Another Method

- construct SCET$_{II}$ operators using SCET$_I$

i) Match QCD onto SCET$_I$ usoft $p_u^2 \sim \lambda^2$
collinear $p_c^2 \sim Q\lambda$

ii) Factorize usoft with field redefinition

iii) Match SCET$_I$ onto SCET$_{II}$ soft $p_s^2 \sim \lambda^2$
collin $p_c^2 \sim \lambda^2$

<u>Notes</u>
- this gives us a simple procedure to construct SCET$_{II}$ ops. (<u>even though they're non-local</u>)
- usoft fields in I are renamed soft for II

eg.
i) $J^I = (\bar{\xi}_n W)\,\Gamma\, h_v$
ii) $J^I = (\bar{\xi}_n^{(0)} W^{(0)})\,\Gamma\,(Y^\dagger h_v)$
iii) $J^{II} = (\bar{\xi}_n W)\,\Gamma\,(S^\dagger h_v)$ as before

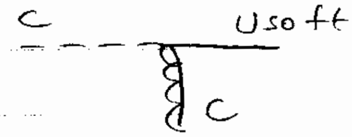
here all T-products in SCET$_I$ & SCET$_{II}$ match up, so matching was trivial

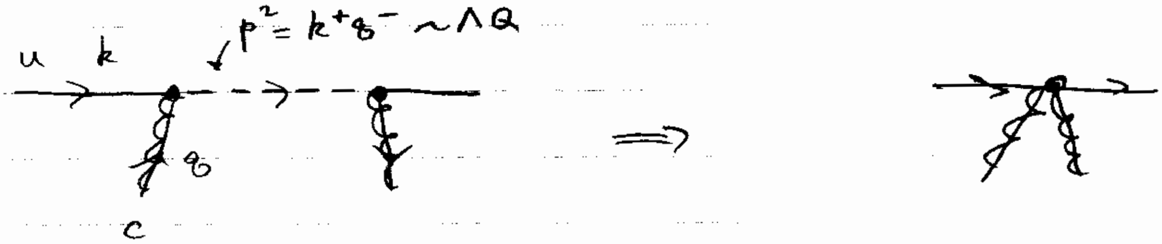
<u>"Thm"</u> • In cases where we have T-products in SCET$_I$ with ≥ 2 operators involving both collin & usoft fields, we can generate a non-trivial coefficient in SCET$_{II}$ (jet-function J)

T eg. $$\int dp_-\, dk_+\, J(p_-, k_+)\, \overbrace{(\bar{\xi} W)_{p_-}\, \Gamma\, (S^\dagger \xi_s)_{k^+}}^{p^2 \sim \lambda^2}$$

SCET$_I$ loops $p^2 \sim Q\lambda$

ind's allow k^+ dependence

eg. two operators 



when we lower offshellness of ext. collin fields
the intermediate line still has $p^2 \sim Q\Lambda$
and must really be integrated out

P.C. $\quad T^{I} \sim \lambda^{2K} \implies O^{II} \sim \eta^{K+E}$

where $\lambda^2 = \eta = \dfrac{\Lambda}{Q}$,

factor $E > 0$ from changing the scaling of ext. fields
eg. $\quad \xi_I \sim \lambda$
$\quad \xi_{II} \sim \eta = \lambda^2$

$\Rightarrow$ No mixed soft-collin $\mathcal{L}$ at leading order
- after field redefn no mixed $\mathcal{L}_I$ ops at LO

- mixed $\mathcal{L}_I^{(1)}$ gives $T\{\mathcal{L}_I^{(1)}, \mathcal{L}_I^{(1)}\} \sim \lambda^2$
matches onto $O_{II} \sim \eta$ or higher

SCET$_I$ $\quad \lambda^\delta$

$$\delta = 4 + 4u + \sum_k \underbrace{(k-4)\, V_k^c}_{\text{rest}} + \underbrace{(k-8)\, V_k^u}_{\text{pure usoft}}$$

