n(x) | 0 \rangle = -i f_\pi\, \bar{n}\cdot p \int_0^1 dz\, e^{i\bar{n}\cdot p\,(zy + (1-z)x)} \phi_\pi(\mu,z)$$

$$\int_0^1 dz\, \phi_\pi(z) = 1$$

momentum space

$$\langle \pi^0(p) | (\bar\xi_{n,p} W) \frac{\bar{n}\!\!\!/\,\gamma_5 \tau^2}{\sqrt{2}} \delta(\omega - \bar{P}_+) (W^\dagger \xi_{n,p}) | 0 \rangle$$

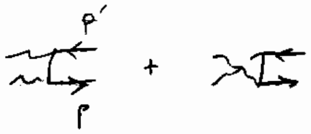
$$= -i f_\pi\, \bar{n}\cdot p \int_0^1 dz\, \delta(\omega - (2z-1)\bar{n}\cdot p)\, \phi_\pi(\mu,z)$$

Plug it into $F_{\pi\gamma}(Q^2)$ and do integral over ω

$$F_{\pi\gamma}(Q^2) = \frac{2 f_\pi}{Q^2} \int_0^1 dz\ C\big((2z-1)Q, Q, \mu\big)\ \phi_\pi(z,\mu)$$

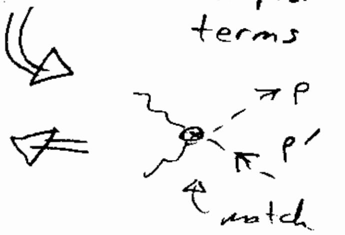
- ϕ_π is universal light-cone dist'n for pions
- C is process dependent (all order factorization in α_s)
- one-dim convolution again

Tree Level Matching

expand $i\Big($  $\Big) = \frac{ie}{2}\,\epsilon_{\mu\nu\rho\beta}\,\epsilon^\nu \bar{n}^\rho n^\beta \left(\frac{\bar{n}\!\!\!/}{2}\gamma_5\right)\hat{Q}^2$

$$* \left(\frac{1}{\bar{n}\cdot p} - \frac{1}{\bar{n}\cdot p'}\right) + \dots$$

non-pion terms



match with $W=1$

So $\quad C = \frac{1}{6\sqrt{2}}\left(\frac{Q}{\bar{p}^+} - \frac{Q}{\bar{p}}\right)$

$$C(\omega = (2x-1)Q) = \frac{1}{6\sqrt{2}}\left(\frac{1}{x} + \frac{1}{1-x}\right)$$

Charge Conj $+1$ for $|\pi^0\rangle$ gives $\phi_\pi(x) = \phi_\pi(1-x)$

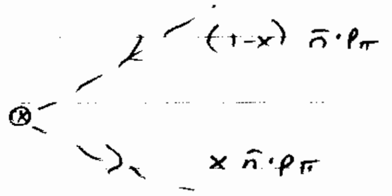
(Hmwk.)

So only $\displaystyle\int_0^1 dx\ \frac{\phi_\pi(x,\mu)}{x}$ appears in our prediction

integrate over all x, much different than DIS $\delta(1-\xi/x) \Rightarrow f_{i/p}(x,\mu)$

Interpretation:

Naively



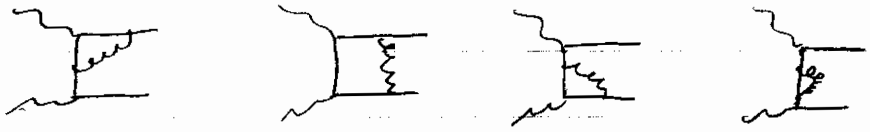
mom fraction of quarks in pion

Really

← mom. fractions at point where quarks are produced. Hadronization process changes "x" carried by valence quarks which is encoded in $\phi_\pi(x)$

Higher Order Matching

full



+ w.fn graphs

SCET

+ w.fn

Difference will be IR finite, and gives C at one-loop

Another Exclusive Example

(hep-ph/0107002)

$B \to D\pi$ $\underbrace{m_b, m_c, E_\pi}_{Q} \gg \Lambda_{QCD}$

QCD Operators at $\mu \simeq m_b$

$$H_W = \frac{4G_F}{\sqrt{2}} V_{ud}^* V_{cb} \left[C_0^F O_0 + C_8^F O_8 \right]$$

