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**PROFESSOR:**

OK. Good. So let us start. So last time, we talked about finding the spacetime geometry produced by D3 brane. Such just based on symmetry, you can write down the metric as the following. So it's some function, and then a Minkowski metric-- a Minkowski metric for the 3 plus 1 direction. And then the path for the transverse direction, which you have rotation symmetry.

So you should think of h-- the D3 brane as a point in  $R^6$ , then there's a spherical symmetry in  $R^6$  around this point. And then there are other 3 plus 1 direction, which is this 3 plus 1 direction. OK?

So when you solve the Einstein equation-- when you find that this  $f, h$ , can be solved explicitly by the following form-- some other function of  $H$  by the same function, with  $H$  given by-- and  $R$  is the constant, which is given by the following form. So  $R$  essentially have the form of some constant, Newton constant times the tension of the D3 brane. And then you have  $N$  of them.

And then this can also be-- if you've written everything in terms of  $G_s$  and  $\alpha'$ , it can be written as  $4\pi g_s$  and  $\alpha'^2$ . OK? So this  $R$ -- so this is just like what you normally see in the Schwarzschild metric. It's now  $G$  times the mass. Except, because of the translation symmetry in this 3 plus 1 direction, so we have the tension appear here.

So  $GN$  is proportional to-- so Newton constant is proportional to  $g_s^2$ . And  $T_3$  is proportional to  $1/g_s$ . So this product is proportional to  $g_s$ . OK? And there's  $N$  of them. So there's  $N$  here. And on dimension of ground, this is dimension 4. So you have  $\alpha'^2$  here. OK? So again, you can write down everything on dimensional ground. And the only thing is just this  $4\pi$ . You have to be careful. OK?

So this is the full D3 brane metric. So this is the full D3 brane metric. So now let's understand a little bit of physics of this metric, which gave us a deeper understanding of the physics of D3 branes.

So first, let's consider various regime of  $r$ . OK? So let's consider, say-- first consider  $r$  goes to

infinity. So in  $r$  goes to infinity,  $Hr$  just become 1.  $H$  just become 1. OK? And then you just recover a 10-dimensional Minkowski metric. OK? So this is very easy to understand. You are infinity away from the D3 brane. Of course, you don't feel anything. And so you just recovered the 10-dimensional Minkowski spacetime. So here is 1. Here is 1. Just is a funny way to write down 10-dimensional Minkowski spacetime.

So this is just a flat  $R^6$  metric. OK? And now-- let's now consider  $r$  go to infinity is just only  $r$  much, much smaller than  $R$ -- than this capital  $R$ . Then this function-- so again, let me call this  $f$ . Let me call this  $H$ . So for example-- did I call them  $H$  or  $g$ ?  $H$ . Yeah.

So for example, the  $f$ -- it's a similar thing with  $g$ , with  $H$ . So the  $f$  will be 1 plus-- so something of order  $R$  to the power of 4 divided by  $r$  to the power of 4. Similar with the  $H$ .

So now you recognize that this  $1/r^4$  is precisely long range Coulomb potential in six dimension. OK? In  $R^6$ . Because only transverse direction matters. And in three spatial dimensions, if you have a point, then you have  $1/r$ . And in six dimensions, then you again-- you go down two powers, and then you get  $1/r^4$ .

So this behavior can simply be understood-- say, essentially, as you have some kind of Coulomb potential. Both due to the-- say the conserved-- both due to the charge of the brane and also due to the gravity. OK? So you have some long range potential.

So now the interesting thing-- things become more interesting is when  $r$  become of order  $R$ . When  $r$  become order  $R$ , then this  $Hr$  start to be significantly different from 1. OK? And then that tells you-- so this is the-- when  $r$  order 1, the deformation-- yeah, at this range, the deformation of spacetime metric from D3 branes becomes significant. OK?

So in other words, this capital  $R$ , this constant, essentially signals the distance from the brane where the gravitational effect becomes strong. OK? The gravitational effect of the D brane becomes strong.

And in particular, you can check yourself. You can calculate yourself the curvature radius now become of order  $R$ . Or the curvature itself-- yeah, let me just say the curvature itself becomes  $R$  to the minus 2. OK?

So essentially, the  $R$  gives you a measure of when the gravity becomes strong, or the scale of the curvature. So we solve this metric in the context of the super gravity-- in the context of low

energy effect of super gravity. So in order for the approximation to work, we said before, we wanted  $\alpha$  prime times the curvature to be much, much smaller than 1. So we said less time. So that means we want this to be much, much smaller than 1 so that the super gravity approximation can be valid.

So now, if you look at expression of the  $R$ , so this tells you that the  $g_N$  must be much, much greater than 1. OK? Yes.  $N$  must be much, much greater than 1. And also, in order to have the string loop crack into small, we also need  $g_s$  to be much, much smaller than 1. So we need  $g_N$  to be much, much greater than 1, but also  $g_s$  to be much, much smaller than 1. So in order for the-- super gravity approximation to work. OK?

So now let's consider-- so when we set up the problem, so this metric is written that  $r$  equal to 0 is the location of the brane. OK?  $r$  equal to 0 is the location of the brane. When we solve this problem,  $i$  equal to 0 is the presumed location of the brane. So now let's see what happens when we-- yeah, that's where you put the source. OK? That's where you put the source.

So now let's consider-- now let's ask the question, what happens to the metric if  $r$  goes to 0? If we try to approach the brane. OK? We try to approach brane. So this is very easy to do. So when  $r$  become much smaller than  $r$  to the capital  $R$ , when  $r$  goes to 0, then in this expression, this term will dominate. OK? We can forget about the 1. This term will dominate.

So in this case, the  $H$  will be given by  $R$  to the 4--  $R^4$ . OK? Then we just plug this into your original metric. OK? So you plug this back. So when you plug this back-- so  $H$  is equal to-- so  $H$  minus  $1/2$  then just become  $r$  squared divided by  $R$  squared. Then we still have  $dt$  squared plus  $dx$  squared. Then plus  $R$  squared divided by  $r$  squared. So it's positive  $1/2$ . Then  $dr$  squared plus  $r$  squared.  $d\omega^5$  squared. OK?

But now something miraculous happened. Something miraculous happened. So let me slightly write this metric in the slightly more explicit way. When I write it-- so let me just separate these two terms. OK? So there are two important thing happened. There are two important thing happened. Just to give you a-- yeah, let me just do it here. Yeah. Let me do it here. Maybe it's good enough. Maybe that would be good, just to be safe.

So previously, let's just draw the product like this. So let's call this  $R^6$ . OK? Just for my purpose-- yeah, let's consider this is the  $R^6$ . And yeah, forget about what's outside. Just imagine the-- yeah. There's no outside. Just the whole thing is  $R^6$ . OK? And when we start solving this problem, the  $D3$  brane--  $D3$  branes are a point in this  $R^6$ . And then, there's some

flux coming out of it. OK? There's some  $F_5$  flux coming out of it. So this is the picture.

**AUDIENCE:** The flux is normal to the  $F_6$ ?

**PROFESSOR:** Hm?

**AUDIENCE:** The flux is normal to  $R_6$ ?

**PROFESSOR:** No. The flux is normal to the  $S_5$  surrounding the point. And just like that. I'm just saying, think about the plane as a whole  $R_6$ . There's nothing outside the plane.

**AUDIENCE:** OK.

**PROFESSOR:** OK. I want to reserve the things outside the plane for other purpose. I just now think about  $R_6$  is the whole thing. This is your whole space. OK? So if you look at this situation, there are two things. First, there's a source at  $i$  equal to 0. You should see a data function which is a source of the flux and is a source of the mass. OK? There's some tension there.

