HONG LIU: So last time we talked about what corresponding to external fundamental quark in [INAUDIBLE] series. So you can see there [? m ?] plus 1 is [? rebrain. ?] Then you separate one of them. You separate one of them. And then you can see there one string connect between, then you can see that the open string connect between this [? m ?] and this 1.

And such object from the point of view, because this end have a unique index. And this end have indices. So this will transform on the fundamental [? representation ?] of this SU(N). Because when you're reshuffling the indices of this [? n, ?] and this will transform as a vector, because there's only one end [INAUDIBLE] on it. So this is the field series picture.

Then in the gravity picture, when you go to AdS, so when you take a [INAUDIBLE] in the gravity side, then this M-Brain just disappeared. And then you have a boundary, say i equal to infinity, or z equal to 0.

And then you find in a suitable [INAUDIBLE] that the single brain still remains there. And then this string become a string just an ending on this brain, but extend all the way to i equal to 0. Because all the other end, this [? rebrain ?] not disappeared, but replaced by the geometry.

So now what you find in the gravity side, this just become a single D-Brain, single [? D ?] [? rebrain ?] now in the AdS 5 Now in the AdS 5. And there's one point on the S5. So depending on the direction. So this will be extended in some direction, in the transverse direction, to this [? rebrain. ?] And so that direction determines the point on the S5. So we're relying on the single point on the S5. And all this special dimension is parallel to the boundary direction.

So this, is the geometry clear? Good.

And now if we want, you can see the external quark which have infinite mass, then you want to put this brain to the boundary, because the mass of this quark is controlled by the length. So you want this length go to infinite. Then you put it to the boundary. And, yeah. Essentially.

So this picture tells you that the external quark in [INAUDIBLE] theory. Then you just mapped to a string ending on the boundary. A string ending on the boundary. Do you have any questions regarding this picture?

And the mass of the string, as we discussed before, either from this picture the mass of the string would be the length divided by 2 pi alpha prime. And that this mass getting [INAUDIBLE] to this picture. And then you can check it as a self-consistency check that the straight string extend all the way say from some radius, from some location r, to the i equal to 0, and precisely have this mass.

So now let me make a note. So as we said before, if you say an ordinary [? K ?] [? series ?], if you take an external quark, say moving in the fundamental [? representation. ?] And when you parallel transport that quark say along some loop, then you get the Wilson loop. Then you get the Wilson loop. But in this case, the parallel transport of this m of such an external quark actually gives something which is slightly different from the standard Wilson loop.

So at the standard Wilson loop is the following. It's you take the trace, [? your ?] path order, and the exponential-- say i, A mu, say supposed S parameterize the length along the path. And then you have A mu. And then you have dx mu ds. So this is the standard Wilson loop.

But in this case, actually you have something more in the exponential for the following reason. So in the [INAUDIBLE] series, you'll not only have a gate field. You'll also have a scalar field. As we [? stated ?] before, the scalar field parameterizes the external fluctuation in the transverse direction of the D-Brains.

So now if you imagine you have a string ending on such D-Brains, just by mechanical thinking, really mechanical thinking, if you have something ending on the D-Brain, and if this thing have a mass, and it got to pull this D-Brain in the direction. Because this thing have a [? finite ?] [? tension. ?] So you want to pull the D-Brain.

So that means that the string actually will deform as a shape of the D-Brain around at the ending point. So what that means is that the string should couple to the scalar field on the D-Brain. Such open string should couple to the scalar field on the D-Brain. The scalar field paramertize the transfer direction and the transfer motion. And the string should pull the D-Brain. So this open string got to couple to the scalar fields.

So this is a rough [? infusion. ?] But you can work this out explicitly. You just take this series. So this is SU(N) [INAUDIBLE] series, N + 1, your [INAUDIBLE] series. And then you break to

SU (N) times u(1).

So this open string goes bounding to the off diagonal degree of freedom, of going into this breaking between the N and the 1, and corresponding to explicit field in this series. And you can just work out the [? dynamically ?] of the field. You can usually check that actually this coupled to the scalar field. So I will not go into detail there. But you can work out more explicitly.

So the bottom line is that such objects, we are not only coupled to the gate field, but also coupled to the scalar fields. And if you work it out explicit;y, it will couple to the scalar field in the following way. So a limit you not [? define. ?] So you have six scalar fields. Let me denote them as a vector.

Yeah. So now let me explain this notation. So this n essentially the direction of the string. So n is a unit vector on S 5, which is a location of that D-Brain, which is enabled the location of this D-Brain is 5, because it's in some transverse direction. And the 5 is just the six scalar fields, which I writed the vector, six fields of [INAUDIBLE] series. And then the dot x squared is just the modulus of this guy. You have any questions on this?

So now, because this is the Wilson loop corresponding to such an external quark. And actually, when you consider the string dynamics on the gravity side, that we will relate to the expectation value, for example, of this Wilson loop. Not the standard one.

So now let's consider such a quark traverses some path C, some loop C on the boundary. So I will make two statements. And from these two statements, then we will be able to guess what the Wilson loop will be corresponding to on the gravity sides.

So the first statement is that this quark, lies under the endpoint of a string in AdS. So that means the C-- so the loop formed by this quark trajectory, there must be the boundary of the corresponding string [? worksheet. ?] So let me call this [? worksheet ?] sigma. So in other words, the C must be corresponding to the boundary of the sigma.

So the second statement is that the expectation value of the Wilson loop-- so this can see considered as the partition function of this quark system.

So you can think about the expectation value of the Wilson loop as the following. So you take the quark, and write down the path integral for this quark. Write down the path of the quark along this loop, and then just integrate all of the dynamics of the quark. And then what you remaining would be just the Wilson loop. So you can just imagine this Wilson loop corresponding to the partition function of this quark system.

So now from our experience with the duality, now we can just make a guess. We can just make a guess, because we always identify one partition function with the other partition function. They should be the same object.

So now we should be able to identify the expectation value of the Wilson loop with the string [? Cartesian ?] function of this particular string, which ending at the boundary, the string partition function, and the host whose [? worksheet ?] ending at the boundary as at C. So this is just, you can just make a natural guess. You have any questions regarding this guess? Good.

So now let me may say a few words about this guy. So suppose an [INAUDIBLE] just parameterize say M sigma alpha to parameterize the [? worksheet. ?] Say sigma alpha is the worksheet coordinates, and [? XM ?] are the coordinate on the AdS. And then this parameterized a string moving AdS.