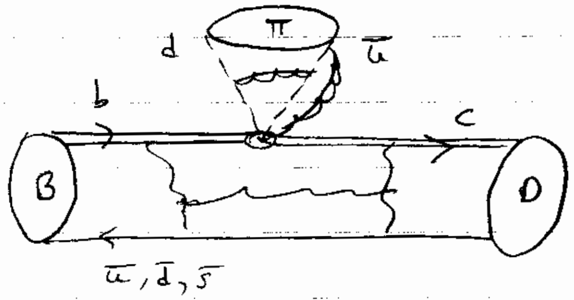
$P_L = \frac{1-\gamma_5}{2}$

where $O_0 = [\bar{c}\gamma^\mu P_L b][\bar{d}\gamma_\mu P_L u]$

$O_8 = [\bar{c}\gamma^\mu P_L T^a b][\bar{d}\gamma_\mu P_L T^a u]$

Want to Factorize $\langle D\pi | O_{0,8} | B \rangle$

ie Show at LO



no gluons btwn B, D and quarks in pion

expect $B \to D$ form factor ξ Isgur-Wise

$\phi_\pi(x)$ distn for pion

B, D soft $p^2 \sim \Lambda^2$

π collinear $p^2 \sim \Lambda^2$ $\Big\}$ SCET$_{II}$

use SCET$_I$ as intermediate step

① Match at $\mu^2 \simeq Q^2$

$$\left.\begin{matrix} O_0 \\ O_8 \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} Q_0^{1,5} = [\bar{h}_{v'}^{(c)} \Gamma_h^{1,5} h_v^{(b)}][(\bar{\xi}_n^{(d)} W)\Gamma_\ell C_0(\bar{P}_+) W^\dagger \xi_n^{(u)}] \\ Q_8^{1,5} = [\ \text{''}\ T^A\ \text{''}\ ][\ \text{''}\ \text{''}\ C_8(\bar{P}_+)T^a\ \text{''}\ ] \end{matrix}\right.$$

↑ usoft SCET$_I$

↑ collinear $p^2 \sim Q\Lambda$

$\Gamma_h^{1,5} = \frac{\not{n}}{2}\{1, \gamma_5\}$

$\Gamma_\ell = \frac{\bar{\not{n}}}{4}(1-\gamma_5)$

② Field redefinitions $\xi_{n,p} = Y \xi_{n,p}^{(0)}$, ...

in $Q_0^{1,5}$ get $\bar\xi_n^{(0)} W^{(0)} \cancel{Y^\dagger Y} W^{\dagger(0)} \xi_n^{(0)}$

$Q_8^{1,5}$ get $\bar\xi_n^{(0)} W^{(0)} Y^\dagger T^A Y W^{\dagger(0)} \xi_n^{(0)}$

$Y T^a Y^\dagger = \mathcal{Y}^{ba} T^b$ $\qquad\qquad$ $Y^\dagger T^a Y = \mathcal{Y}^{ab} T^b$

↑ adjoint Wilson line

$T^a \otimes Y^\dagger T^a Y = Y T^a Y^\dagger \otimes T^a$

↑ moves usoft Wilson lines next to h_v fields

③ Match $\mathrm{SCET_I}$ onto $\mathrm{SCET_{II}}$ (trivial here again)

$Y \to S$

$\xi_n^{(0)} \to \xi_n$ in II etc.

$$Q_0^{1,5} = [\bar h_{v'}^{(c)} \Gamma_h h_v^{(b)}] [\bar\xi_n^{(d)} W \Gamma_\ell C_0(\bar P_+) W^\dagger \xi_{n,p}^{(u)}]$$

$$Q_8^{1,5} = [\bar h_{v'}^{(c)} \Gamma_h S T^a S^\dagger h_v^{(b)}] [\bar\xi_n^{(d)} W \Gamma_\ell C_8(\bar P_+) T^a W^\dagger \xi_{n,p}^{(d)}]$$