$u = 1$ no c., else $u = 0$

$\underline{SCET_{II}}$

$$\delta = 4 + \sum_k (k-4)\left(V_k^c + V_k^s + V_k^{sc}\right) + L^{sc}$$

pure c — pure s — mixed — $p \sim (\eta^2, \eta, \eta)$ loops

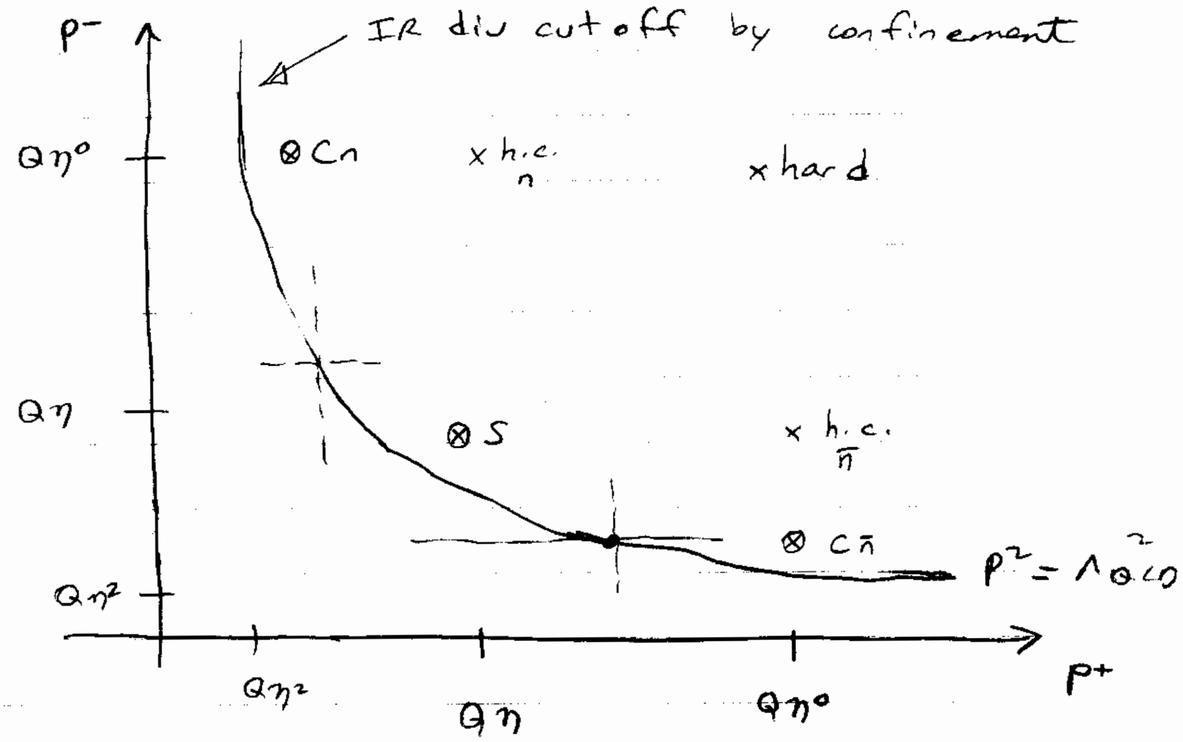
$$\delta = 5 - N_c - N_s + \sum_k (k-4)\left(V_k^s + V_k^c\right) + (k-3)\, V_k^{sc}$$

connected soft, collin components

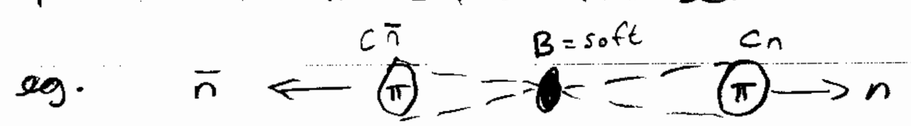
[in eg. $SCET_I$ $\quad \lambda^3\lambda \,\frac{1}{\lambda^2}\, \lambda^3\lambda \sim \lambda^{6-4} \sim \lambda^2$ $\Rightarrow$ $\left(\eta^{3/2}\eta\right)^2 \frac{1}{\eta} = \eta^{4-3} = \eta$]

or $\lambda * \lambda \sim \lambda^2$

$$\mathcal{L}_{SCET^{II}} = \mathcal{L}^{(0)}_{soft}[q_s, A_s] + \mathcal{L}^{(0)}_{collin\text{-}n}[q_n, A_n] + \mathcal{L}^{(0)}_{collin\text{-}\bar n}[q_{\bar n}, A_{\bar n}]$$



Non-pert d.o.f in different sectors $\qquad B \to \pi\pi$



Exclusive

eg. $\gamma^* \gamma \to \pi^0$ hard - collin factorization

[Breit frame: soft modes have no active role so this does not really probe difference between $SCET_I$ & $SCET_{II}$]

QCD has

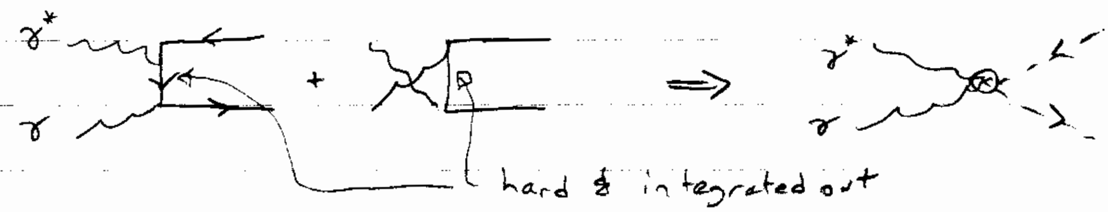
$$\langle \pi^0(p_\pi) | J_\mu(0) | \gamma(p_\gamma, \epsilon) \rangle = ie\, \epsilon^\nu \int d^4z\, e^{-ip_\gamma \cdot z} \langle \pi^0(p_\pi) | T\, J_\mu(0)\, J_\nu(z) | 0 \rangle$$
$$= -ie\, F_{\pi\gamma}(Q^2)\, \epsilon_{\mu\nu\rho\sigma}\, p_\pi^\nu \epsilon^\rho q^\sigma$$