And then the string partition function, you can just write it as DX. You integrate of all possible X. And then the string action with the boundary condition that the worksheet of the boundary should ending on C. So this essentially is the right-hand side.

So according to your convenience, you can either choose the string action say to be a lambigoto or [? Poliakoff ?] as you want. A Poliakoff. For example, let me just write down the remind you what is the [? lambigoto. ?] For the [? lambigoto, ?] you just 2 pi alpha prime delta h. And the h is the induced the matrix on the [? worksheet. ?] H is induced matrix on the [? worksheet. ?] Yeah. So in principle we have a precise mathematical formulation we can work with. Then you can check whether this proposal makes sense, et cetera.

Of course, again, this object is very hard to calculate. Yeah, I should say you may also include the fermions, depending on superstring [INAUDIBLE] you may have other things. But this is the most basic object, So again, this object in principle very difficult to calculate, but then become simple if we can see the g string goes to the 0 limit, and the alpha prime goes to the 0 limit.

So in the g string goes to 0 limit, you remember, then we connect a different topologies. You can just stick to the lowest topology. So in other words, you can, when g string goes to 0, you can neglect splitting or joining on the string. So you can just can see that the worksheet with the simplest topology.

And in alpha prime goes to 0 limit. So alpha prime appears as a prefactor. 1 over alpha prime appears as a prefactor in this [? worksheet ?] series. Whether you do the [? Poliakoff ?] or you do the [? lambogotto, ?] it's the same thing. So alpha prime essentially plays to the coupling constant on the [? worksheet. ?]

So the alpha prime goes to 0 limit. So alpha prime is essentially the h bar on the [? worksheet, ?] is the [? factive ?] h bar on the [? worksheets. ?] So when alpha prime goes to 0, you can evaluate, then the path integral. Just LIKE in the gravity formulation, we can evaluate the path integral when the g Newton goes to 0. So here we can evaluate path integral using the sadale point approximation. The sadale point.

So this is [INAUDIBLE]. It implies, in the sadale point especially, essentially you just evaluate the classical action. Just find the classical solution, classical string solution. And then evaluate the action on that classical solution. And so this is corresponding to [INAUDIBLE] fluctuation of the [? worksheets. ?]

So as I said, this alpha prime appears in a [? worksheet ?] action just as the place which the h bar appears. And so when you take alpha prime goes to 0 limit, then corresponding to your, you neglect the fluctuation of the [? worksheet. ?] Then you can just can see that the classical string dynamics.

Then in this limit, then we can do this easily. So that means in the g string goes to the 0 limit, and the alpha prime goes to 0 limit, which again translate on the field series side to include infinity. And the lambda goes to infinite limit.

Then we can write the expectation value of the Wilson loop just as the classical action-- the action for the string evaluated as a classical solution. So S cl is the action evaluated at a classical solution. So essentially you just have a [? regions ?] [? worksheet. ?] Any questions on this?

So now we can talk about some examples. So the simplest example is that it's just considered a static quark. A static quark does not move. So if this is a time direction, then the [INAUDIBLE] line of the quark is just a straight line moving in the time direction. And say from minus infinity, plus infinity. And then you can imagine close to the loop at infinity. Anyway, this is just a straight line.

So this is a simple situation. And so a level is convenient to give a length to this line, the time direction. So they record the total length as T-- capital T.

And then without calculating, essentially by definition the expectation value of the Wilson loop should be just minus i MT. So this is just the phase factor associated with [INAUDIBLE]. This [? thing ?] under the quark does not move. And the M is the total energy of the quark, the mass of the quark. And then you just have the standard [INAUDIBLE] [? i ET. ?]

So now let's try to calculate on the gravity side. Now let's try to calculate what is the corresponding object to this on the gravity side. So now let me just remind you the AdS [? metric. ?] So we can use the two difference way, say whether using this R coordinate, or using this Z coordinate.

And they're related by a very simple coordinate transformation. Just 1 over. Remind you of that. So on the gravity side, so let's now draw our boundary. So this is r equal to infinity, or z equal to 0. So the r increase in this direction, and the z increase in that direction.

So such objects on the gravity side, just from the description we said earlier, was just corresponding to a string. Does not move at all. Extends from the boundary all the way to the interior. And because this is just a single quark and does not move. This is just our previous picture.

So we can parameterize it. So as we discussed before, this action is reparameterization environment. So that means we can choose the sigma and the tau at this 2 sigma coordinate. So sigma alpha. And we'll call the tau on the sigma. So I can choose the tau on the sigma as we want according to the convenience.

So for this one, let me choose tau equal just t at the real space time t. So that's t. And then take the sigma along the radial direction, this r.

So this straight string just corresponding to the solution, which x i t r equal to constant. So this is a string which does not move. So the other directions, it just a constant. If you put it at some point, it stays there. So this is the obvious solution of that configuration.

So you can check. This is indeed a solution to the [? lambogoto ?] action. I will not check there, because this is the essentially started with. So you can immediately tell this must be a solution.

So low let's try to find the classical action corresponding to this solution. So for this purpose, we have to work the [INAUDIBLE] [? metric. ?] So the easiest way to work out this the reduced [? metric ?] as follows. So the [? worksheet ?] [? metric ?] would be h alpha beta. So it is sigma alpha to sigma beta. And h alpha beta is given by this guy.

And the easiest way to work out is that you just write down the AdS [? metric. ?] And now you think that [? ht ?] and X and r as a function of the sigma tau. And you just substitute the functions in.

So t just equal to tau. So this just becomes t tau squared. But X is independent of t and r. So when I write the space time coordinates, I use x. But when I write it as a coordinate as a function of the [? worksheet ?], then I use the capital. So this is just to emphasize, this is a function.

Anyway, and here I replace by that X which is independent of [? ht ?]. So there's nothing here. And now the R square become sigma squared. So it become sigma squared, because i just equal to sigma.

And then this is my [? worksheet ?] [? metric. ?] Sigma square, r square minus d tau square plus r square sigma square [? d sigma ?] squared. So this is my induced [? worksheet ?] [? metric ?] for this special worksheet. And if you imagine, there's a time direction, which is this string just translate in the time direction. Does not move.

So now we can easily work out what is the determinant. This is diagonal [? metric. ?] You see these two cancel. So the determinant just equal to one.

So now we can write down the [? worksheet ?] action. Let me erase here. So the [? worksheet ?] action S N [? g ?] will be just minus 1 over 2 pi alpha prime. Now the theta h is just equal to 1. Then I just have integration of dt. Then I have integration of dr, because tau equal to t and sigma equal to r. And r should be from 0 to infinity.