④ Take Matrix Elements

$$\langle \pi_n^- | \bar\xi_n W \Gamma C_0(\bar P_+) W^\dagger \xi_n | 0\rangle = \frac{i}{2} f_\pi E_\pi \int_0^1 dx\, C(2E_\pi(2x-1)) \phi_\pi(x)$$

$$\langle D_{v'} | \bar h_{v'} \Gamma h_v | B\rangle = N' \xi(w_0, \mu)$$

↑ $w_0 = v \cdot v'$

B, D purely soft → no contractions with collinear fields

π " collinear → no " " soft fields

which is why it factors into two matrix element

F - O_8:

$$\langle D_{v'} | \underbrace{\bar h_{v'} Y T^a Y^\dagger h_v}_{\text{color octet operator between color singlet states}} | B_v \rangle = 0$$

Find Factorization Formula

$$\langle \pi D | H_W | B \rangle = i \underset{\substack{\uparrow \\ \text{prefactors}}}{N} \; \xi(w_0, \mu) \int_0^1 dx \; C(2E_\pi(2x-1), \mu) \, \phi_\pi(x, \mu) + O(\Lambda/Q)$$

- $\xi(w_0, \mu)$ is Isgur-Wise function at max. recoil
  $w_0 = \frac{m_B^2 - m_D^2}{2 m_B}$ (measured in $B \to D e \nu$ recall)
- This applies to type-I (& III) decays
  $\bar{B}^0 \to D^+ \pi^-$ $\bar{B}^0 \to D^{*+} \pi^-$, $\bar{B}^0 \to D^+ \rho^-$, ...
  $B^- \to D^0 \pi^-$ $B^- \to D^{*0} \pi^-$ $B^- \to D^0 \rho^-$, ...
  predicts type-II decays are suppressed by Λ/Q
  $\bar{B}^0 \to D^0 \pi^0$, ... (we could derive fact. thm. for these too)

Another inclusive example . $B \to X_s \gamma$ [case where usoft modes matter]

Here we will need both usoft & collinear d.o.f. in $SCET_I$

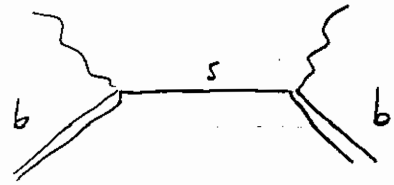
$$H_{eff} = \frac{-4G_F}{\sqrt{2}} V_{tb} V_{ts}^* C_7 O_7 \quad , \quad O_7 = \frac{e}{16\pi^2} m_b \bar{s} \sigma^{\mu\nu} F_{\mu\nu} P_R b$$

photon $q^\mu = E_\gamma \bar{n}^\mu$

$$\frac{1}{\Gamma_0} \frac{d\Gamma}{dE_\gamma} = \frac{4E_\gamma}{m_b^3} \left(\frac{-1}{\pi} \right) \operatorname{Im} T$$

$$T = \frac{i}{m_B} \int d^4x \, e^{-iq\cdot x} \langle \bar{B} | T \, J_\mu^\dagger(x) J^\mu(0) | \bar{B} \rangle$$

$$J^\mu = \bar{s} \, i\sigma^{\mu\nu} q_\nu P_R b$$



looks like DIS

Consider endpoint region

$$m_B/2 - E_\gamma \lesssim \Lambda_{QCD}$$

$$p_X^2 \simeq m_B \Lambda$$

B-rest frame $\quad p_B = \frac{m_B}{2} (n^\mu + \bar{n}^\mu) = p_X + q$

$$p_X = \frac{m_B}{2} n^\mu + \frac{\bar{n}^\mu}{2} \underbrace{(m_B - 2E_\gamma)}_{\Lambda}$$

collinear

so quarks and gluons in X are <u>collinear</u> with $p_c^2 \sim m_B \Lambda$

B has <u>usoft</u> light d.o.f.

match onto LO SCET operator

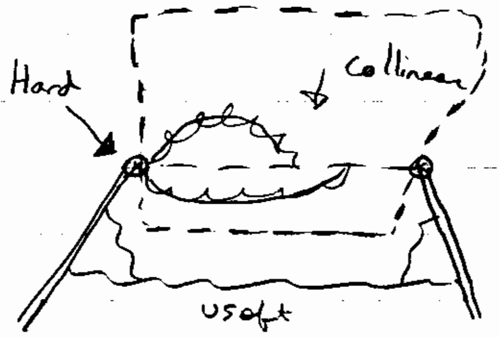
$$J_\mu = -E_\gamma \; e^{i(\bar{P}\frac{n}{2} - m_b v)\cdot x} \quad \bar{\xi} W \gamma_\mu^\perp P_L h_v \; C(\bar{P}^+, \mu)$$