And the other thing is that if you look at  $S_5$  around this point-- and this 5, when you go to the point, then  $S_5$  will become 0 sides. OK? So around this point. Yeah just in flat space, around the-- if you write in the polar coordinates-- so the sides of this  $S_5$  surrounding this point become 0 sides when you go to that point. OK? Yeah.

But now, if you look at this metric-- now if you look at this metric, now actually when you go to the-- when you go to the location of the brane, actually, you never reach  $i$  equal to 0. Actually, this  $S_5$  will now become a constant side. OK?

Also, if you look at this-- so there are two things happening. So first this is 1. This is 2. So the first thing is that  $S_5$  direction-- let me call  $S_5$ -- now has a constant radius given by this  $R$ . So this is the first thing. And the second thing is that, because of this, the  $R$  squared-- over  $R$  squared structure-- say if you write as the proper distance-- write it as a  $dl$  squared, so  $dl$  will be the proper distance. Then when you solve this, then you find  $l$  is essentially  $\log r$ , say, plus some constant.

Then as  $r$  goes to 0, the  $l$  goes to-- when  $r$  goes to 0, the  $l$  goes to minus infinity. That means that  $i$  equal to 0-- yeah, let me just write it here. This means  $i$  equal to 0-- see it's at infinite proper distance away.

So for all practical purposes, the brain has disappeared. OK? So the geometric picture is that

when you include this-- when you include the gravity of this source, so now I have to draw something called the embedding diagram just to use this extra direction to show the curvature. OK? So when you go-- when you approach this point, what do you find? You don't find the point. Actually, you find there's an infinite throat.

And so when you try to go approach this point, and this is the cross section of this throat, which I can now draw, is  $S^5$  with the constant radius. And then this throat has an infinite long distance. OK? So I and here and have an infinite distance. OK? So that's what's happening here. And yeah, the sides of this sphere, of course, is  $R$ , of radius  $R$ .

So in this picture-- so when you include, what we say, the back reaction of the D3 branes, essentially the D3 branes themselves have disappeared. The only thing left are this curved geometry and the fluxes. Yeah, they're still-- the fluxes won't go away. So you will still see the  $S^5$  flux-- the  $F_5$  flux through this  $S^5$ . OK? So there's still flux through this  $S^5$ . OK?

Under this metric-- so this is  $S^5$ , which completely decoupled from the rest of the parts. So this have a constant radius. So this  $S^5$  have decoupled from the rest part. So it's natural just to look at this part. And this is actually at  $AdS_5$  So let me call it 1. So 1 is just  $AdS_5$ . It's the metric of-- 5-dimensional anti-de Sitter spacetime times an  $S^5$ . OK? And the idea is 5 have the metric. So let me just copy here. Essentially just this part--  $r^2 - dt^2 + dx^2 + R^2 dr^2$ . OK? So this is the  $S^5$ .

So something magic has happened. When you try to reach the brane, then you find the brain is no longer there. And actually develop an infinite throat. And this throat, the metric is given by the  $S^5$  times  $S^5$ . OK? And of course, at infinity, you still see a flat space. And infinity, you still see a flat Minkowski space. OK?

So now, we have to take a little bit conceptual leap. So now, we have to take a little bit conceptual leap. Yes?

**AUDIENCE:** So we just basically bound the most general solution of  $g_r$  with the given symmetries given by the brane?

**PROFESSOR:** Yeah. With-- what do you mean by most general? We have specific D3 branes.

**AUDIENCE:** Well, so we fixed the symmetry given by the D3 brane, and then--

**PROFESSOR:** No, you also fixed the D3 brane charge and D3 brane tension.

**AUDIENCE:** OK.

**PROFESSOR:** That fix everything. Yeah. Yes?

**AUDIENCE:** So what is the nature of the source? Originally, in D3 brane, it's like electromagnetic--

**PROFESSOR:** Yeah.

**AUDIENCE:** --but now it's like a mass.

**PROFESSOR:** No, now the mass-- even the mass is gone, you only see a curved space time and with some fluxes.

**AUDIENCE:** But I mean, [INAUDIBLE].

**PROFESSOR:** Yeah. But the mass now is completely gone. It's, in some sense, sitting at an infinite distance away. You can never reach it. And for all practical purpose, if you leave this spacetime, you will never see the source. What you will see-- what you will observe is just a curved spacetime with some fluxes. And this R also, perfect solution of the Einstein equation. Yeah, because away from the source, it's also perfect solution of Einstein equations. Yeah, you have first?

**AUDIENCE:** Yeah, so I was just trying to understand. So the point is that-- so is this somehow protecting us from these issues that-- even electromagnetism or even quantum electromagnetic field has issues of like the divergence, of kind of going towards a point charge?

**PROFESSOR:** Right, right.

**AUDIENCE:** So is this somehow getting around that by making it go far away? Or not really?

**PROFESSOR:** Yeah. No, this is not getting around that. This is-- the solution presents itself. We did not dictate the solution. This is pure mathematics. And we're just trying to interpret these mathematics.

**AUDIENCE:** But it does achieve that-- I mean, we didn't--

**PROFESSOR:** Yeah. Yeah, achieved that. Right, right. Yeah, just the solution presented itself.

**AUDIENCE:** That's crazy. OK.

**PROFESSOR:** Yeah. Yes?

**AUDIENCE:** So all the electromagnetic charge, wouldn't this tear the space like this, but at least [INAUDIBLE]. What about other [INAUDIBLE]?

**PROFESSOR:** Yeah. Yeah. Yeah, something similar will happen, but the story is even more complicated. And so the D3 brane, the metric is simplest. Yeah, for reasons that I will mention later. So the reason I choose D3 brane is just because of the-- the metric is the simplest one. Other questions?

**AUDIENCE:** So the long-range Coulomb potential, that's from the charge of the brane?

**PROFESSOR:** Both.

**AUDIENCE:** OK. You can't really separate them I guess.

**PROFESSOR:** Yeah, in principal, you can separate. It'll depend on your probe. Yeah. But from the metric-- from the metric, actually, from the metric it's just from the assumption that come from the-- from metric, of course, you should just interpret that as the gravitational part. Yeah, this is just gravitational part.

**AUDIENCE:** I have a question. So there, you'll draw the flux as flowing out of the cylinder or along the cylinder? I guess it's along the cylinder.

**PROFESSOR:** No, no. Not along the cylinder. Along the cylinder is the  $r$  direction. The flux is in the  $S^5$ . Just think of a hollow  $S^2$ . Think of a hollow square. Think of a hollow  $S^2$ . Then  $S^2$  you can have magnetic flux.

**AUDIENCE:** Yes. And the flux is all over the radial direction.

**PROFESSOR:** And then there's no radial direction. If you only have  $S^2$ , then there's no radial direction. Just only have  $S^2$ .

**AUDIENCE:** Oh, OK.

**PROFESSOR:** And then you can have a flux on that  $S^2$ .

**AUDIENCE:** Yes.

**PROFESSOR:** Yeah. Something proportional to  $F_{\theta\phi}$ , say in the natural magnetism. Yeah, so this would be a magnetic flux in the-- yeah.

**AUDIENCE:** Oh, yes. But you write it like in R6. If you write it in R6, it is along the radius direction.

**PROFESSOR:** Yeah. Just forget about the-- in this picture it's back in the radial direction. But once we're in this picture, and then this becomes completely coupled.

**AUDIENCE:** Oh, oh. I see.

**PROFESSOR:** And then you just have separate S5.

**AUDIENCE:** I see, yeah.

**PROFESSOR:** Yeah, yeah. OK. So now, we have to do a little bit conceptual leap. It's not too big. It's that now we have two of the three braes. OK?

So in the description one, you just have a D brane. You really just have some Dirichlet boundary conditions in flat Minkowski 10-dimensional spacetime, and where open strings can end. Where open strings leave, say.