So this integration of dt just give us a capital T factor. And then this is divergent. So normally when we see something divergent, as we always do, we just say this is not at infinity. Let's put a small cutoff here at i equal to lambda. So if I do that, and then this here become lambda.

So now if you read from here, this should be identified with wc. I times of that object should be identified with wc. And that should be equal to minus iMT. So we conclude that the mass must be lambda over 2 pi alpha prime.

But this is actually what I just advertised earlier, said you can check it explicitly, that the mass of the object is precisely just the radial location divided by r, divided by 2 pi alpha prime. And I just confirmed that expectation. I just confirmed that expectation.

So this is infinite by design, because I want to have an infinite mass quark so that I don't have a fluctuation. And so in principle I can take lambda. It's not just I want. But actually I can show the lambda. So it actually make sense. So physically you should think there's some kind of [? disequilibrium ?] here which I can show the location. And in the end, I take lambda go to infinity, because I want the mass go to infinity.

So it's also instructive to rewrite this in terms of the z. So in terms of the d, and the z is related to capital R by this way, So in this corresponding to a small thing in z. So z will be R squared. So this corresponding to a cut off z at epsilon. At epsilon. So we can write this M in terms of z. Then equal to become 2 pi alpha prime R squared divided by epsilon.

So now let me do one more step. So as we said before, that in terms of z-coordinate, so you'll also have checked yourself by doing this holographic bound exciter. So this epsilon can be conceived as a short distance cutoff because of the boundary, [INAUDIBLE] cut off at boundary. So this can also be conceived as the mass of the quark. And due to that, you put some short distance cutoff. Say epsilon distance away from the quark.

And so now let's remember the dictionary R square divided by alpha prime is related to on the gravity side to what? Do you remember? Yeah. A limit, right? The big box here.

So R to the power of 4 divided by alpha prime squared equal to what? Equal to lambda, which is the g [INAUDIBLE] square N [INAUDIBLE] g square N. And then the five-dimensional Newton constant divided by R cubed, that is related to the N. So this is-- I think it's pi divided by 2 N squared.

You don't need to remember the precise prefactors. But you should remember that R squared divided by alpha prime is related to lambda. So lambda is the root coupling. So now we can rewrite this as the square root lambda divided by 2 pi, then 1 over epsilon.

So there's something interesting here. He said there's a square root lambda. It's the square root lambda appearing here. And this is a square root, g squared times N.

So if you remember in the [? QED ?], so if you try to calculate what the [? self ?] energy of the electron in the most naive way. So electron naively have an infinite potential energy, because if you go closer to the electron, and then the energy will blow up. But if you put a small cutoff around the electron, then what would be energy. You would you get?

AUDIENCE: [INAUDIBLE]

HONG LIU: Yep. Say if you put two electrons. Two electrons.

AUDIENCE: [INAUDIBLE]

HONG LIU: Yeah. You use a lot of electrons. You probe it. Then you would get e square. You would get e square. But here, interesting thing is that it's proportional to the square root of the e square. This is proportional to the square root of e square. So this square root lambda is essentially the strong coupling prediction. We will say more about this.

So we have finished. This simple example seems to make sense. So this just corresponding to a mass of the quark.

So now let's do something a little bit more [? nontrivial ?]. So let's consider the static potential between a quark and the anti-quark.

So from the field series side, we can see that such a configuration. So suppose this is some x 1 direction. So this is a time direction. So we can see there's such a loop.

So this is L in the x 1 direction. Say this is L divided by 2. And this is minus L divided by 2. And in the T direction, again the length is capital T. The length is capital T. And we always take the T to be much, much greater than L. Essentially you can imagine this T as infinite.

So as we mentioned last time, if you can see the loop like this, when the T become much, much greater than L, you can forget about the beginning part and the end part. Essentially, you have two parallel line. And the parallel lines in the opposite direction in time, then you can see that this is this corresponding to a quark, this corresponding to the anti-quark moving parallel in time.

And then again, the expectation of the Wilson loop should give you-- so this is a static system. This is a system with translation variance in time. If I take T go to infinite, you can imagine as a translation variance in time.

And then due to this translation variance in time, you can write it to [INAUDIBLE]. It must have the falling form, just on general ground. And e totally is just the total energy of the system. Just in quantum mechanics. In quantum mechanics, any translation variance system, the whole thing just minus iE T.

And the total should corresponding to the mass over the two external quark, then plus the potential energy between them [INAUDIBLE] energy between them.

So if you calculate this Wilson loop in say [? QCD. ?] So that will give you the force between the quark and the anti-quark, et cetera. But in general, if you have a strongly coupled [? gate ?] series, we don't know how to do such a calculation. It's very hard to do.

But in this case, equipped with this, equipped by this, then the gravity will help us to do this calculation. So the gravity can help us find V(L) and strong coupling.

So now we want to calculate the string [? worksheets ?] corresponding to such configuration. Do you have any questions regarding our set-up?

Good. So now let's write. You can see that the gravity side again, this is the boundary of AdS. And now let's imagine this is our x 1 direction. Then we have a quark here at L divided by 2. And then have anti-quark here at minus L divided by 2.

So you can see that these two lines, it's one from quark, one from anti-quark. And now this is translates in time. So this the r equal to infinite, or z equal to 0.

So if we just have two isolated quark, if each of them are isolated, then the string [? worksheet ?], we are just as we said before, just two straight line. Just two straight line. And that's corresponding to an isolated quark.

But now these two quark can interact. Can interact. So the magic expectation is that this string [? worksheet ?] will get deformed, and the [INAUDIBLE] expectations that they will just connect to each other. So you expect the string [? worksheet ?] will be like this.

So if the string [? worksheet ?] is like this, that means there's no interaction between these two quarks. And now if the two string, yeah, so this is the quark. And then this is the anti-quark. So they should have opposite orientation. And then the string [? worksheet ?], we have orientation like this. So it will go off like this.

So now you expect that the [? worksheet ?] will be like this. And again, it's transition variant in time.

Any questions on this? Is this clear to you? So geometrically, this is only way which these two quark can have interactions is by joining their [? worksheet ?] together.

AUDIENCE: So generally, we would be doing the path integral over all the closed strings that have those boundary emissions. Sorry. Open string.

HONG LIU: Yeah. That's right. That's right. Yeah. Good.

So now with this realization, then the rest become mechanical. Not quite mechanical. Yeah. So we can choose, as before, tau equal to T. So T equals infinity. So we can consider the system translation variant in time.