e.m. current $J^\mu = \bar\Psi \hat{Q} \gamma^\mu \Psi$, $\hat{Q} = \frac{\tau_3}{2} + \frac{1}{6} = \begin{pmatrix} 2/3 & \\ & -1/3 \end{pmatrix}$

For $Q^2 \gg \Lambda^2$ $F_{\pi\gamma}$ simplifies (a la Brodsky-Lepage)

frame $q^\mu = \frac{Q}{2}(n^\mu - \bar n^\mu)$, $p_\gamma^\mu = E\, \bar n^\mu$

$p_\pi^\mu = p + p_\gamma = \frac{Q}{2} n^\mu + (E - \frac{Q}{2}) \bar n^\mu$



SCET Operator at Leading - order (for T-product) is

$$\mathcal{O} = \frac{i\, \epsilon^\perp_{\mu\nu}}{Q} \left[\bar\xi_{n,p} W \right] \Gamma\, C(\bar{P}, \bar{P}^\dagger, \mu) \left[W^\dagger \xi_{n,p'} \right]$$

order λ^2 (" twist -2 ")

- obeys current conservation
- dim analysis fixes $\frac{1}{Q}$ pre-factor for C dimless

- Charge Conj: $T\{J, J\}$ even so $\mathcal{O}$ even
  so $C(\mu, \bar{P}, \bar{P}^\dagger) = C(\mu, -\bar{P}^\dagger, -\bar{P})$

• flavor & spin structure $\qquad \Gamma = \underbrace{\bar{n}\!\!\!/\,\gamma_5}_{\text{for pion}} \; 3\sqrt{2}\; \underbrace{\hat{Q}^2}_{\text{2nd order e.m.}}$

• color singlet, purely collinear (again) so soft gluons decouple

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ SCET$_{II}$

equate $\quad \dfrac{Q^2}{2} F_{\pi\gamma} = \dfrac{i}{Q} \langle \pi^0 | (\bar{\xi} W)\, \Gamma\, C\, (W^\dagger \xi) | 0 \rangle$

write $\quad \bar{P}_\pm = \bar{P}^\dagger \pm \bar{P}$

now $\bar{P}_-$ gives total mom of $(\bar{\xi} W)\Gamma(W^\dagger \xi)$ operator

ie momentum of pion

$$\bar{P}_- = \bar{n}\cdot p_\pi = Q$$

$\longrightarrow$ total mom

$$F_{\pi\gamma}(Q^2) = \frac{2i}{Q^2} \int d\omega\, C(\omega,\mu)\, \langle \pi^0 | (\bar{\xi} W)\, \Gamma\, \delta(\omega - \bar{P}_+)(W^\dagger \xi) | 0 \rangle$$

<u>Non-perturbative Matrix Elt</u> $\qquad$ finite Wilson line $\left(P\exp ig\int_x^y ds \ldots\right)$

position space $\qquad$ Fourier Transform of $\bar{n}\cdot p$ label

$$\langle \pi^0(p) |\, \bar{\xi}_n(y)\, \frac{\bar{n}\!\!\!/\,\gamma_5 \tau^3}{\sqrt{2}}\, W(y,x)\, \xi_n(x) | 0 \rangle = -i f_\pi\, \bar{n}\cdot p \int_0^1 dz\, e^{i\bar{n}\cdot p\,(zy + (1-z)x)}\, \phi_\pi(\mu,z)$$

$$\int_0^1 dz\, \phi_\pi(z) = 1$$

momentum space

$$\langle \pi^0(p) | (\bar{\xi}_{n,p} W)\, \frac{\bar{n}\!\!\!/\,\gamma_5 \tau^2}{\sqrt{2}}\, \delta(\omega - \bar{P}_+)\, (W^\dagger \xi_{n,p}) | 0 \rangle$$

$$= -i f_\pi\, \bar{n}\cdot p \int_0^1 dz\, \delta(\omega - (2z-1)\,\bar{n}\cdot p)\, \phi_\pi(\mu,z)$$

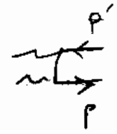
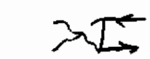
Plug it into $F_{\pi\gamma}(Q^2)$ and do integral over ω

$$F_{\pi\gamma}(Q^2) = \frac{2 f_\pi}{Q^2} \int_0^1 dz \; C\big((2z-1)Q, Q, \mu\big) \; \phi_\pi(z,\mu)$$