So in this picture, you just-- you specify some plane, some surface in the Minkowski 10-dimension. And so this is R3. Say 01, 3 if you include the time direction. So you just specify some plane in the spacetime, in the 10-dimensional spacetime where, on this plane, you can have open string citations, OK?

And then the system, of course, can also have closed strings. Then this close string can interact with the brane, can interact with the open strings, et cetera. And the only-- so everything is in the flat space. Everything is in the flat space. And you have open string, you have close string, open string. Even on the brane, the close string can interact with the brane, can-- a close string can interact with open string. In particular, for example, you can send the close string and absorb by the brane, and then decay into open string. It's possible. So this just goes one into say, yeah. Or you can have two open strings collide, then become a closed string, and then jump out of the brane. Yeah, you can have all kinds of this kind of interaction. The key point is that in this description, the only thing you did is Dirichlet boundary conditions specified on some component your worksheet fields, for the string. OK?

So this is-- but now, from here, we have also found the different, second description. Inside, we have spacetime metric. One which is some curved spacetime, then plus F5 flux on s5. OK? So here, there's no D branes anymore. We just have some curved spacetime with some fluxes. So here, the only thing leaves is a closed string.

So only closed strings. But those closed strings see a curved geometry. OK? So now I can include all dimensions. To contrast these two, let me draw this picture again. Now let's draw a 10-dimensional picture. So this is  $R, \text{Minkowski } 10$ . So this is  $\text{Minkowski } 10$ .

So when you now go to smaller radius  $r$ , and then you develop an infinite throat-- so this becomes  $\text{AdS } 5 \text{ times } S^5$ . OK? 5 down the throat. And here, only closed string can propagate. There's no D brane, but the closed string at infinity in  $\text{AdS}_5$ , so you can have closed strings. OK? So is this clear?

**AUDIENCE:** What would have closed strings? Because in low energy cases, no strings would only have effective field.

**PROFESSOR:** But you have-- yeah. But this is a geometry. This provides the geometry. In principle, the expectations would be closed string. Now you can do string theory in this geometry.

**AUDIENCE:** Oh. Oh, I see.

**PROFESSOR:** So now essentially--

**AUDIENCE:** [INAUDIBLE] as a target. New targets space--

**PROFESSOR:** Exactly.

**AUDIENCE:** Oh, and do new string theory--

**PROFESSOR:** Exactly. So we started with a D brane. And then we solved the geometry due to the D branes-- geometry due to that D branes. And then now you can quantize the string theory in that new geometry. Then that will describe how things interact with the D brane.

**AUDIENCE:** Oh, OK.

**PROFESSOR:** Yeah. But in this picture, there's no D brane anymore. There's only closed strings. OK? So we just have-- yeah. Good? Yes? Hm?

**AUDIENCE:** What was  $F_5$ ?

**PROFESSOR:**  $F_5$ , it's this  $F_5$  form of the type IIB string, which this D3 brane carry the charge. So this generalized charge I discussed last time. Other questions?

**AUDIENCE:** But in last class, we know the F5 is from the effective theory.

**PROFESSOR:** Yeah, yeah.

**AUDIENCE:** Yeah. And the effective theory is from the open string spectrum.

**PROFESSOR:** No. No, no. F5 is close string spectrum.

**AUDIENCE:** Oh, it's closed string spectrum. So why can't we apply string theory twice? The first time we get a flux.

**PROFESSOR:** Yeah.

**AUDIENCE:** And then we get to modify the targets with--

**PROFESSOR:** Yeah, right. Yeah. Yeah, you can. Yeah. String theory, you can work with any field of spacetime. You can think about propagating a string in the generic closed spacetime.

**AUDIENCE:** OK. Because it seems like the second application of string theory is like a higher order correction.

**PROFESSOR:** No. No this picture-- this picture, I'm just looking at-- forgot about D branes. I just look at some charge object. I look at some charge object in string theory. And that object have some charge, have some mass. Then I will call this metric. I will call this metric. And the string must be able to live in that-- must be able to live in that metric. And if I want to work out the property of the string, then the metric-- I quantize the string in this geometry. And quantum string geometry.

So this point, if you were just purely to view the D3 brane as some charged object in the string theory. You just solve this geometry. OK? Any other questions? So this is a very important point. It's a very important point. So now, the key is that consistency A must equal to B. So these two descriptions must be equivalent. OK?

So this can be generalization as the simple exercise we described earlier. That if you want to describe the interaction between the two D branes, you can do it in two equivalent way. One is to consider some loops of open strings, which only need to consider the flat space. Just consider loops of open strings.

Or you can see that the graviton exchange between the two D branes. And the graviton

exchange between the two D brane is to view that the other D brane have already curved your spacetime and that the other D brane live in that curved spacetime.

So this are a precise generalization of that equivalent. OK? Now we have two descriptions of the D branes which are completely different-- this one involving flash space and open and closed strings, and this one only involving closed string but in some curved spacetime. OK?

And in principle, both descriptions can in principle be valid for all value of alpha prime and g string. OK? So the two parameters in the string theory is alpha prime and-- by all alpha prime, just consider for all energies. Yeah. OK?

**AUDIENCE:** Why?

**PROFESSOR:** Hm?

**AUDIENCE:** It seems like they're constrained, no?

**PROFESSOR:** Yeah. Yeah. So first, in this picture, there's no restriction what is the alpha prime or g string. And you just impose the Dirichlet boundary condition. And when you have a g string, you just need to include the higher loop diagrams, et cetera. So when we work out this geometry, so this statement is more try to apply to b. In b, when we work out this geometry, and also that AdS5, we were working using the approximation that the alpha prime should be small and the g string should be small.

**AUDIENCE:** Yes.

**PROFESSOR:** But once we have worked out that solution in these regime, in principle, if you're technically powerful enough, you should be able to extend this to general values of g string and alpha prime. It may not be precisely this metric, but with some kind of formation of that from other prime corrections and g string corrections. But you should be able to do that. Yeah.

Yeah, so that's why I said in principle.

**AUDIENCE:** Got it.

**PROFESSOR:** OK? So we must have these two descriptions. And these can be valid for all range of the parameters you can be interested in. OK? And so this is a very surprising statement, this A equal to B. But there's not much to do about it, because of this is a complicated thing. And this is also a complicated thing. This is even more complicated, doing string theory on some

curved spacetime. In particular-- say if you consider g string is a very small, et cetera, then maybe you can do something. But then it's not that interesting, because on both sides, you do some perturbation theory, et cetera. Anyway.

But the remarkable thing-- which Maldacena realized in 1997 is that you can actually take a limit of this statement. And when you take a limit of this statement, then the statement becomes very interesting. Then you can actually do a lot. OK? And the limit is the low energy limit.

So if you consider the low energy limit, then you derive what is now called AdS/CFT. We will now explain how to do that. Yes?

**AUDIENCE:** It seems like this correspondence between A and B only exists in the low energy limit. Because here, the closed string is traveling in the base spacetime. That spacetime is only the low energy corrections from the D brane, not the higher corrections. Because we only solved the low energy effective theory.

**PROFESSOR:** No, that's-- this is a good question, but that's what I was trying to say earlier. So we find this metric by working in this regime. But now if I want to go away from this regime-- if I want to go away from this regime, then if I want to go away from this regime, then I have to take into account other prime corrections, et cetera. And this is a complicated process. But in principle, I can do that. In principle, I can do that. So that geometry may get modified, the schedule. But, in principle, I can do.

So now it turns out, if you consider lower energy limit of this A equal to B, then you can actually derive a very powerful statement. OK?

**AUDIENCE:** Is this actually the lower energy limit?

**PROFESSOR:** No, this itself is not low energy limit. This is a classical gravity limit.

**AUDIENCE:** What's the difference?

**PROFESSOR:** Hm?

**AUDIENCE:** What's the difference?

**PROFESSOR:** I will explain that. No, this is the classical gravity limit.

**AUDIENCE:** OK.

**PROFESSOR:** And at least the low energy limit does not involve this. Yeah, I will explain what this low energy limit. Yeah. Any other questions? Good? OK.