$\uparrow$ our heavy-to-light current from earlier $\equiv J_{eff}^\mu$

The coefficient $C(\bar{P}^+)$ has $\bar{P}^+ = m_b$ since this is total momentum of s-quark jet in $\bar{n}\cdot P_X$

Factor with Field redefn

$$J_{eff}^\mu = \bar{\xi}_n^{(0)} W^{(0)} \gamma_\mu^\perp P_L Y^\dagger h_v$$



factored

$$T_{eff} = i\int d^4x \; e^{i(m_b\frac{\bar{n}}{2} - q)\cdot x} \langle \bar{B} | T\, J_{eff}^{\dagger\mu}(x)\, J_{eff\,\mu}(0) | \bar{B}\rangle$$

$$= i\int d^4x \; e^{i()} \quad \langle \bar{B} | T\, (\bar{h}_v Y)(x) \;\; (Y h_v)(0) \; | \bar{B}\rangle$$
$$* \;\; \langle 0 | T \; (W^{\dagger(0)} \xi^{(0)})(x) \;\; (\bar{\xi}^{(0)} W)(0) \; | 0 \rangle$$

$\leftarrow$ spin & color indices & structures $\gamma_\mu^\perp P_L$ suppressed

$$= \frac{1}{2}\int d^4x \int \not{d}^4k \;\; e^{i(m_b\frac{\bar{n}}{2} - q - k)\cdot x} \;\; \langle \bar{B} | T\, (\bar{h}_v Y)(x) \, (Y^\dagger h_v)(0) | \bar{B}\rangle$$
$$* \;\; J_P(k)$$

$$\langle 0 | T\, (W^\dagger \xi)\, (\bar{\xi} W) \, | 0\rangle = i \int \not{d}^4k \; e^{-ik\cdot x} \; J_P(k) \; \frac{\bar{\not{n}}}{2}$$

$\uparrow$ only depends on k^+!

so do $k^-, k^\perp$ integrals

in Teff we then get

$$S(\ell^+) = \frac{1}{2}\int \frac{dx^-}{4\pi} \; e^{-i/2\, \ell^+ x^-} \; \langle \bar{B} | T[\bar{h}_v Y)(\tfrac{n}{2}x^-) \;\; (Y^\dagger h_v)(0) | \bar{B}\rangle$$

$\leftarrow$ $Y(\frac{n}{2}x^-, 0)$ $\uparrow$

$$= \frac{1}{2} \langle \bar{B}_v | \; \bar{h}_v \; \delta(in\cdot D - k^+) \, h_v | \bar{B}_v\rangle$$

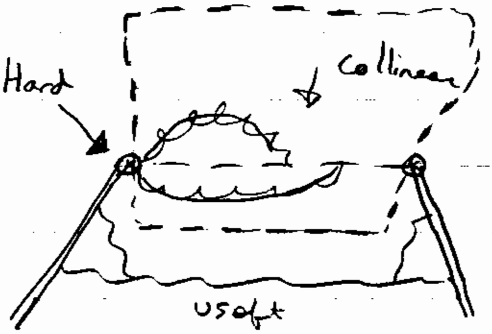
match onto LO SCET operator

$$J_\mu = -E_\gamma\, e^{i(\bar{P}\frac{n}{2} - m_b v)\cdot x} \quad \bar\xi W \gamma_\mu^\perp P_L h_v \quad C(\bar P^+,\mu)$$

$\uparrow$ our heavy-to-light current from earlier $\equiv J^\mu_{\rm eff}$

The coefficient $C(\bar P^+)$ has $\bar P^+ = m_b$ since this is total momentum of s-quark jet in $\bar n\cdot P_X$