So let's choose again tau equal to T. So we can also chose the sigma still equal to R. But now let me do it slightly differently. So let me do it sigma equal to x 1. So you can do both. But just to give you a variety, [INAUDIBLE] sigma equals to x 1, because x 1 [? direction also ?] become nontrivial. And let me choose sigma equal to x 1.

And then this [? worksheet ?] then is parameterized just by z, which is only a function of sigma. So sigma runs from minus half L to L. And then this string would be parameterized by some function z as a function of sigma.

And because of the translation variance, you cannot have dependence on t. Cannot have dependence on t. And also, all the other directions are not equal to 1. They should be just constant, because they're not moving in the other directions. So they should be just constants. So the only non-trivial thing would be in the x 1 direction.

You can also choose the sigma to be z, and the x 1 would be a function of z, then the x 1 would be a function of sigma. You can do it either way. Doesn't matter.

So the boundary condition would be z re-evaluated at plus minus 1/2 L. Then this of course should be at the boundary. So it should be equal to 0. So this is the boundary condition that this starting from the boundary.

So now let's try to write down the [? worksheet ?] action. Let's try to write down the [? worksheet ?] metric under the [? worksheet ?] action. So the [? worksheet ?] metric is again as what we were doing before. You can just write down i equal to z square minus dt square plus dx square plus tz square.

So now t equal to tau. So dt squared just become d tau square. And the [? dx 1 ?] square just become [? d ?] sigma square. under the others. We don't need to care. So this just become d sigma square.

And the z is a function of sigma. So this just become z prime sigma square d sigma square. And then you just find this is equal to R square z square d tau square plus 1 plus z prime squared d sigma square. So this is the induced metric on the [? worksheet ?].

And then you can now write down the [? lambagoto ?] action. So it just 1 over 2 pi alpha prime d 2 sigma.

And then you take the determinant of this metric, which is easy to do. You get R square factor out. And then you get 1 over d square. And it just square root of 1 plus z prime squared. So this is the square root determinant.

So we can do a little bit of [? massage. ?] So this d 2 sigma is dt d sigma. And the sigma is from minus L to 1/2 L, because this is a sigma, is x 1 from minus L to 1/2 L.

So [INAUDIBLE] depend on t. So we can just replace it by a factor of capital T. And obviously this system have symmetry between the x and the minus x. And then I can just choose the integration from half of it. So I do it from 0 to L divided by 2. And then I eliminated 2 on the [? downstairs. ?]

So this is my action. Then what I need to do is I need to [? extremize ?]. This is action respect to z sigma. Then you solve for z sigma, which satisfy the boundary condition. And then solve for z sigma. And then find S classical action.

So now this reduce to a freshmen problem-- literally, little cute freshman problem. So which is actually quite fun to do with a little bit twist. With a little bit twist. So do you want to do it immediately, or do want to have a break?

So now let's try to do this. Try to do this. So first, just by consistency we expect this thing must be divergent, because this contains 2M. But 2M in our set-up, if you start something from the boundary, and the M is infinite, then we would expect this thing to be divergent. We would expect something to be divergent.

And indeed, you see the dangerous thing is that when the sigma approach [INAUDIBLE] over 2, then you have a 1 over d square here, and at this z of 1 over 2, this is 0. So there is some potential divergences here. But of course you also have to look at the behavior of this term. By this you see there is some potential divergence here.

And this is expected, because we know that this must contain the 2M. And if this whole story is consistent, there must be a divergent.

So let us just write with that in mind. So let me just subtract the 2 MT from here. So this [? UNG ?]. So this is supposed to be minus i E total T. So from this expectation, we would write V(L).

So now again I replace the R square divided by alpha prime by square root lambda. So we have square root lambda pi. Then L divided by 2 d sigma 1 over z square 1 plus z prime square. And the minus now 1 over epsilon. Because the [? twice ?] the M is just the square root of pi divided by the square root lambda divided by pi times 1 over epsilon. So I take out this factor. And so we expect the V(L) [INAUDIBLE].

So if this guess we have said earlier is correct, that the partition function. of the Wilson loop should be related to the partition function of the string, and then reduced to a classical action, et cetera-- if that idea is right, we should find by self consistency that this thing should be finite. This thing should be finite, and should also be [? inactive, ?] because the quark and the anti-quark have an attractive force. So we should find this thing to be finite and negative.

You see, this guy is actually positive. And no matter what is z, this guy is actually positive. So that means that this guy must be smaller than 1 over epsilon. And so you get something active. So this guy must be some 1 over epsilon lambda something.

So now let me call this to be my Lagrangian. So now we had to do a small variation of the problem. Yeah. Maybe not confused by that L. Let me put this script L. So this is my Lagrangian.

And then this is a one-dimensional Lagrangian problem. And this Lagrangian does not depend on sigma. So we treat the sigma as time. So this is like a one-dimensional problem with the sigma as time. So if the Lagrangian does not depend on time, what's going to happen?

AUDIENCE: Energy conservation.

HONG LIU: Exact. The energy's conserved. The energy's conserved. So we can write down the energy corresponding to this guy. And the energy is z prime pi z minus L. So this must be constant. And the pi z, just the economic momentum conjugates to the z.

So all this is very elementary. You can immediately write down this equation, [? h of 1 over ?] d square. So you find the pi you plug here. Then you find this equation. This equation just become that.

So now let's try to parameterize this constant. So the way to do it is just look for the special point on the trajectory. So the special point on the trajectory is this point. By symmetry, it should be corresponding to sigma equal to 0.

So this point, it should corresponding to sigma equal to zero. And so let me call this point a z 0. So this is essentially how deep this string are going into the [? back. ?] So let's call that 2 [? bz ?] 0.

So it's clear at sigma equal to zero, the z prime should be equal to zero. So this should be just equal to 1 over z 0 squared. Should be 1 over z 0 squared. And then you can just immediately rewrite this as ordinary differential equation.

So now you can immediately integrate this equation. And say z 0 is found, it's determined at the boundary condition by requiring z L equal to 0. So I will not do it here, but you can do it in 30 seconds to find the z 0. You actually don't need to actually integrate this equation. There's a simple trick to do it.

Anyway so let me just write down the answer. You can find the z 0 equal to L. It's proportional to L, under the square root of pi divided by 2 comma 1/4, and become 3/4.

AUDIENCE: [INAUDIBLE]

HONG LIU: Oh, this funny number.

AUDIENCE: 30 seconds.

HONG LIU: Yeah. Yeah. Mathematica will tell you. Take you 25 seconds. You type into the Mathematica, and the four seconds to check the error, and the Mathematica one second to give you the answer.