- ϕ_π is universal light-cone dist'n for pions
- C is process dependent (all order factorization in α_s)
- one-dim convolution again

Tree Level Matching

expand

$i\,($  $+$  $)$

$$= \frac{ie}{2}\, \epsilon_{\mu\nu\rho\beta}\, \epsilon^\nu\, \bar n^\rho n^\beta \left(\frac{\bar{n}\!\!\!/}{2}\gamma_5\right)\hat{Q}^2 \times \left(\frac{1}{\bar n\cdot p} - \frac{1}{\bar n \cdot p'}\right) + \ldots$$

non-pion terms

So $C = \dfrac{1}{6\sqrt2}\left(\dfrac{Q}{\bar p^+} - \dfrac{Q}{\bar p^-}\right)$

$$C(\omega = (2x-1)Q) = \frac{1}{6\sqrt2}\left(\frac{1}{x} + \frac{1}{1-x}\right)$$

match with $W=1$

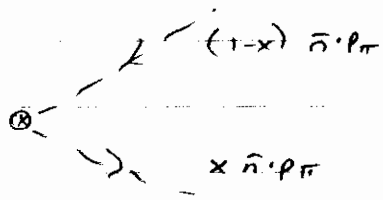
Charge Conj $+1$ for $|\pi^0\rangle$ gives $\phi_\pi(x) = \phi_\pi(1-x)$ (Hmwk.)

So only $\displaystyle\int_0^1 dx \, \frac{\phi_\pi(x,\mu)}{x}$ appears in our prediction

integrate over all x, much different than DIS $\delta(1-\xi/x) \Rightarrow f_{i/p}(x,\mu)$

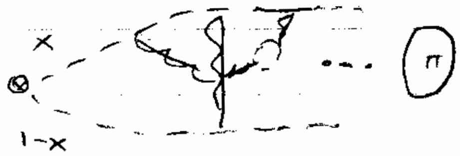
Interpretation:

Naively



mom fraction of quarks in pion

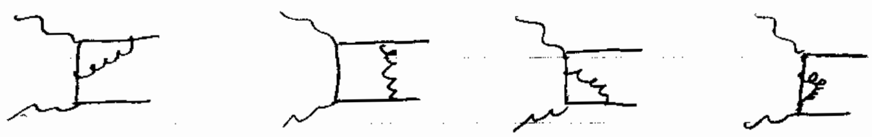
Really



mom. fractions at point where quarks are produced. Hadronization process changes "x" carried by valence quarks which is encoded in $\phi_\pi(x)$

Higher Order Matching

full



+ w.fn graphs

SCET

+ w.fn.

Difference will be IR finite, and gives C at one-loop

Another Exclusive Example

(hep-ph/0107002)

<u>$B \to D\pi$</u> $\qquad \underbrace{m_b, m_c, E_\pi}_{Q} \gg \Lambda_{QCD}$

QCD Operators at $\mu \simeq m_b$

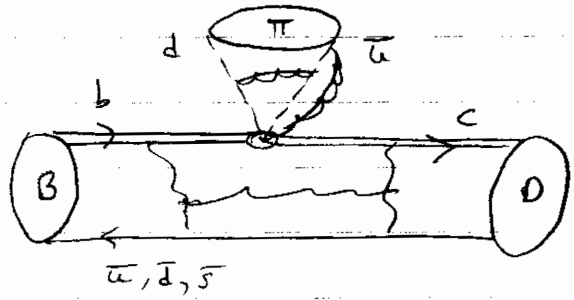
$$H_W = \frac{4G_F}{\sqrt{2}} V_{ud}^* V_{cb} \left[C_0^F O_0 + C_8^F O_8 \right]$$

$$P_L = \frac{1-\gamma_5}{2}$$

where
$$O_0 = [\bar c \gamma^\mu P_L b][\bar d \gamma_\mu P_L u]$$
$$O_8 = [\bar c \gamma^\mu P_L T^a b][\bar d \gamma_\mu P_L T^a u]$$

Want to Factorize $\langle D\pi | O_{0,8} | B \rangle$

ie show at LO



no gluons btwn B, D and quarks in pion

expect $B \to D$ form factor ξ Isgur-Wise

$\phi_\pi(x)$ distn for pion

$$\left.\begin{array}{lll} B, D & \text{soft} & p^2 \sim \Lambda^2 \\ \pi & \text{collinear} & p^2 \sim \Lambda^2 \end{array}\right\} SCET_{II}$$