So now let's look at low energy limit. In particular, here, I don't do any restriction on the  $g$  string. So  $g$  string can, in principle, be strong here. I don't impose any constraint on the string happening now. I only consider low energy. OK?

So low energy limit-- because when I fix the energy,  $E$ -- some process I'm interested in, some typical energy I'm interested in. Then I want to take the  $\alpha'$  go to 0 limit. OK? Or you fix-- equivalent, you can fix  $\alpha'$ . Then you take  $E$  equal to 0. OK? Because only the dimension is product of the matters. And I can describe either way. OK?

So physicality, what is more convenient-- mathematically, normally what is more convenient is that we think we fix  $E$ , and then take  $\alpha'$  goes to 0. But you can think either way. OK? So the low energy limit is essentially just the limit of  $\alpha'$  times  $e$  squared goes to 0. That's the dimension statement. OK? So this is the low energy limit.

I only consider those physical process with  $E$  that would satisfy this region. OK? Good? And I'm doing no restriction on the  $g$ s.

So now, let's consider  $A$ . What is the low energy limit for  $A$ ? For  $A$ , we know-- so  $A$ , we have two factors. We have open string factor, which is live on the  $D$  brane. Then we also have close string factor. OK? Which live outside the brane. For the open string factor, we just recover. So we discussed before-- on the  $D$  brane, when you go to low energy, what do you get?

**AUDIENCE:** Super Yang-Mills?

**PROFESSOR:** Yes. You just recover your Mills series. It just recover super Yang-Mills series. And in this case-- let me give a special name for the  $D3$  brane. It turns out you get to the  $n$  equals 4 super Yang-Mills. Just the specific super Yang-Mills theory in four dimension. And then we sketch group  $un$ . OK? Because we have  $D$  branes.

And in particular, as we said before, the Yang-Mills coupling is related to the  $g$ s. OK? It's proportional to the  $g$ s. And so the  $D3$  brane give rise to a 3 plus 1 dimensional field theory. In the 3 plus 1 dimensional field theory, the Yang-Mills coupling is dimensionless. So there's no

other dimensional parameter here. So there's only-- so there's only numerical factor,  $4\pi$ , which you can work out by explicit calculations.

So this is key, because if there's a scale here, then the story is more complicated, because you have to take  $\alpha$  goes to 0, et cetera. OK? So the key is that this is a dimensionless coupling. So they have a simple relation.

So in open string, when you take low energy limit, you just get a Yang-Mills series and some finite coupling. OK? Which is determined by the string coupling constant. But in this series, there's also closed string factor. In a closed string factor, again, when you take a low energy limit, you get the massless modes. You get graviton, dilaton, et cetera. But only massless modes. OK?

But now, there's a very important point. The coupling between the closed and open string are mediated through gravitational effect. OK? So the coupling between the massless-- so only massless modes are left. So the coupling between the massless closed and open string, or closed string themselves, those interactions just-- it's always controlled by  $G$  Newton. This we know, because this is a low energy limit. Graviton, it's controlled by  $G$  Newton. It's gravitational by gravitational interactions.

Another key is that  $g$  Newton, in contrast to this Yang-Mills coupling, which is dimensionless,  $G$  Newton is dimensional. And the  $g$  Newton is proportional to  $g_s^2 \alpha'$  to the power of 4. So that means, in the low energy limit, then the dimension is coupling-- then the dimensionless parameter is  $g$  to the power, say, 8. Because this have dimension 8. So these is dimension-- this one goes to 0. OK?

So that actually tells you, in a low energy limit, the interaction between open and closed string will decouple. OK? This is our familiar fact. That electron-- when we talk about electron, you don't have to talk about gravity. Because their energy is so low, their gravity is so weak. OK? And the gravity only become strong if their energy become big. OK? It's just because of this. In 3 plus 1, we just  $G_N$  times  $E$  squared. But here we are in 10 dimensions,  $E$  to the power 8 doesn't matter.

So now that means, in the  $E$  goes to 0 limit, what we find is we find that interacting  $n$  equals 4 super Yang-Mills theory plus three gravitons plus other massless modes. But they all become free, because the interaction essentially switches off. Any interaction in this factor, they switch off. In particular, there's no interaction between these two factors. OK?

So this is what we get on the A picture.

**AUDIENCE:** So the top really is actually  $G_{\text{Newton}} E^8$ . Times  $E^8$ .

**PROFESSOR:** No, I'm just saying this is a dimensionless combination of them.

**AUDIENCE:** It's the actual coupling?

**PROFESSOR:** No, the actual coupling can be  $1/2$ . It can be square root of these. Controlled by the  $G_{\text{Newton}}$ . And the key thing, it's controlled by  $G_{\text{Newton}}$ . And  $G_{\text{Newton}}$  is a dimensional parameter. That's just the key. Yes?

**AUDIENCE:** Why can't they direct through gauge boson?

**PROFESSOR:** No the gauge bosons interact with themselves. The interaction with the gauge boson with graviton is gravitational. Have to go through  $G_{\text{Newton}}$ . The gauge boson will interact with themselves. It's captured by here. So anything involving gravity, involve dimensional couplings, then they will switch off in the low energy limit. Just like we don't have to worry about the gravitational interaction of an electron when we talk about hydrogen atom.

**AUDIENCE:** But we know that from [INAUDIBLE], but how do we see from here that--

**PROFESSOR:** No, no. It's just from here. This is just the same. You can calculate it carefully. You can calculate-- you can calculate it precisely, work out the sketching amplitude between the photon and the graviton, take the low energy limit, and you find that sketching goes to 0. You can do that calculation explicitly and take the limit. But this argument tells you that you don't have to do the calculation. It will always work.

OK. So now let's look at Picture B, when we take the low energy limit. And now let's go back to this metric. Now let's go back to this metric. Actually, it's handy. So now, let's look at picture B, the low energy limit. Let's look at this metric.

So this is a curved space time. So when we take low energy limit, we have to be careful. Because one important thing you always keep in mind when you have a curve of spacetime, the time depends on where you sit. You have to always specify a reference time. Because time is no longer the absolute thing. OK? Because at the local time, et cetera, you have to specify your time frame. OK?

And so we have to ask what is this  $E$ ? In picture A, translate into this picture. In Picture A, everything is in Minkowski spacetime. And that  $E$  is essentially  $E$  for the Minkowski spacetime at the infinity in the picture B. OK? So in other words, the  $E$ -- in other words-- so here, let me just say the curve of spacetime must be careful to specify what energy we are interested in, what energy we are talking about. OK?

So the energy depends on now your choice of time, or it depends on your observer. OK? But in order to compare a with b, we have to translate what is  $E$  in A. This translated into B is defined with respect to the  $t$ -- this  $t$  there. OK? In other words, it's the time at  $i$  equal to infinity. OK? So that energy scale is defined to that specific time.

But now, when we talk about the physics happening in this spacetime-- for example, we're talking about the string theory string  $g$  excitations of the mass of the string, et cetera, you have to refer to the local proper time at whatever place-- at whatever place you are talking about your physical process. OK?

So that means-- oh, actually today we are going to 6:30. Yeah, right. So suppose you are at some location,  $r$ . Then the local proper time, which I will call  $\tau$ -- the local proper time, the  $\tau$ , then it's related to the time at infinity by this factor. OK? By the factor. This-- the local observed equals this, the  $\tau$  squared. So they're related to this by this factor.

So now you can easily convert-- then the local proper energy is related to the energy we are talking about here, by  $H$  to a power of  $1/4$   $E$ . OK? OK? So now, it's very important, when we take the low energy limit, we take this  $E$  small. But due to the gravitational redshift effect, this  $E$   $\tau$  does not have to be small. OK? And so this is the thing will be important here.