So you find this. So we will comment on this implication a little bit later. So now we can plug this into this action. So let me see. Yeah. So now let me plug this equation back into this equation. [INAUDIBLE] [? star ?].

So there's again a simple trick to do it. You don't need to know the expressive form of z, because what you can do is you can rewrite this V(L). You can rewrite this integral as follows-- V(L), say this square root of lambda divided by pi.

So this d sigma, you can write it as dz divided by z prime. So this is the d sigma. You can change the variable from sigma to z. And then this just integration become z 0 to z 0. And then you have the 1 over z square, 1 plus z prime square.

So now you can substitute what is the z prime into here. And now this just become integral over z, and just pure integral. We still need to minus [INAUDIBLE].

So if you plug into the expression for the z prime, and then do a scaling-- say take z equal to z 0 y-- because there you see this all have the nice scaling form. So it's good to scale the z 0 out.

So let's do a scaling, z equal to z 0 y. Then you could rewrite that integral. Sorry this is no V(Z). This is V(L). So you can rewrite that integral as V(L) equal to square root of pi, square root of lambda divided by pi 1 over z 0.

So now I have to put the epsilon in here anticipating that this integral will be divergent here. So this is the same epsilon we put in there when we do the single quark. So we put it there.

Yeah. Maybe let me just write one more step so that you can see it more explicitly. Yeah, so I just plug this in, plug this into here. Then what you get square root lambda z zero square epsilon. Then I put a cutoff in the epsilon. And this becomes [? tz ?] square.

And now I can do this scaling to express in terms of y. And then this becomes square root lambda pi 1 over z 0 t y over 1 all over y square. [? of ?] [? 4. ?] So minus 1 over epsilon.

For simplicity, I will slightly rewrite this 1 over epsilon into the following thing just as a contrast as from infinite, epsilon divided by z zero dy divided by y squared. You can easily do this integral. This integral you integrate as 1 over y. And the other end it give you 0. Lower end they give you z 0 divided by epsilon. And then z 0 cancels with z 0. So you just get 1 over epsilon.

But the reason to write in this form is to see that this two singularity indeed cancel. So these have a singularity. As y goes to 0, this is non-integrable. And here you have the same singularity. They cancel.

So when [? d ?] over [? y ?] equal to 0, they cancel, and the next order term come from by expanding this term. Then you get the y to the power 4 when you expand to this term. [? In the upstair ?] you get 1 over y power 4. [? Other ?] than that, will give you something finite. So that just tells you immediately see that the singularity have canceled. So you will guaranteed to get something finite. You are guaranteed to get something finite.

And another reason to write in this form is that this is [? something ?] integrate to 1, and this is something integrate to infinity, even though the difference between them is negative, because you have agreed to do the calculation. But this is because integrand is not exactly the same. But the fact that this is going to infinite give you a chance that this become active. And when you calculate it, it indeed negative.

And the [INAUDIBLE], literally you can do it. Yeah. So you can write down the answer. You find the answer is given by something like this. It's minus square root lambda divided by pi, then some numerical factor, which is not very important, but let me just write it down. To the power of 4. So this answer is nice.

So then now let me make some final remarks. So this is finite and negative. So this pass the minimal consistency check that we are doing something sensible. We are doing something sensible. If we find something infinite, then that proposal must be wrong. You can immediately rule it out. And if you find something positive, then you can also immediately rule it out. And so this is a self-consistency check.

Oh, I forgot the 1 over y at z 0. Oh, actually, if you plug in the 1 over z 0, I think you just-- no no, no, no. I write it wrong. This is not pi. This is L. So I have already plugged in z 0. So z 0 is given by this. It's proportionate to L. And then this is 1 over L.

So this 1 over L is also makes sense physically, because 1 over L is just the cooling potential. And we do expect the cooling potential, because [INAUDIBLE] as we said is a scale [? invariant ?] theory. And in four-dimensional scale, [? invariant ?] series, potential just [INAUDIBLE] the [? ground ?] it will be 1 over L, just by scaling symmetry. It just exactly [INAUDIBLE] [? why ?] [? electromagnetism ?] is 1 over L.

And so this just from the scale invariance. There's no other scale. So the dependence on L must be 1 over L, because we have dimension mass. So they cannot have other L dependence.

So the second point is now we again see this proportional to square root lambda. Again, this is a strong coupling with that. Because at weak coupling, we know it must be [? proportional ?] to lambda divided by L. And lambda is just the analog of the fine structure constant. And then this is the cooling potential.

So 1 over L is always true, no matter the value of the coupling. So you see, when you [INAUDIBLE] the coupling from lambda equal to 0, then it should be linear in lambda. Then when lambda goes to infinity, then become the square root in lambda.

So now the final remark. [INAUDIBLE] we see that z 0 is proportional to L. So now let's look at the geometrical meaning of this z 0. So this z 0 is how far a string extends into the [? back. ?]

So if you say L is small, then z 0 is small. Now if L is large, if I make L large, then z 0 become bigger. That means that the string extends further into the [? back. ?] And when you take L goes to infinity, and this go all the way to the AdS.

And again this is the refraction of IR/UV. Refraction of IR/UV, because L corresponding to whether you measure UV physics or IR physics in the field series. So L small is a short distance. [INAUDIBLE] is long-distance physics. And you see when you probe long-distance physics, then you are probing much deeper in the interior in AdS. And if you're looking at short distance, then you're probing the region very close to the boundary. Any questions on this? Yes.

AUDIENCE: So are there any interesting experimental implications of these sorts of things that we couldn't get otherwise? Could we have known these four facts basically without doing this calculation by doing more standard techniques?

HONG LIU: You mean this thing?

AUDIENCE: Yeah.

HONG LIU: No. Yeah, so this is really strong coupling prediction.

AUDIENCE: So what would that correspond? What sort of thing could you measure in nature that would sort of--?

HONG LIU: Yeah. So then what you would do in your p-set. So in your p-set, in the last problem, you will calculate this thing at the finite temperature, which we will talk of finite temperature in a few minutes. You do it in the finite temperature.

And then you can essentially calculate this potential at the finite temperature. And as you will do in the p-set, there's a [? screening ?] length. And the finite temperature is no longer scale invariance. Beyond a certain length, the potential will go to zero. And you can measure the screening length. And people on the [? lattice, ?] [? QCD ?], that precisely match with that.

So you can compare AdS calculation to the strongly coupled [? QCD ?] calculation, spend a huge amount of computing time, large amount of money, large amount of manpower on the computer to do that kind of Wilson loop calculation on the computer. And then you see the answer is very similar. Yeah.