use $SCET_I$ as intermediate step

① Match at $\mu^2 \simeq Q^2$

$$\left.\begin{array}{l} O_0 \\ O_8 \end{array}\right\} \Rightarrow \left\{ \begin{array}{l} Q_0^{1,5} = \left[\bar h_{v'}^{(c)} \Gamma_h^{1,5} h_v^{(b)}\right] \left[(\bar\xi_n^{(d)} W) \Gamma_\ell C_0(\bar P_+) W^\dagger \xi_n^{(u)} \right] \\ Q_8^{1,5} = \left[\;\text{"}\; T^A \;\text{"}\; \right] \left[\;\text{"}\quad \text{"}\; C_8(\bar P_+) T^a \;\text{"}\; \right] \end{array}\right.$$

usoft $SCET_I$ $\qquad$ collinear $p^2 \sim Q\Lambda$

$$\Gamma_h^{1,5} = \frac{\not n}{2}\{1, \gamma_5\}$$
$$\Gamma_\ell = \frac{\not{\bar n}}{4}(1-\gamma_5)$$

② Field redefinitions $\xi_{n,p} = Y\, \xi_{n,p}^{(0)}\,, \ldots$

in $Q_0^{1,5}$ get $\bar\xi_n^{(0)} W^{(0)}\, \cancel{Y^\dagger Y}\, W^{\dagger(0)} \xi_n^{(0)}$

$Q_8^{1,5}$ get $\bar\xi_n^{(0)} W^{(0)}\, Y^\dagger T^A Y\, W^{\dagger(0)} \xi_n^{(0)}$

$Y T^a Y^\dagger = \mathcal{Y}^{ba} T^b \qquad\qquad Y^\dagger T^a Y = \mathcal{Y}^{ab} T^b$

↑ adjoint Wilson line

$T^a \otimes Y^\dagger T^a Y = Y T^a Y^\dagger \otimes T^a$

↑ moves usoft Wilson lines next to hv fields

③ Match $SCET_I$ onto $SCET_{II}$ (trivial here again)

$Y \to S$

$\xi_n^{(0)} \to \xi_n$ in II etc.

$$Q_0^{1,5} = \left[\bar h_{v'}^{(c)} \Gamma_h h_v^{(b)}\right]\left[\bar\xi_n^{(d)} W \Gamma_e\, C_0(\bar P_+)\, W^\dagger \xi_{n,p}^{(u)}\right]$$
$$Q_8^{1,5} = \left[\bar h_{v'}^{(c)} \Gamma_h\, S T^a S^\dagger h_v^{(b)}\right]\left[\bar\xi_n^{(d)} W \Gamma_e\, C_8(\bar P_+)\, T^a W^\dagger \xi_{n,p}^{(u)}\right]$$

④ Take Matrix Element

$$\langle \pi_n^- |\, \bar\xi_n W\, \Gamma\, C_0(\bar P_+)\, W^\dagger \xi_n |0\rangle = \frac{i}{2} f_\pi E_\pi \int_0^1 dx\; C(2E_\pi(2x-1))\, \phi_\pi(x)$$
$$\langle D_{v'} |\, \bar h_{v'}\, \Gamma\, h_v\, | B\rangle = N'\; \xi(w_0,\mu)$$

↑ $w_0 = v\cdot v'$

B, D purely soft → no contractions with collinear fields

π " collinear → no " " soft fields

which is why it factors into two matrix element

F O8:

$$\langle D_{v'} |\, \underbrace{\bar h_{v'}\, Y T^a Y^\dagger\, h_v}\, |B_v\rangle = 0$$

color octet operator between color singlet states

Find Factorization Formula

$$\langle \pi\, D \,|\, H_W \,|\, B \rangle = i \underset{\substack{\uparrow \\ \text{prefactors}}}{N} \, \xi(w_0, \mu) \int_0^1 dx \; C(2E_\pi(2x-1), \mu)\, \phi_\pi(x, \mu) + O(\Lambda/Q)$$

- $\xi(w_0, \mu)$ is Isgur-Wise function at max. recoil
  $w_0 = \frac{m_B^2 - m_D^2}{2 m_B}$ (measured in $B \to D e \bar\nu$ recall)

- This applies to type-I (& III) decays
  $\bar B^0 \to D^+ \pi^-$ $\bar B^0 \to D^{*+} \pi^-$, $\bar B^0 \to D^+ \rho^-$, ...
  $B^- \to D^0 \pi^-$ $B^- \to D^{*0} \pi^-$ $B^- \to D^0 \rho^-$, ...
  predicts type-II decays are suppressed by Λ/Q
  $\bar B^0 \to D^0 \pi^0$, ... (we could derive fact. thm. for these too)

Another inclusive example - $B \to X_s \gamma$ [case where usoft modes matter]