So when  $r$  is much, much greater than  $r$ -- so when we are sitting around here, then  $H$  is 1. OK? Then the limit  $E$  square  $\alpha$  prime goes to 0 really just tells you that all massive closed string decouple. All massive closed string decouple. OK?

In particular, though, the interaction among closed string themselves. By the same argument, they also become weaker and weaker when you go to low energy limit. So essentially, again, they become free gravitons. OK? But now let's look at the other regime and  $r$  around here. OK?

So this is for that. But now, let's consider  $r$  smaller than  $R$ , capital  $R$ . And then, in this regime, we go to this  $AdS_5$  times  $S^5$ . Then  $H$  becomes  $R$  to the power of 4,  $r^4$ . And now this

statement, and the  $E^2$ ,  $\alpha'$ , go to 0, translates. Now you have a redshift factor. And  $\tau^2$ . If I translate it in  $\tau$ , the local energy scale.

In particular, now, let's remember this  $r^4$  is given by that. So this can be written  $e^{\tau^2} r^4$ . So  $4\pi g_s N$  goes to 0. So now, important thing happened. Important thing just happened. Then you find this  $\alpha'$  actually canceled with each other.  $\alpha'$  here and  $\alpha'$  there, they cancel each other. And here, there's nothing in  $\alpha'$  left. OK?

So when you take this limit-- when you take this limit and it's-- then this quantity goes to 0. And now,  $E\tau$  can actually be anything. As far as  $r$  sufficiently small. OK?

So that means for any  $E\tau$ , you can even have huge energy-- local proper energy. You can satisfy the low energy limit if that mode live at  $r$  very close to 0. Means that they live on these throat deep enough. So this is a very simple. Physically, this is very simple. Because of this curved space effect, because of  $H$ , this curved space effect, there's a huge amount of red shift between here and there-- between the local proper time here and there. Very big energy here. Viewed from here, can become very, very small.

In particular, you go deeper into this throat, the redshift factor is bigger. So normally, the higher the energy, if you go deep enough, will always appear within the low energy limit here. OK? OK? So that means, when you sit deep enough, that actually anything is allowed, including a massive string mode. OK? Including massive string mode.

So now, we remember, when  $r$  goes to 0, essentially you get this AdS5 times S5. So in  $r$  equal to 0, we just get this geometry, which is AdS5 times S5. So now what we find is that in a low energy limit, in the B, we get free gravitons at  $r$  equal to infinity. And then plus full string theory AdS5 times S5. OK? Full string theory AdS5 times S5.

And again, the two factors decouple, because when you take the low energy limit, then those strings in mode have to live very far down in the throat. And then they have low overlap with the graviton and live at infinity. OK? And they have low overlap with graviton and live at infinity. Yes?

**AUDIENCE:**

So I'm confused about the way that you wrote that. So you say three-- so there's three gravitons that are at infinity. But also full string theory. But doesn't full string theory include gravitons?

**PROFESSOR:** No, no, no. Yeah. This is-- there's some different graviton ideas, 5 times S5. Full string theory in AdS5 times S5. Those are Minkowski space graviton. This is Minkowski space graviton. So this is a Minkowski space graviton. I think we should leave here. The natures are very different. OK? Yes?

**AUDIENCE:** Is full string theory only for closed strings?

**PROFESSOR:** Only closed string. There's no open string in this picture. They're only closed strings. OK?

**AUDIENCE:** [INAUDIBLE]?

**PROFESSOR:** Sorry?

**AUDIENCE:** So [INAUDIBLE]?

**PROFESSOR:** No, no. There's no artificial separation. That's what I just explained. No free graviton live here. And the string theory live infinite down the throat. Throat, in the low energy limit. There are infinite-- there are essentially as large proper distance as you want to separate them. Depends how low energy you want to go.

**AUDIENCE:** But there is something living at finite  $r$ , right? There is always [INAUDIBLE].

**PROFESSOR:** Yeah. I'm just saying, when you take the low energy limit, then the separation then becomes infinite.

**AUDIENCE:** I just don't understand why [INAUDIBLE].

**PROFESSOR:** No, no. They have nothing to do with each other.

**AUDIENCE:** Oh.

**PROFESSOR:** Geographically, one is in this side of the galaxy. The other is living on the other side of galaxy. So their wave functions have low overlap with each other.

**AUDIENCE:** But with the [INAUDIBLE] clearly something between there.

**PROFESSOR:** Yeah. I'm just saying when you take the low energy limit, then the only mode which can survive, other than the massless mode, which lives around here-- just in the finite energy mode that cannot live in this place. Yeah?

**AUDIENCE:** Because the string theory, or [INAUDIBLE], flux as well?

**PROFESSOR:** Yeah. Yeah, yeah. Yeah, with the flux, yeah. Yeah. OK? Then these two factors should decouple. So now we have derived-- see from this  $A$  equal to  $B$ , then we take the low energy limit. So here, we have  $n$  equals 4 super Yang-Mills series plus three gravitons.

And here, you have type II B string and AdS5 times S5 plus free graviton. OK? And then the free graviton part is the same. Then you conclude-- and they're just free. And they interact, in part, of both sides, then you just equate them. You find the input for super Yang-Mills series with  $uN$ , then it's equal to Type II B strong AdS 5 times S5.

So this was a surprising statement, saying  $A$  equal to  $B$ . But  $A$  and  $B$  are both, say, complicated objects. Both are complicated objects. But now, when we take the low energy limit, and then you reach something very surprising, because on this side, there's no gravity. On this side, there's no gravity. But this side is a full string theory. So now you equate some field theory with the full string theory. OK? Yes?

**AUDIENCE:** So the super Yang-Mills is living just on the D3 brane.

**PROFESSOR:** Yeah.

**AUDIENCE:** OK.

**PROFESSOR:** Any questions regarding this?

**AUDIENCE:** So for example, [INAUDIBLE].

**PROFESSOR:** Yeah. Yeah. But that's always valid proper distance away. They always valid proper distance. So the key is when you talk about the general mode, you can-- anything you talk about is finite distance away. If we are taking a limit here, then this throat is going to infinity, you are living infinite down the throat.

**AUDIENCE:** So anything finite is free.

**PROFESSOR:** Yeah, anything-- when you talk about the finite energy, than the story is always complicated. And then all this geometry will matter. And the thing only becomes simple-- you really take the  $E$  goes to 0 limit. It's the same thing here.

Same thing, in this picture, when we say you go to the  $n$  equals 4 super Yang-Mills series, you

always have to take that limit--  $E$  goes to 0 limit. Then you completely decouple the other things. Because any tiny, tiny non-zero  $E$ , there's always some gravity left. So you really, really have to take that energy, go to 0 limit. And then you rescale the energy to be finite. You understand what I mean? You always have to take that singular limit. It's just like, when you take the-- [INAUDIBLE] you have to take the infinite volume limit, and then things become simple. And here, it's the same thing. Yeah. Whenever you have some finite energy, there's always some coupling between them.

Good? So do you want to have a break?

**AUDIENCE:** Yeah.

**PROFESSOR:** This is like a physicist's guess that, somehow, these two must be the same. OK? These two must be the same. So this is a surprising statement. Because on the one side, it's a field theory. And then on the other side, it's a full gravity theory.

**AUDIENCE:** I have a question. So you said that, in principle, you can generalize the solution for any metric in the full string theory for any regime.

**PROFESSOR:** Yeah.

**AUDIENCE:** But in that case, maybe you will probably not get a geometry as same as AdS5.

**PROFESSOR:** Yeah. That's a very good question. That's a very good question. It turns out, indeed, logically, you may not. But actually, AdS 5 times S5 is a highly-- I said I will explain. It's a very highly symmetric space. It turns out actually it is exact string theory. At least we believe it's exact string city geometry.