AUDIENCE: That's interesting. [INAUDIBLE] the strong coupling I guess is proportional to square root lambda. But the square root of lambda was what you said from the beginning, because [INAUDIBLE] of the alpha prime. It does not involve the calculation at all. So this [INAUDIBLE] produce a weak coupling result.

HONG LIU: Oh sure. Sure. Because we are calculating a strong coupling. Because we are calculating the strong coupling. Yeah. So this whole set-up is for the strong coupling.

AUDIENCE: So this [INAUDIBLE].

HONG LIU: No. But weak coupling you can do [INAUDIBLE] diagrams. You don't need this. But strong coupling, we have no other way to do this kind of calculation, other than doing massive computer simulation. And for some things, you cannot even do computer calculation. Good. Any other questions?

Yeah. So this is a good question. So this actually a very good point. So this tells you on the gravity side, essentially anything involving these [? always ?] [? pointing ?] to the square root of lambda, just related one of alpha prime. Yeah. It just come from the kinematics. Yeah, but [INAUDIBLE] the system the very nontrivial prediction of the strong coupling behavior.

Good. We need to go to the finite temperature. Because I want you to be able to do your p-set. So now we have talked about how to calculate the various observables. Now we can pause a little bit to talk about other generations.

So the first generalization is [INAUDIBLE], let's do finite temperature, do a [? langio ?] temperature. So so far, we, can see that so far we discussed AdS 5 times [? AdS ?] 5 string series AdS [INAUDIBLE] is 5 string. In AdS 5 times [? s ?] 5 is related to [INAUDIBLE] series.

So we talked about this in terms of small perturbations. The normalizable mode on this side should map to some state. Say you have some normalizable mode, should do [INAUDIBLE] some states.

And for example, the pure AdS 5. So there's nothing excited. This is example of this [INAUDIBLE] kind of normalizable mode. So this is should be due to the vacuum, because nothing excited on the field series side. So this must be due to the vacuum.

So now start with this pure AdS 5. Now as you start putting things inside, excited the system, then you will say in some sense excite some normalizable modes. Then you go to excited states in your [INAUDIBLE] series.

And then one of the obvious excited states is the finite temperature states. So the obvious question is what does the thermal state correspond to? Yeah. corresponding to just save time.

So the gravity description should have satisfied the following conditions. To satisfy the following features. So now we want to guess what should be the thing on the gravity side.

First he said it should be asymptotic AdS 5. That means that there should be only normalizable modes. There should be only normalizable mode, because the finite temperature state is a specific state of the field series. So there should be no non-normalizable modes excited.

And of course if the field series have a finite temperature, the other side must also have a finite temperature. So it must have a finite temperature. So now I use T means the temperature. Now the capital T means the temperature. And in particular, this should satisfy all laws of thermodynamics.

And the third one is that you should have of course all of the symmetries of the finite temperature system. So you should have translation variance, rotational symmetry, et cetera. Of course, a Lorentz symmetry, Lorentz boost would be broken. So you still should have translation symmetry and the rotational symmetries. But the boost will be broken, because now when you add finite temperature, you no longer have the Lorentz boost.

So you need to find some kind of gravity solution which will satisfy those criteria. So there are two obvious candidates. So there are two candidates.

So the first is just you can see there's some kind of thermal gas in AdS. So just imagine I have some thermal gas in AdS. And the second possibility is that you have a black hole. Because we also know a black hole is a thermal object. We also know black hole is a thermal object.

So it turns out so we will only consider the [INAUDIBLE] [? patch. ?] So we only consider the boundaries is really the r [INAUDIBLE] rather than the [INAUDIBLE]. So the thermal gas.

So let me first describe the thermal gas, a description. So normally in the field series, how do you describe a thermal system? So how would you describe a thermal system?

AUDIENCE: [INAUDIBLE]

HONG LIU: In field series.

AUDIENCE: [INAUDIBLE]

HONG LIU: Yeah. A partition function is one observable of a thermal system. Say if you want to describe the thermal system in field series, what do you do? Say if you want to calculate correlation function, you want to calculate many other things?

AUDIENCE: [INAUDIBLE] Euclidean [INAUDIBLE]?

HONG LIU: Exactly. So normally in the field series, what you do is that you go to Euclidean signature, and then periodically identify time. And the period of time is the inverse temperature. So we can do it here. The thermal gassing idea is essentially is what we do.

You say we go to the Euclidean signature. Yeah, let me call it t E. We go to Euclidean time. We go to Euclidean time. And then we identify Euclidean time periodically with inverse temperature, which we always call beta. And so in particular, the fermions is anti-periodic. Anti-periodic. That means anti-periodic. Of course, the Boson is periodic.

But actually, this metric is bad for the following reason. So in gravity as a general rule, whenever you see a compact space go to 0 sites, then there's a danger of having a curvature singularity. It's almost always you have a curvature singularity. Only for some special situations it does not have it.

And so this is bad, because now when you compactify this Euclidean time, this become a compact direction. This is a circle. And when you go to z goes to infinity, this circle goes to zero sides. You can check that actually this have more curvature singularity.

So this is actually bad. So when t is uncompact, then this is a coordinate singularity. But when t become compact, and then this become a [? general ?] singularity. This become [? general ?] singularity. So first this metric's bad, because I have singularity. At z equal to infinity. At deep in the interior of AdS.

There's also a second reason, which is more stringent reason, which you don't need to care. Let me just write it down for completeness. So if you actually analyze a string theory in such kind of space time.

So also there's a generic rule in string theory is that if you have a very small circle, then the string can wind around the circle. The string can wind around the circle. And then when that circle become very, very small, then it takes less and less energy for string to wind around that circle.

And then normally, so if you have supersymmetry, and also you have a string wind around circle, there's a [? casmir energy. ?] And the [? casmir ?] energy is typically is inactive for the Boson, and the fermion is positive. And if you have a supersymmetry, then that [? casmir ?] energy cancels. And then when the circle shrink to zero size, then the string become massless. So that's typical in the superstring.

But if you in the situation where you just saw a symmetry's broken, then typically then there's an active [? casmir image. ?] And then when the circle side becomes smaller and smaller, and then it take lower and lower energy for string to wrap around circle. And then anyway, if you include the [? casmir image, et cetera, ?] then you find the string actually become [? negative image. ?]

And you find that the strings winding around t E actually develops negative mass square. So as we discussed before, this is called [INAUDIBLE], and signals instability. And here there's a circle which become a small size. And here there's no supersymmetry, because when you have a finite temperature, your supersymmetry is broken, because both on fermion have different boundary conditions.