Here we will need both usoft & collinear d.o.f. in $SCET_I$

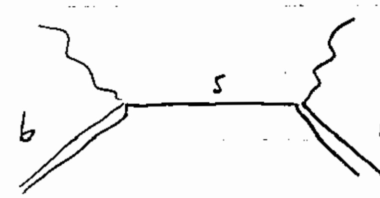
$$H_{eff} = \frac{-4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \, C_7 \, \mathcal{O}_7 \,, \qquad \mathcal{O}_7 = \frac{e}{16\pi^2} m_b \, \bar{s} \, \sigma^{\mu\nu} F_{\mu\nu} P_R \, b$$

photon $q^\mu = E_\gamma \bar{n}^\mu$

$$\frac{1}{\Gamma_0} \frac{d\Gamma}{dE_\gamma} = \frac{4E_\gamma}{m_b^3} \left(\frac{-1}{\pi}\right) \mathrm{Im}\, T$$

$$T = \frac{i}{m_B} \int d^4x \; e^{-iq\cdot x} \langle \bar{B} | T \, J_\mu^\dagger(x) \, J^\mu(0) | \bar{B} \rangle$$

$$J^\mu = \bar{s} \, i\sigma^{\mu\nu} q_\nu P_R \, b$$



looks like DIS

Consider endpoint region

$$m_B/2 - E_\gamma \lesssim \Lambda_{QCD}$$
$$p_X^2 \simeq m_B \Lambda$$

jet 

B-rest frame $\quad p_B = \frac{m_B}{2}(n^\mu + \bar{n}^\mu) = p_X + q$

$$p_X = \frac{m_B}{2} n^\mu + \frac{\bar{n}^\mu}{2} \underbrace{(m_B - 2E_\gamma)}_{\Lambda}$$

collinear

so quarks and gluons in X are <u>collinear</u> with $p_c^2 \sim m_B \Lambda$

B has <u>usoft</u> light d.o.f.

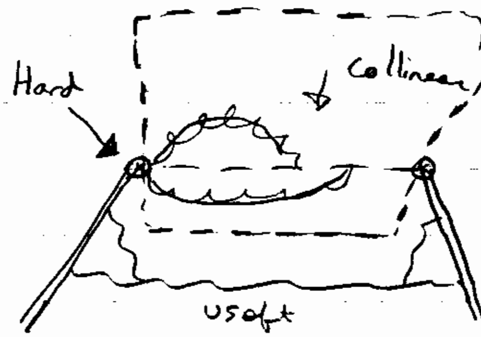
match onto LO SCET operator

$$J_\mu = -E_\gamma \; e^{i(\bar{P}\frac{n}{2} - m_b v)\cdot x} \; \bar{\xi} W \gamma_\mu^\perp P_L h_v \; C(\bar{P}^+, \mu)$$

↑ our heavy-to-light current from earlier $\equiv J_{eff}^\mu$

The coefficient $C(\bar{P}^+)$ has $\bar{P}^+ = m_b$ since this is total momentum of s-quark jet in $\bar{n}\cdot p_X$

Factor with Field redefn



$$J_{eff}^\mu = \bar{\xi}_n^{(0)} W^{(0)} \gamma_\mu^\perp P_L Y^\dagger h_v$$

$$T_{eff} = i \int d^4x \; e^{i(m_b\frac{\bar{n}}{2} - q)\cdot x} \langle \bar{B} | T \, J_{eff}^{\dagger\mu}(x) \, J_{eff\,\mu}(0) | \bar{B} \rangle$$

$$= i \int d^4x \; e^{i(\,)} \langle \bar{B} | T \, (\bar{h}_v Y)(x) \; (Y h_v)(0) \; | \bar{B} \rangle$$
$$* \; \langle 0 | T \, (W^{\dagger(0)} \xi^{(0)})(x) \; (\bar{\xi}^{(0)} W)(0) \; | 0 \rangle$$

spin & color indices & structures $\gamma_\mu^\perp P_L$ suppressed

$$= \frac{1}{2} \int d^4x \int đ^4k \; e^{i(m_b \frac{\bar{n}}{2} - q - k)\cdot x} \langle \bar{B} | T \, (\bar{h}_v Y)(x) \; (Y^\dagger h_v)(0) \, | \bar{B} \rangle$$
$$* \; J_P(k)$$

$$\langle 0 | T \, (W^\dagger \xi) \, (\bar{\xi} W) \, | 0 \rangle = i \int đ^4k \; e^{-ik\cdot x} \; J_P(k) \, \frac{\bar{n}\!\!\!/}{2}$$

↑ only depends on k^+! so do $k^-, k^\perp$ integrals

in T_{eff} we then get

$$S(\ell^+) = \frac{1}{2} \int \frac{dx^-}{4\pi} \; e^{-i/2\, \ell^+ x^-} \langle \bar{B} | T [\bar{h}_v Y)(\tfrac{n}{2} x^-) \;\; (Y^\dagger h_v)(0) | \bar{B} \rangle$$