Factor with Field redefn



$$J^\mu_{\rm eff} = \bar\xi_n^{(0)} W^{(0)} \gamma_\mu^\perp P_L Y^\dagger h_v$$

factored

$$T_{\rm eff} = i\int d^4x\, e^{i(m_b\frac{\bar n}{2} - q)\cdot x} \langle \bar B | T J^{\dagger\mu}_{\rm eff}(x) J_{{\rm eff},\mu}(0) |\bar B\rangle$$

$$= i\int d^4x\, e^{i()} \langle \bar B| T (\bar h_v Y)(x)\ (Y h_v)(0)\ |\bar B\rangle$$
$$* \langle 0 | T\, (W^{\dagger(0)}\xi^{(0)})(x)\ (\bar\xi^{(0)} W)(0)\ |0\rangle$$

$\leftarrow$ spin & color indices & structures $\gamma_\mu^\perp P_L$ suppressed

$$= \frac{1}{2}\int d^4x \int \bar{d}^4k\ e^{i(m_b\frac{\bar n}{2} - q - k)\cdot x} \langle \bar B | T (\bar h_v Y)(x)\, (Y^\dagger h_v)(0) | \bar B\rangle$$
$$* \ J_P(k)$$

$$\langle 0 | T\, (W^\dagger \xi)\, (\bar\xi W)\ |0\rangle = i\int \bar{d}^4k\ e^{-ik\cdot x}\, J_P(k)\, \frac{\bar{\not n}}{2}$$

$\uparrow$ only depends on k^+!

so do $k^-, k^\perp$ integrals

in $T_{\rm eff}$ we then get

$$S(\ell^+) = \frac{1}{2}\int \frac{dx^-}{4\pi}\, e^{-i/2\,\ell^+ x^-} \langle \bar B | T[\bar h_v Y)(\tfrac{n}{2}x^-)\ \ (Y^\dagger h_v)(0) | \bar B\rangle$$

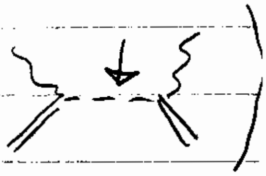
$\nwarrow \nearrow$ $Y(\frac{n}{2}x^-, 0)$

$$= \frac{1}{2}\langle \bar B_v | \bar h_v\, \delta(in\cdot D - k^+)\, h_v | \bar B_v\rangle$$

imaginary part is in jet function

let $J(k^+) \doteq -\frac{1}{\pi}\, \mathrm{Im}\, J_p(k^+)$

(tree level $J(k^+) = \delta(k^+)$ from



)

All order's factorization

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_\gamma} = N\, \underset{\underset{p^2\sim m_b^2}{\uparrow}}{C(m_b,\mu)} \int_{2E_\gamma - m_b}^{\bar{\Lambda}} d\ell^+\, \underset{\underset{p^2\sim\Lambda^2}{\uparrow}}{S(\ell^+)}\, \underset{\underset{p^2\sim m_b\Lambda}{\uparrow}}{J(\ell^+ + m_b - 2E_\gamma)}$$

$\uparrow$

Shape function is seen in the data

Final example   two - jet production

How do we define a jet ?

Consider $e^+e^- \to q\bar{q}g$

$$q = p_1 + p_2 + p_3$$

$$2 = x_1 + x_2 + x_3$$

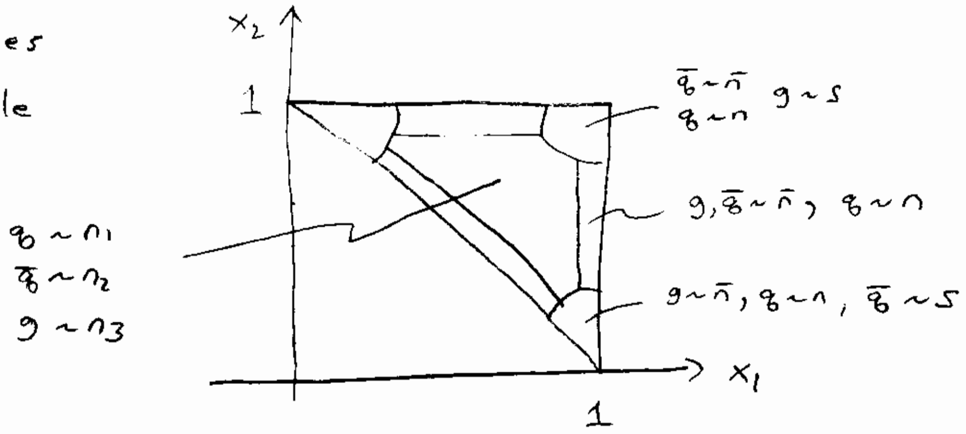
for $x_i = \frac{2 p_i \cdot q}{q^2}$

gives

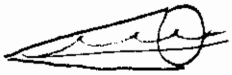
$$\frac{1}{\sigma_0}\frac{d\sigma}{dx_1 dx_2} = \frac{C_F \alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Two jets along edges