Anyway, so that means that this option is gone, cannot work. So the only thing we know is to do a black hole. So now we need to find a black hole with the right symmetry.

So the reason I emphasized with the right symmetry is because when we say black hole, it's because a black hole is a hole, is a [INAUDIBLE] symmetric hole. But here we need to have a translation symmetry, because this is a [INAUDIBLE].

So here now we need to have a black brain, black the whole horizon. It's actually not a hole. It's actually a plane. So I did the black hole in this topology for the horizon topology. Topology should be r to the d minus 1, rather than a sphere.

Once you've realized that, then it's easy to do. Once you realize that, it's easy to do. So you can just write down the answers, because this system have lots of symmetries. And you can just write down the answers, and then just plug into the Einstein equation to solve it.

Say for example you can write down the answers. So you can write down say r square. So you can write down answer's ds square equal to r squared divided by z square.

So now you still get translation symmetry, so you can put something before dt square. But you still have rotation symmetry. Yeah, so anyway, you can write down answers like this. Put in some two functions here. Put in some two functions here. So this is a most general solution, which is consistent with the translation symmetry in the t and x direction, and the rotational symmetry in the t and x directions.

And then you have just two functions which can only depend on z. You cannot depend on t and x. Otherwise you'll break translation symmetry. So these are the most [? general ?] [? metric ?], you have two undetermined functions. Then you plug into Einstein equation. Then you solve it.

So you find in fact h actually equal to 1 over g equal to something like this. It's in d-dimension. In d-dimension, say if I have AdS d plus, 1 and then you find a very simple [? metric ?] like this.

So let me again give you a slight quiz. So why I don't put a function here? So why in principle, I can also put a function here, right? So why I only put a function here and the function here, but not put the function here?

AUDIENCE: [INAUDIBLE] [? Because it redefines ?] [? z. ?]

HONG LIU: Yeah, exactly. Because you can imagine this just as a definition of z. But then I don't have freedom to choose this z. Or I can choose here to be 1, and then I need to put the function there.

So in this metric, the horizon we set z is equal to z 0. z 0 is some constant. So z 0 is some constant. Zero some constant. Right. Is z 0. And indeed I have a topology of the R t minus 1, because [? the d, ?] z equal to 0, the topology is at this r. It's not at the sphere.

And also you see that when you go to infinity, the f and the g approach 1 with a very fast fall-off with d to the power d. So this is the normalizable. And this is normalizable. So this behavior is normalizable.

So this is metric satisfy all our criterions. And under the standard black hole thermodynamics, we are sure this is a thermodynamical system. So this must be [? due ?] to our finite temperature [INAUDIBLE] series.

So now let's work out the temperature in terms of z 0. So it's just using the standard trick going to the Euclidean signature which we have discussed before. And the required the metric is regular at the horizon. When you go to Euclidean signature require the metric is regular at the horizon. And then you can deduce that this should have some specific periodicity. Anyway, I hope you still remember that.

So you can deduce a temperature of the black hole. So beta is equal to 4 pi to the d times z 0. So you can deduce. So this you remember, right? OK. Good.

So emphasize this is the temperature matched in t. Because whenever, as we emphasized many times before, but whenever you talk about temperature, you have to refer to a time, time units to define the temperature. And this temperature is defined with respect to this time.

So in other words, this is boundary temperature. So in the [? back ?], of course, depending on the value of z, the local temperature is different. And this is the temperature corresponding to the boundary. So this relation is again very interesting.

So this is z equal to 0 boundary. And now on the gravity side now there's a horizon at z equal to 0, z equal to z 0. And this horizon according to this formula is proportional to 1 over t. It's proportional to 1 over t.

Again, this is the refraction of the IR/UV connection. Because if you can see the very high temperature, it means you are probing very high energy process. Then the z 0 becomes small. Then the horizon moves to the boundary. And the horizon moves to the boundary. And when t becomes small, then z 0 becomes big, then the horizon move down.

So that means that depending on the value of t, you can probe different regions in the gravity side. So for very high temperature, you probe very tiny region near the infinity. And for a small t, you can probe one or more regions. Again this is a refraction of IR/UV. Any questions on this?

Now with this black hole solution, we can work out [INAUDIBLE] thermodynamics. And then we should be able to work essentially by definition. Then, for example, the black hole entropy should be identified with the field series entropy, or black hole free energy should be identified with the free field series for energy, et cetera. It just essentially, if there's a relation, if black hole describe the field series at finite temperature, then you should be able to equate the thermodynamic quantities.

And in this way, we can actually use the black hole thermodynamics now to calculate the thermodynamical behavior of a strongly coupled [INAUDIBLE] series. So we can now obtain thermodynamical behavior, thermodynamical quantities at strong coupling in the [INAUDIBLE] series in a black hole.

So now let's do that. So now let's specify to d equal to 4. So that's corresponding to the four-dimensional boundaries, and that's [? 2t ?] equal to 4.

So now let's first do the black hole entropy, which is the easiest thing to do. So the black hole entropy, just the area of the horizon divided by Newton constant. So we have to use the factor of five-dimensional Newton constants. So we work with AdS 5. So we dimension reduction on the S 5. So we work with the five-dimension Newton constant.

And then the area here is just infinite volume. It is just dx 1, dx 2, dx 3, because this is just parallel to the boundary. It doesn't matter. It doesn't matter. So what this defines, or [INAUDIBLE] times R cubed to the z 0 cubed, because you have to multiply the area at the horizon.

So you essentially compute the determinant of the spatial part at the horizon, you evaluate it at the z 0. You evaluate it at z 0. So you have this factor. And then you have this infinite factor.

But this infinite factor, it doesn't matter. We can just take it out. Then what you get is the entropy density. So then we find the entropy density is essentially just given by this R cubed divided by z 0 to the cube 4G 5.

So now you can just plug in the numbers. So now you have to use this equation. G 5 divided by R cubed is related to the [INAUDIBLE] related to pi divided by 2 N square. And then you plug in this relation to relate z 0 to t. And then, after a tiny bit of algebra, so you find N squared T cubed.

Again this answer makes sense. Again, you can do some very minimal consistency check. So first thing you said is that this is proportional to T cubed. It makes sense, because we are working with a four-dimensional scale invariant series.

And entropy by itself does not have a dimension. And entropy density should have dimension cubed because of the volume. You divide it by the volume. Should be 1 over volume.