$Y(\frac{n}{2}x^-, 0)$

$$= \frac{1}{2} \langle \bar{B}_v | \; \bar{h}_v \, \delta(in\cdot D - k^+) \, h_v | \bar{B}_v \rangle$$

match onto LO SCET operator

$$J_\mu = -E_\gamma \, e^{i(\bar{P}\frac{n}{2} - m_b v)\cdot x} \; \bar{\xi} W \gamma_\mu^\perp P_L h_v \; C(\bar{P}^+, \mu)$$

↑ our heavy-to-light current from earlier $\equiv J_{eff}^\mu$

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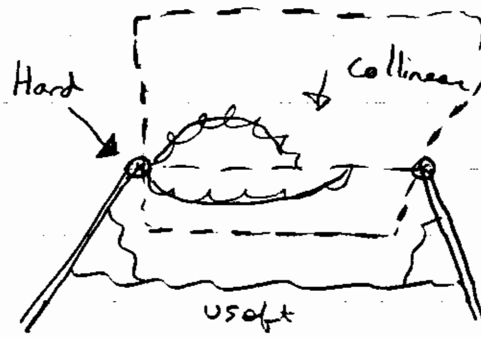
Factor with Field redefn



$$J_{eff}^\mu = \bar{\xi}_n^{(0)} W^{(0)} \gamma_\mu^\perp P_L Y^\dagger h_v$$

$$T_{eff} = i\int d^4x \; e^{i(m_b\frac{\bar{n}}{2} - q)\cdot x} \langle \bar{B} | T J_{eff}^{\dagger\mu}(x) J_{eff\,\mu}(0) | \bar{B}\rangle$$

factored

$$= i\int d^4x \; e^{i()} \langle \bar{B} | T (\bar{h}_v Y)(x) \; (Y h_v)(0) \; | \bar{B}\rangle$$
$$* \langle 0 | T (W^{\dagger(0)} \xi^{(0)})(x) \; (\bar{\xi}^{(0)} W)(0) \; | 0 \rangle$$

← spin & color indices & structures $\gamma_\mu^\perp P_L$ suppressed

$$= \frac{1}{2} \int d^4x \int đ^4k \; e^{i(m_b\frac{\bar{n}}{2} - q - k)\cdot x} \langle \bar{B} | T (\bar{h}_v Y)(x) \; (Y^\dagger h_v)(0) \; | \bar{B}\rangle$$
$$* \; J_P(k)$$

$$\langle 0 | T (W^\dagger \xi) \; (\bar{\xi} W) \; | 0 \rangle = i \int đ^4k \; e^{-ik\cdot x} \; J_P(k) \; \frac{\bar{n}\!\!\!/}{2}$$

↑ only depends on k^+! so do $k^-, k^\perp$ integrals

in T_{eff} we then get

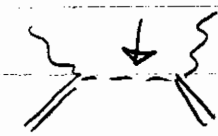
$$S(\ell^+) = \frac{1}{2} \int \frac{dx^-}{4\pi} \; e^{-i/2\, \ell^+ x^-} \langle \bar{B} | T [\bar{h}_v Y)(\tfrac{n}{2}x^-) \; (Y^\dagger h_v)(0) | \bar{B}\rangle$$

$Y(\frac{n}{2}x^-, 0)$

$$= \frac{1}{2} \langle \bar{B}_v | \; \bar{h}_v \, \delta(in\cdot D - k^+) \, h_v | \bar{B}_v \rangle$$

imaginary part is in jet function

Let $J(k^+) \doteq -\frac{1}{\pi} \, \mathrm{Im}\, J_p(k^+)$

(tree level $J(k^+) = \delta(k^+)$ from )

All order's factorization

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_\gamma} = \underset{\substack{\uparrow \\ p^2 \sim m_b^2}}{N\, C(m_b,\mu)} \int_{2E_\gamma - m_b}^{\bar{\Lambda}} d\ell^+ \underset{\substack{\uparrow \\ p^2 \sim \Lambda^2}}{S(\ell^+)} \underset{\substack{\uparrow \\ p^2 \sim m_b \Lambda}}{J(\ell^+ + m_b - 2E_\gamma)}$$

Shape function is <u>seen</u> in the data

Final example   two-jet production

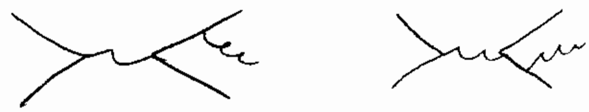
How do we define a jet?