Three jets in middle



Sterman - Weinberg Definition of 2-jets



if gluon has $p_3^0 < \epsilon Q$ or

if gluon has angle $\cos\theta_{13} > 1 - 2\delta^2$ or $\cos\theta_{23} > 1 - 2\delta^2$

$$\frac{d\sigma}{de} = \frac{1}{2Q^2} \sum_N \Big| \langle N | \underset{\substack{\uparrow \\ \bar{\psi}\gamma\psi}}{J^\mu_{QCD}(0)} | 0 \rangle \underset{\substack{\uparrow \\ \text{leptons}}}{L_\mu} \Big|^2 (2\pi)^4 \delta^{(4)}(q - \Sigma p_N) \underset{\substack{\uparrow \\ \text{event shape variable}}}{\delta(e - e(N))}$$

eg. jet energy $E_J = \sum_{\substack{i \text{ in} \\ \text{cone}}} E_i$

eg. Thrust $T = \max_{\hat{t}} \sum_{i \in N} |\vec{p}_i \cdot \hat{t}| \Big/ \sum_{i \in N} |\vec{p}_i|$

hep-ph/0212255

Two-jets ($SCET_I$)

$$\bar\psi\Gamma\psi \to (\bar\xi_n W_n)\,\gamma_\perp^\mu\, C(\bar P, \bar P^\dagger, \mu)\,(W_{\bar n}^\dagger \xi_{\bar n}) = J^\mu_{SCET}$$

Matching ensures only 2-jets

Decouple U-soft

$$\xi_n \to Y_n\, \xi_n^{(0)} \qquad\qquad Y_n = \bar P \exp\left(-ig \int_0^\infty ds\, n\cdot A_{us}\right)$$

$$\xi_{\bar n} \to Y_{\bar n}\, \xi_{\bar n}^{(0)}$$

$$J^\mu = (\bar\xi_n W_n Y_n^\dagger)\,\gamma_\perp^\mu\, C\,(Y_{\bar n} W_{\bar n}^\dagger \xi_{\bar n})$$

State: $|N\rangle = |X_n X_{\bar n} X_u\rangle$

all the soft particles, not observed

we will not bother to observe this jet, $e(N)$ indep of it.

Schematically

always bigger than usoft

big momenta

$$d\sigma = \int d^4p_{\bar n}\, \delta^{(4)}(q - \overbrace{p_n - p_{\bar n}}^{\text{always bigger than usoft}})\quad |C(p_n^-, p_{\bar n}^+)|^2$$

$$\sum_{X_n, X_{\bar n}, X_u} \underbrace{\delta^{(4)}(p_n - \Sigma p_{X_n}^i)}_{\int d^4x\, e^{i x\cdot(p_n - \Sigma p_{X_n}^i)}}\ \underbrace{\delta^{(4)}(p_{\bar n} - \Sigma p_{X_{\bar n}}^i)}_{\int d^4y\, e^{i y\cdot(p_{\bar n} - \Sigma p_{\bar n}^i)}}\ \langle 0| J^{\mu\,\dagger}_{(0)} |X_n X_{\bar n} X_u\rangle\langle X_u X_n X_{\bar n}| J^\mu_{(0)} |0\rangle$$

recall $p_n^+ \sim p_{\bar n}^- \sim$ usoft momentum

$$\underset{p_n^-}{(\bar\xi_n W_n Y_n^\dagger)(y)}\,\gamma_\perp^\mu\,(Y_{\bar n} W_{\bar n}^\dagger \xi_{\bar n})(x) \quad \cdots$$

No Time for this

In lecture I defined what a jet is in terms of operators and discussed how it relates to our example of a jet in b->s gamma.