And then when you include all those corrections, this will not change. Yes?

**AUDIENCE:** So with this in mind, that means that people don't-- they only know this heuristically.

**PROFESSOR:** Yeah.

**AUDIENCE:** So there's no actual proof of this, besides the fact that everyone believes it works.

**PROFESSOR:** Yeah. It depends on what-- the label. What do you mean--

**AUDIENCE:** Both.

**PROFESSOR:** So the physicist, the way it works is that 1 works, 2 works-- yeah,  $n$  equal to 1 works,  $n$  equal to 2 works,  $n$  equal to 3 works,  $n$  equal to 4 works, then we say it works. And under this, I'm way past that. So maybe this works to  $n$  to the 10,000, something like that. But it's not a proof.

**AUDIENCE:** Sure.

**PROFESSOR:** Yeah. But it worked to the  $n$  to the 10,000. Something like that.

**AUDIENCE:** OK.

**PROFESSOR:** OK. Good? So now let's talk about this duality. So this is normally called AdS/CFT duality. OK? So to understand this duality, let's first understand the object in this relation. OK? So let me just say a little bit about the object in this relation. First, let me say a little bit about anti-de Sitter space, since that may not be something you have worked before. OK?

So one way to write the metric is this metric. So this is AdS<sub>5</sub>. So this is AdS<sub>5</sub>. So this is AdS<sub>5</sub> metric. So you can actually write it as AdS-- yeah, let me just copy it down. So you can generalize to any number of dimensions. You can generalize any number of dimensions. If we make-- if you make this part into a 3-dimensional Minkowski spacetime, OK? And then this would be the metric of AdS-- say  $D$  plus 1 dimension. So this is a  $D$  plus 1 dimension, Minkowski. Then you have another  $r$  direction. So let we call this  $m$ -- so now let me relabel this to be 1. OK?

So  $r$  can be considered as a curvature radius of this spacetime. So this  $r$  is a constant. And this radius,  $r$ , goes from 0 to infinity. OK?  $r$  goes to 0 from infinity. So roughly, a geometric picture here is that-- so you can see that you have half space. OK? You have a half space,  $r$  from 0 to infinity. And that each value of  $r$ -- at each value of  $r$ , there's a Minkowski spacetime.

There's a  $d$  dimension Minkowski spacetime. So if you fix  $r$ , then you just get some  $d$  dimension Minkowski spacetime. And then with some pre-factor to define the scale of that Minkowski spacetime. OK? So just like this, you have a half line--  $r$  from 0 to infinity. Then at each point of this half line, it's  $d$  dimensional Minkowski spacetime. So that's essentially what this does.

So the key thing is that the Minkowski time is what we normally called warped. It's that the scale factor of this Minkowski spacetime depends on where you sit in the radial direction. OK? So this  $r$ , we normally call radial direction. It depends on where you sit in the radial direction.

So when your  $r$  goes to infinity, the scale size becomes infinite. So this, we'd normally call it to be the boundary. So this we call to be the boundary. So  $r$  equals infinity, we call to be the boundary of the spacetime. So this is similar to, say, when we say-- when we write-- a Euclidean metric in this form, when  $r$  goes to infinity, the sphere size become infinite, and then this is essentially the spacetime boundary, roughly. And so this can be similarly understandable. OK?

So this idea is spacetime-- idea is spacetime of constant curvature-- constant negative curvature. OK? In fact, this is the most symmetric spacetime of [INAUDIBLE] constant curvature. So you can easily check. You can easily check this satisfy the Einstein equation with the [INAUDIBLE] constant. So you can easily check if you satisfied the Einstein equation. So now, because I'm using this  $r$  to denote this radius, this curvature radius. So now I will quote my curvature [INAUDIBLE] always using the script now.

So also,  $MN$  now refers to all directions in this AdS. So you can easily check if you satisfy this Einstein equation with some cross-module constant. And you can just compute the-- it's easy to compute the curvature of this guy. So let me just write down the answer.

So yeah, at first you can-- yeah, you can compute the curvature of this guy. You can just find the answer. You find the curvature, the rich scalar is given by minus  $D$  times  $D$  plus 1  $R$  squared. OK? So this is a rich scalar. This  $R$  is the same as this  $R$ .

And then similarity, you find the cross-module constant. Then you can find here. It's given by minus  $1/2 D$  minus 1 divided by  $R$  squared. Sorry, here it should be minus 2. OK?

And I said that this is the most symmetric homogeneous negative curvature spacetime. So this is reflected in that actually you can write the Riemann tensor-- the Riemann tensor of this spacetime purely in terms of this metric itself. Without using derivative-- OK?

So this is essentially the definition of the most symmetric [INAUDIBLE] active curvature spacetime, is that the Riemann tensor can be written solely in terms of metric itself. OK?

So these are just some facts of this space. So often, we use a lot of coordinates. Sometimes it's often convenient to use-- instead of using this  $r$ , we will introduce another coordinate. We introduce a  $z$ . You go to  $R$  squared divided by-- capital  $R$  squared divided by small  $r$ . Essentially, it's just the  $1$  over  $R$ . And because the capital  $R$  goes from 0 to infinity, then it's more  $z$  also going from 0 to infinity.

But now, the boundary-- but now it's equal to 0 is the boundary, rather than equal to infinity. And then the metric can be written in the slightly different way in terms of this  $z$  coordinate.

We can in this form. OK? You can see there is a  $d$ . So again, here, you just change the label  $r$  by  $z$ . And now it's equal to 0, is the boundary. And they're going to interior, because one increases  $D$ . So  $D$  equals 1 into a different direction. OK?

**AUDIENCE:** [INAUDIBLE]?

**PROFESSOR:** Yeah. That's a very good question. I'm waiting for some of you to ask. So this is the physicist's metric which we described. Yeah, this is the physicist's metric. So yeah, let me just say it this way. So you have an infinite throat.

So going to low energy limit, we should just go very much down in the throat. OK? Because once you decouple those decreased freedom, then you can just rescale your energy scale to whatever you want. You go to finite energy, et cetera. And that's why you go into AdS5.

Yeah. Once you decouple those things, then you are living AdS. Then you are living in the AdS, and then you can talk about the energy scale, or any radius as you want. It just becomes relative to itself. You see what I mean?

**AUDIENCE:** That's kind of normalization?

**PROFESSOR:** No. This is not normalization. This is just-- normally, when we take a low energy limit, the reason we take a low energy limit is we want to decouple those massive modes. As once we decouple those massive modes, then we consider the finite energy. And we include the finite energy. Because we consider finite energy relative to low-- we still consider a finite range of energy. It's the same thing as that. Is this clear to everybody? This is a very important point. Yeah. Good. Yeah?

**AUDIENCE:** What was the question?

**PROFESSOR:** The question is that previously, in order to decouple those high energy modes, when we take a low energy limit, we have to take  $r$  equal to 0. And why here we worry about  $r$  goes to infinity--  $r$  equals infinity here? OK? Why are we worried about  $r$  equals infinity. It seems like this is going outside our previous regime.

But the answer is that, once we decouple those high energy modes, then you can just extend

to whatever energy range you want. Because you have served the purpose of discover-- decouple those-- yeah, because those things already went to infinity. You don't have to worry about them. Is it clear?

Just like in the Yang-Mills theory, we said we take the low energy limit. We decouple those massive modes. But eventually, in the Yang-Mills theory, we would consider very high energy processing in the Yang-Mills theory. OK? But those things we decouple have even higher energy scale. OK? Good, good, good.

So we call this-- so this is 1 and this is 2. So in fact, actually 1 and 2, they're not covering the full anti-de Sitter spacetime. So actually, they only cover a part of anti-de Sitter spacetime. So they only cover, say, what is so-called Poincare patch, a patch of de-Sitter spacetime, anti-de Sitter spacetime.