Consider $e^+e^- \to q\bar{q}g$

$$q = p_1 + p_2 + p_3$$

$$2 = x_1 + x_2 + x_3$$

for $x_i = \dfrac{2 p_i \cdot q}{q^2}$

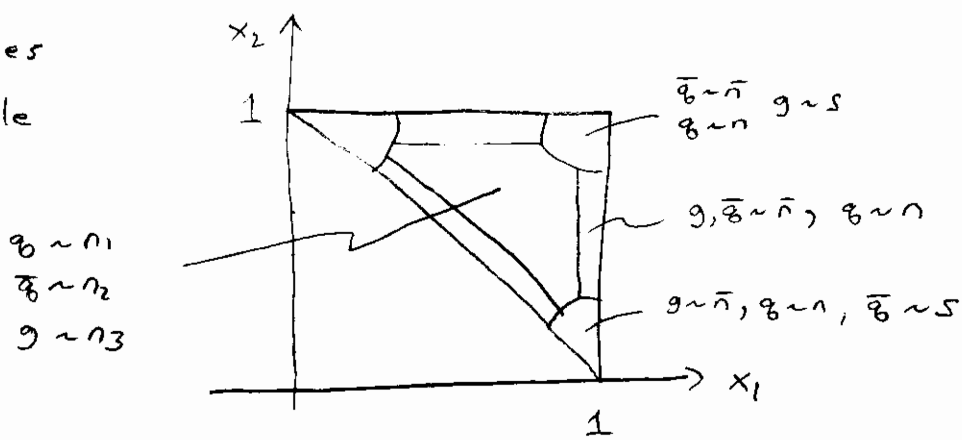


gives

$$\frac{1}{\sigma_0}\frac{d\sigma}{dx_1 dx_2} = \frac{C_F \alpha_s}{2\pi}\,\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Two jets along edges

Three jets in middle



Sterman-Weinberg Definition of 2-jets

if gluon has $p_3^0 < \epsilon Q$ or

if gluon has angle $\cos\theta_{13} > 1 - 2\delta^2$ or $\cos\theta_{23} > 1 - 2\delta^2$

$$\frac{d\sigma}{de} = \frac{1}{2Q^2}\sum_N \left|\langle N \,|\, J^\mu_{QCD}(0)\,|\,0\rangle\, L_\mu\right|^2 (2\pi)^4\,\delta^{(4)}(q - \Sigma p_N)\,\delta(e - e(N))$$

$\bar{\psi}\Gamma\psi$ leptons   event shape variable

eg. jet energy $E_J = \sum_{i \text{ in cone}} E_i$

eg. Thrust $T = \max_{\hat{t}} \sum_{i\in N} |\vec{p}_i \cdot \hat{t}| \Big/ \sum_{i \in N} |\vec{p}_i|$

Two-jets (SCET$_I$)

$$\bar{\psi}\Gamma\psi \to (\bar{\xi}_n W_n)\,\gamma_\perp^\mu\, C(\bar{P}, \bar{P}^+, \mu)\,(W_{\bar{n}}^+ \xi_{\bar{n}}) = J^\mu_{SCET}$$

Matching ensures only 2-jets

Decouple U-soft

$$\xi_n \to Y_n\, \xi_n^{(0)} \qquad Y_n = \bar{P}\exp\left(-ig\int_0^\infty ds\, n\cdot A_{us}\right)$$

$$\xi_{\bar{n}} \to Y_{\bar{n}}\, \xi_{\bar{n}}^{(0)}$$

$$J^\mu = (\bar{\xi}_n W_n Y_n^+)\,\gamma_\perp^\mu\, C\,(Y_{\bar{n}} W_{\bar{n}}^+ \xi_{\bar{n}})$$

State: $|N\rangle = |X_n X_{\bar{n}} X_u\rangle$ — all the soft particles, not observed

we will not bother to observe this jet, e(N) indep of it.

Schematically

always bigger than usoft; big momenta

$$d\sigma = \int d^4p_{\bar{n}}\, \delta^{(4)}(q - p_n - p_{\bar{n}})\, |C(p_n^-, p_{\bar{n}}^+)|^2$$

$$\sum_{X_n, X_{\bar{n}}, X_u} \underbrace{\delta^{(4)}(p_n - \Sigma p_{X_n}^i)}_{\int d^4x\, e^{ix\cdot(p_n - \Sigma p_{X_n}^i)}}\ \underbrace{\delta^{(4)}(p_{\bar{n}} - \Sigma p_{X_{\bar{n}}}^i)}_{\int d^4y\, e^{iy\cdot(p_{\bar{n}} - \Sigma p_{\bar{n}}^i)}}\ \langle 0|\, J^{\mu+}_{(0)}\, |X_n X_{\bar{n}} X_u\rangle\langle X_u X_n X_{\bar{n}}|\, J^\mu_{(0)}|0\rangle$$

recall $p_n^+ \sim p_{\bar{n}}^- \sim$ usoft momentum

$$\underbrace{(\bar{\xi}_n W_n Y_n^+)(y)}_{p_n^-}\ \gamma_\perp^\mu\ (Y_{\bar{n}} W_{\bar{n}}^+ \xi_{\bar{n}})(x) \quad \ldots$$

No Time for this

In lecture I defined what a jet is in terms of operators and discussed how it relates to our example of a jet in b->s gamma.