Do you follow? So entropy density should have dimension. Entropy density should have dimension M d minus 1. Because at the d minus 1 come from because you divide it by entropy by the volume, and the volume have, yeah, 1 over volume have dimension M d minus 1.

But here in the scale environment you don't have any of the scale factor. You only have T. So it must come from T to the d minus 1. So you get T cubed. And also, this is a proponent to N square. And of course in the [INAUDIBLE] series should be [INAUDIBLE] to N square.

So now you can also obtain, say for example, the energy density and the pressure, et cetera, by calculating the T m [INAUDIBLE] mu. So you can calculate say one [? point ?] function of the stress tensor Then you can read what is the energy density, and what is the pressure, et cetera.

And so this is the same as what we did. It's very similar procedure as what we did before for the scalar field series. The one point function of the stress tensor, you can read it from the counterpart of the [? B ?] modes, a counterpart of the [INAUDIBLE] modes.

So here the [INAUDIBLE] mode is just this z to the power d. And the [? corruption ?] is 1 over 0, 1 over z 0 to the power d. And so that tells you you should be proportional to 1 over z to the 0 to the 4. Because that's an analog of this [? B ?] mode-- the normalizable mode-- to the 1 over z to the 0 4.

And again, this will be proportional to T to the power of 4. Again, this is consistent on dimensional ground, because energy density have dimension m to the power d. And so it's T to the power 4.

But you actually work out the prefactor request [INAUDIBLE] effort. Yeah, in a scalar case, we say it's 2 [? mu ?] times b. So in the gravity case, in the [? metric ?] case, it gets a little bit more complicated. Anyway, but you can read the scale. I will not calculate the prefactor using this method.

So to calculate the prefactor, it's easier just to use the thermodynamics. So because S just equal to the free energy divided by T. So now everything here, we can see that the density is S is entropy density, and F is the free energy density [? exciter. ?]

So we know now the entropy. You can just integrate the free energy density. So this is equal to pi square divided by 8 N squared T to the 4. And then you can find out the energy density of plus TS. So you find this 3 pi square [INAUDIBLE] squared [INAUDIBLE]. Now if you work hard, the one point function of the stress tensor carefully of the [? free ?] factor, you will find this.

So this is, as a result, at lambda equal to infinity. So these are the strong coupling we got. All these are the strong coupling we got.

AUDIENCE: [INAUDIBLE]

HONG LIU: The thermodynamical behavior?

AUDIENCE: [INAUDIBLE] How do you get N?

HONG LIU: No. You just integrate this equation. Yeah. Just integrate that equation.

So this is the behavior, that infinite lambda. So now let's compare with the free series. So this is the undergraduate thermodynamics. So let's just calculate the entropy density with zero coupling.

So I have to remind you one thing. It said in each massless [INAUDIBLE] degree of freedom contribute to the entropy density 2 pi square divided by 45 to the power T cube. I don't know whether you still remember this thing.

So in four dimension, a single massless degree of freedom contribute to the entropy density. Yeah, this come from-- doing a single massless [INAUDIBLE] degree of freedom contributed entropy like this. This is a good thing to remember.

And now in the [INAUDIBLE] series we have eight Boson, as we said before. We have eight massless Boson degrees of freedom. And then we have eight fermionic massless degrees of freedom.

And each fermionic degrees of freedom contribute to how much of the Boson? Do anybody know? A massless fermion compared to the massless Boson. What is the ratio for each degrees of freedom contributed to the entropy density in the free series, free particle?

AUDIENCE: One half?

HONG LIU: No [INAUDIBLE]

AUDIENCE: One.

HONG LIU: No. 7/8. And now in the SU(N) [? Gate ?] series, we have N squared minus 1. We have N squared minus 1 in the join [? representation. ?] So now you can calculate this is a [INAUDIBLE] free series. So let's take [INAUDIBLE], forget about the 1. So then you get 2 pi over 3, 2/3 pi squared N squared T to the power cubed.

So now you find something remarkable. So now you find something remarkable. You find S lambda equal to infinity divided by S lambda equal to 0 is equal to 1/2 divided by 2/3. So it becomes 3/4. So you go from zero coupling just to infinite coupling, and the entropy change by 3/4.

So there's a long story. There are several long story I can tell, a good story I can tell about this thing. But now I run out of time. But let me mention a couple words on your p-set problem.

So in the p-set problem, you want to compute that this Wilson loop, the same Wilson loop, the L and the T at the finite temperature. So now instead of the pure AdS, you use this black hole geometry. You use this black hole geometry. Use this black hole geometry.

So what you will find, so physical expectation, physically the V(L). So you expect the potential between the quark and anti-quark will go to 0 when L is large. So physically for this reason, so here this is the same thing happens even in the [? QED ?].

So this is just the story of the screening. Suppose you have electrons, anti-electrons. So of course they have interaction between them. But now if you have a plasma between them, you have many positive and [INAUDIBLE] charge of particles [INAUDIBLE] running around.

And then they interacting will be screened. The interaction will be screened, because there will be slightly more active particle here, and slightly more positive particle here. Then they will be screened.

Similarly, if you have a quark and an anti-quark which [INAUDIBLE] object, and we input them at the finite temperature, in the finite temperature, you can cite many other massless [INAUDIBLE] degrees of freedom. Then they running around [INAUDIBLE] quark and the anti-quark. Then they will screen them. So when they take the distance sufficiently large, then you will find the potential between them goes to 0.

So on the gravity side, what you will see-- so now you cannot do everything analytically. But you can derive the qualitative behavior analytically. It said if the L is small-- so suppose here is the horizon. When the L is small, then of course we just connect a string. We're close-by. Nothing changes. Nothing much changes from our zero temperature calculation.

And this makes sense, because at a short distance, you probe the UV physics. And the probe, the UV physics, of course you don't see the temperature. So for L is much, much smaller than 1 over temperature. Then you don't see the effect of the temperature, and [INAUDIBLE] will be just like small corrections to the vacuum [? we got. ?]

But now when L become of [? all ?] the 1 over the temperature, then you find something interesting happens. Then the L, then this [? worksheet ?] will roughly go to the horizon.

Then what we will find in the end when the L become say larger than some value say of all the 1 over t, the only solution you can find is that this string just two string heading down. That's the only solution. And then that means that the quark, anti-quark no longer interact with each other. And then they are screened. Then they are screened.

So a short distance is the same as in the vacuum. But when you go to long distance, of all the 1 over temperature, and then eventually it would get screened. So this is another cute calculus problem which I'm sure you will enjoy. Yeah. That's all for today.