So now let me just mention a little bit what is normally called a global AdS. OK? So the AdS, or global AdS-- so let me first write down this sentence. So 1, 2-- 1 over 2 only covers a part of full AdS. OK? So the global AdS, the so-called global AdS, can be defined-- can be described as a hyperboloid in the Lorentz spacetime. OK? Hyperboloid in Minkowski--  $d + 2$  Minkowski spacetime of signature  $2, d$ . OK? So to make this precise, say, for example, if you go to the [INAUDIBLE]-- for example, the book by Hawking and Ellis called *The Large Scale Structure of Spacetime*, the way they define the anti-de Sitter as follows.

So you consider  $d + 2$  dimension of Minkowski spacetime with signature  $2, d$ . So you consider a Minkowski spacetime like this. OK? So now you have  $d$ , positive signature, and the two negative signature. So  $x$  starts from minus 1 index to 0, which gives you the negative signature. And then you have-- so this is Minkowski spacetime of the  $2, d$  signature.

So now you can see that the following hyper surface in that spacetime. OK? So this is a hyperboloid in that spacetime. And this defines your AdS. OK? So this surface defines AdS. This surface defines AdS.

So, of course, this is a very different way to parametrize AdS. Then either this metric or that metric. And you can actually parametrize AdS in many, many different ways. So now let me just mention a few ways to-- a few choices of different coordinates to parametrize this hyperboloid. OK?

So the first is-- one is called the Poincare coordinates. It's this one. So first is what we call the

Poincare coordinates. So in this Poincare coordinates, you introduce  $r$ . So  $r$  there is given by  $x$  minus 1 plus  $xd$ . So you take the 1 over the negative signature and the 1 over the positive signature. You take  $x$  minus  $1xd$ , add them together. So this is like some kind of [INAUDIBLE]. And then you write  $x^\mu$ . So there, we always write  $x^\mu$  equal to  $tx$ . OK? So this is a three-dimensional Minkowski coordinate. You add  $x^\mu$  equal to  $r$  times capital  $\mu$  divided by  $r$ . OK?

So in this hyperboloid, you make this definition. And then you restrict to the region of  $r$  greater than 0. Then you recover. So 1 corresponds to  $r$  greater than 0 part of this metric, of this spacetime. And you can easily check yourself. Just make this definition-- this is the exercise you should do, which I don't have time to do here. Just plug this in. Plug this into that metric. Plug this parametrization into that metric, and then you find you will recover 1.

Of course, 1 is only restricted to  $r$  equal to 0 to infinity. So you only restrict to  $r$  greater than 0 part of this. And if you just introduce this in [INAUDIBLE], of course  $r$  can be smaller than 0. OK? So these are Poincare coordinates. So once you go to that metric, you then go to this metric. It's the same.

Now also let me describe something called the global coordinates. So this is-- if I notice the following structure. So this has the form of the difference of two squares. So I can write-- so let's write  $x_i$  to 1 to  $d$   $x_i$  squared to be some function,  $r$  squared and  $r$  squared-- times  $r$  squared. So this  $r$  have nothing to do with that  $r$  now. We are doing a different set of coordinates. I'm just using the same notation for convenience.

So let's introduce a new coordinate,  $r$ , which is just the sum of all these squared. And then I introduce-- then from this equation, the  $x_1$  minus  $x_0$  squared should be equal to  $r$  squared plus 1 times  $r$  squared. OK? So now I can-- so this is a circle. So for fixed  $r$ , this is a circle. For fixed  $r$ , this is a  $d$  minus 1 dimensional sphere. OK? So I can introduce the  $d$  minus 1 spherical coordinates on this and then introduce a circular coordinate on this. So if I do that-- so let me just call the circular coordinate on here  $\tau$ , the polar coordinate in the  $x$  minus  $y$ . Polar coordinate in the  $x$  minus 1 and the  $x_0$  plane. So let me call this  $\tau$ . And let me call [INAUDIBLE] coordinate here  $\omega$   $d$  minus 1. OK? Then you plug this into here. Then you find the following metric.

So you find the following metric. You find  $ds$  squared equal to  $r$  squared minus 1 plus  $r$  squared  $d\tau$  squared plus the  $r$  squared plus 1 plus  $r$  squared plus  $r$  squared  $d\omega$   $d$  minus 1 squared. OK?

So remember, tau is the polar coordinate in this  $x$  minus 1 and  $x_0$  plane. And the omega d minus 1 is the polar coordinate in the  $x_1$  to  $x_d$  OK? And then you get this. Again, it's an exercise you should go through yourself. Yes?

**AUDIENCE:** So I have a question. So I'm just trying to visualize it. So there's these-- at least the way it works in [INAUDIBLE]. So you have these two sheets which are disjoint from each other. So how is it that this enables us to go smoothly between them? It seems that there's-- so maybe I'm confused about what you did.

**PROFESSOR:** What do you mean?

**AUDIENCE:** So we're trying to describe this object, which is two disjoint components.

**PROFESSOR:** We only look at one component.

**AUDIENCE:** So what's the point of global coordinates? Is it just a different--

**PROFESSOR:** No, no, no. No, that only covers part of the one component.

**AUDIENCE:** Oh, I see. OK. Yes.

**PROFESSOR:** Yeah. OK, good. So you look at here-- yeah. Again,  $r$  goes from 0 to infinity. But this  $r$  is very different from this  $r$  now. OK?

So in this definition, tau is a polar coordinate. So in this definition, tau will go from 0 to  $2\pi$ . OK? But once you write it in this form, nothing depends on tau. So you can extend tau to any range you want. So once we write this form, we take tau goes to minus infinity to plus infinity. So this is [INAUDIBLE], rather than from 0 to  $2\pi$ , we take tau from minus infinity to plus infinity.

So sometimes, this is also called extended AdS. Sometimes people don't just bother to do that. You just extend it automatically. OK? So strictly speaking, this, as a definition of AdS, only corresponding to the part of tau from 0 to  $2\pi$ . OK? And when you extend tau from minus infinity to plus infinity, you actually have an infinite copy of this.

OK. So now, with one more thing, we can-- so this coordinate is nice. Now it covers everything. Covers everything, but even actually a little bit more. Because now you extend the tau to minus infinity. We can visualize this metrically. It might be better by doing the following. Allow the transformation. So now let's take-- yeah, before I do that, let me say a few words about--

let me just do this first.

Let's take tau again. Take  $r$  equal to tangent rho. Rho is some other coordinate. And  $r$  from 0 to infinity. Then rho would be from 0 to  $2\pi$ ,  $\pi$  over 2. OK? 0 to  $\pi$  over 2. And you park this into here with a little bit tiny-- so after two seconds, you find the following metric. OK?

So this is easy to see. Because  $1 + [\text{INAUDIBLE}] \rho^2$  is just  $1 / \cos^2$ , et cetera. So the reason this metric is nice, because other than this conformal factor, this thing we can easily rationalize. So this is just a cylinder. So this is just a cylinder. Let me draw it here.

So this is a cylinder. OK? So this direction can be considered-- the radial direction is the rho direction. And this direction is the tau direction. And this direction is the  $\omega d - 1$ , or the polar coordinate direction. I'm only drawing one of them. OK?

So essentially, this I have a structure of a cylinder. And at the center here, I have rho equal to 0. And here, I have rho equal to  $\pi$  over 2. OK? Took rho from 0 to  $\pi$  over 2. OK? And when you go to  $\pi$  over 2, its quotient factor goes to 0. So the overall scale blows up. But here is your boundary.

So  $\pi$  over 2-- yeah, so  $\pi$  over 2 is the boundary. OK? Is this clear? So  $\pi$  over 2 is the boundary. Now the boundary map. So the boundary of the cylinder-- so the boundary now is sitting at rho equal to  $\pi$  over 2, has the topology of  $r \times s d - 1$ . OK?

So the boundary itself, it's-- so the boundary itself, it's a cylinder. The whole space is a solid cylinder. It's a field cylinder. OK? But the scale factor blows up when you approach the boundary, because of this prefactor. And the  $r$  is the time and the  $sd - 1$  is the-- now, to visualize this space a little bit better, let's look at the cross section. So imagine you put a plane in this space. OK?

So now let me call this circular direction, it's called theta. And then now let's consider the cross section across this solid cylinder. So then we just have a plane. OK? A plane. I should not have-- let's consider this cross section. OK? In the center of this cylinder. Cut half. Cylinder cut in half.

So for example, in this notation, so this will be rho equal to 0. So here is rho equal to  $\pi$  over 2. And I introduced this angular coordinate. So that equals 1 into theta equal to 0. And here will

be, again,  $\rho$  equal to  $\pi/2$  of  $\theta$  equal to  $\pi$ . OK?

And again, this is the  $\tau$  direction. Is it clear what I'm doing? I cut the cylinder in half, and that's that cross section. So now, because of the structure of this times a conformal factor overall, a scale factor. So the propagation of a light ray does not depend on this overall factor. The propagation of that ray can be seen from here. OK?

So now, let's consider radial light ray, which only goes into the  $\rho$  direction. There's no in this angular direction. Let's consider radial light ray. Then from that picture, it's very clear-- let's say this is  $\tau$  equal to 0. Then it actually takes only  $\pi/2$  for light ray to reach the boundary. OK? Because  $\rho$  only goes from  $\pi/2$ , and light ray always travel in 45 degrees. So this is just like-- if you look at the [INAUDIBLE] plus the  $\rho$  squared, this is Minkowski. And so light ray travel 45 degree.

And then you reach the boundary. And then take log of  $\pi/2$ . Yeah, so this is  $\pi/2$ . So this is  $\tau$  equal to 0. And then this would be  $\tau$  equal to  $\pi/2$ . Then suppose that you put some reflecting wall here. And by the time  $\tau$  equal to  $\pi$ , it has bounced back. OK?

And now, if you go another round, then at  $\tau$  equal to  $2\pi$ , then precisely go back to the origin. Then go through a period. OK? And that actually precisely is  $\tau$  equal to  $2\pi$ , which would define the angular coordinates. If you follow that, then that equals 1 into precisely one period of the light ray traveling. OK?

But the key thing is that the light ray will travel  $\pi/2$  to the boundary. And now you can check. If you look at-- not the light ray. So light ray travels along the light magnitude axis. But if you look at the time magnitude axis, which is travelled by massive particle, then you can show that the massive particle never reach the boundary. We'll go into a motion like this. OK? So this is a massive particle.

So the massive particle will never reach the boundary. It will always be reflected back intuitively, from the gravity potential. OK? So you will show this fact yourself. And your P-set show that actually the massive particle will never reach the boundary.

So in a sense, so if you look at this metric, at  $\rho$  equal to 0, at  $\rho$  equal to 0, this is essentially like a flat metric. Because at  $\rho$  equal to 0, this is just a sphere. So this becomes 1. This is a sphere. Yeah. Yeah.  $\rho$  equal to 0, this is just like Euclidian space. Because the sine of  $\rho$  squared become  $\rho$  squared. OK? So this just becomes like a Euclidian space in

polar coordinates. And then you have a time. And then this scale factor becomes 1.

So at  $\rho$  equal to 0, just like you have ordinary Minkowski spacetime. And then the curvature comes in when you go to finite  $\rho$ , and then those factors become important. And then you can show-- yeah, just because of that, you can show that the massive particle will never actually travel to the boundary. OK? Always reflected back.

In some sense, you can think that the massive particle is always pulled back by the gravitational potential. OK? Pulled by gravitational potential. So from a different point of view, it's that another way to say it is a familiar statement, which people often make and often can give you great intuition, physical intuition in many physical problems, is that AdS is like a confining box. Just like a box. So things cannot escape from it. OK? OK?

And in particular, the typical curvature radius is  $r$ . So you can roughly imagine this-- because in the center, it's like a flat space. So it's more like a box of size  $r$ . OK? You can heuristically think of it as a box of size  $r$ .

**AUDIENCE:** Yeah. Does it have finite or infinite volume?

**PROFESSOR:** Hm?

**AUDIENCE:** Does it have finite or infinite volume?

**PROFESSOR:** Yeah, it has infinite volume.

**AUDIENCE:** It's infinite volume. OK.

**PROFESSOR:** Yeah. But because it's confining, most of the particles will only see the sides  $r$ .

**AUDIENCE:** So one other question. So is it correct to say that it's like an antiharmonic trap? So if I'm in the space, and I throw a ball that it's going to come and hit me back in the head or something like that? It seems like--

**PROFESSOR:** Yeah. Right, right. Yeah.

**AUDIENCE:** It'll bounce back?

**PROFESSOR:** Yeah. Yeah, actually, our own friend, Frank Wilczek, in the '80s, have tried to use these ideas, this feature, to say, maybe this actually describes confinement. Because if you put [INAUDIBLE] there, they're confined. Anyway. Yeah, but keep this heuristic picture in mind.

Because many physical problems actually, if you think in terms of this picture, become very intuitive. OK? OK.

And often-- so you say this is an exception for the light ray. But often, we impose the reflecting boundary coming on the light ray. And from the light ray, this is also like a box. And you go to the boundary, and you bounce back, et cetera. OK?

So now let me again just say it. But you should go back to check yourself. Now, by looking at the relation between here-- so here is a specific relation between  $r$  and  $x$ . And here is a specific relation between this  $x$  and this  $\tau$  and this  $r$ . You can find the relation between this Poincare coordinate and this global coordinate. OK? Use a little bit effort. And then you can show yourself-- let me just draw the picture.

Again, using the same picture. You can show that the Poincare patch cover a wedge-- say in this cross section. The Poincare patch cover a cross section of this global AdS. OK?

So here is  $\tau$  equal to minus  $\pi$ . And here is  $\tau$  equal to plus  $\pi$ . And here, say  $\tau$  equal to 0. And in this Poincare patch-- and then this  $\theta$  become  $r$  equal to 0 and  $t$  equal to minus infinity. And this line become  $r$  equal to 0 and  $t$  plus infinity. OK? Plus infinity.

So the time essentially goes around like this. Time goes around like this. And this line equals  $t$  equal to 0. OK? So  $t$  equal to 0 and  $\tau$  equal to 0 coincides, and then goes to minus infinity rotating into that. And the constant  $r$  surface-- so this is a constant  $r$  surface in the Poincare coordinates-- goes like this. So the time goes up. OK? And you can check yourself just by working out the relation between those coordinates through the  $x$ .

**AUDIENCE:** You mean this is wrapped on the cylinder?

**PROFESSOR:** No, no, no. No, I'm just saying--

**AUDIENCE:** This picture?

**PROFESSOR:** Just within this cross section, I can look at this cross section, I see how this translates into Poincare coordinates. And that's what's happening. A Poincare coordinate covers this part. OK? It cover this part.

I can also draw a more fancy picture here. Then roughly, what you can show is that in this general picture, the Poincare patch covers something like this. OK? And the region between

these two lines. So this is a spacial infinity of the-- in this Poincare patch, the boundary is Minkowski spacetime.

So you see-- so the Poincare patch cover like that. And then this is a spatial infinity over the Minkowski spacetime. This is times infinity and times a path in the Minkowski spacetime. OK? So I urge use to do it yourself. Because this kind of thing, you really have to work through yourself to have intuitive feeling for it. But this the picture. Once you work hard yourself, you compare with my picture, and then you will understand. OK?

So I will stop here today. So in some sense-- yeah, let me just say one more sentence. This extended ideas contain infinite copies of this Poincare patch. OK?