HONG LIU: OK, so let me first summarize what we did at the end of last lecture. So we can see that the Rindler space can be separated from the right column into into four different patches. In particular, there's a left patch and the right patch, and this is a constant row slice in the Rindler.

So what we showed is that the Minkowski vacuum, the standards, the vacuum we defined, the Minkowski quantum field theory, can be written as an entangled state, and we express it in terms of the Hilbert space of the left and the right patch. So this n sum over all possible in the [INAUDIBLE] state. So this n and n are eigen vectors, eigen values, and eigen vectors of the Rindler Hamiltonian, which we called HR. And so this, similarly, is an L. So an R means the eigen vector in the right patch and an L is the eigen vector for the left patch.

And now when you trace out, for example, the left patch, suppose you're only interested in the physics in the right patch when you trace out the left patch, then you find the sum of density matrix for the right patch. [? Under ?] the [? sum ?] of the density matrix is the partition function for the full Rindler minus 2 pi h and so you conclude that the temperature is 1 over 2 pi.

So that's what we did at the end of last lecture. So there are several key elements here. So there are several key elements here.

So the first key element is that the Minkowski [? ground ?] state turned out to be a particular kind of entangled state between the left and the right patch. And then you get the [? sum ?] of density matrix when you have a Lorenz above the left patch. So these are two of the basic elements. So any questions on this? Yes.

AUDIENCE: [INAUDIBLE] left patch. Is there any physical meaning to the left patch?

HONG LIU: It's the same physical meaning to the right patch. For the observer in the right patch, of course, you don't see the left patch, and so that's why you get the [? sum ?] of density-- yeah, that's why you have to integrate them out. Yeah, they play a very important physical role.

AUDIENCE: [INAUDIBLE] observable [INAUDIBLE]

HONG LIU: This is just pure Minkowski space. Yeah, this is just pure Minkowski space. This is Minkowski space. So this kind observer can only observe the right patch, so to this kind of observer then you must integrate out all the physics in the left patch. So that's why they see a similar physics. Yeah, so this is just a physical explanation why this is similar physics.

AUDIENCE: Why do we need to introduce a harmonic oscillator during [INAUDIBLE] HR can be anything? I forgot the reason why we introduced harmonic oscillator.

HONG LIU: Oh, I just gave you a simple example to explain this kind of physics using a simple example. just in case you're not familiar with this kind physics I gave you a simple example to build up your intuition. the reason we considered the harmonic oscillator is not an important reason. Say if you [? quantize ?] the scalar field theory-- so any theory on this space, say if you [? quantize ?] a free quantum field theory, that just reduce the harmonic oscillator. So the harmonic oscillator in fact have the exact same physics as general quantum field theory you consider. Any other questions?

AUDIENCE: May I repeat my question? The right patch, the [INAUDIBLE] patch, corresponds to the exterior of the black hole, right?

HONG LIU: In the black hole problem, the counterpart of the right patch is exterior over the black hole. That's right.

AUDIENCE: So the upper patch is the interior?

HONG LIU: That's right.

AUDIENCE: So what are the kind of [INAUDIBLE]?

HONG LIU: The left patch is [INAUDIBLE] asymptotical region over the black hole, which is also outside the horizon. So the extended black hole has two asymptotical regions. Yeah.

AUDIENCE: So is there any kind of [INAUDIBLE] way with the left patch [INAUDIBLE]?

HONG LIU: Yeah, as I said. Yeah, I'm going to talk about this a little bit. I'm going to talk about black hole in a little bit. Any other questions? Good?

So let me make some further remarks. The first is that this Minkowski vacuum-- this entangled state between the left and the right-- is invariant under the [? axim-- ?] Or maybe I should say-- let me just say it. It's invariant under HR R minus HR left. By invariant under this I mean-- so this state is [INAUDIBLE] by this, and this invariant under any translation generated by this combination. I'm not using very precise English, but I think you understand what I mean.

Anyway, so this you can see immediately from there. I just [INAUDIBLE] this harmonic oscillator example, and if you act this on that-- so this R means Rindler, and this R means the right patch and left patch. So act this on that state, then they just get an E because of the minus sign. They just canceled. And so this minus sign is related. What I said last time is that you can think of the left patch half the time running over the direction. By time I mean this [INAUDIBLE], OK?

So what this operator gener-- it does is the generator flow-- there's a generate flow in eta. So the eta goes running to so this goes running to [? comes ?] in the low surface. And the flow in the right quadrant is going up, but in the left quadrant it should go down so the time should be moving [? normally ?] direction, and that's what this minus sign means.

And then this transformation, we'll leave this state invariant. And you [? graph ?] the same thing like we did before for the harmonic oscillator. Any questions on this?

So you can immediately see from that equation this operator [INAUDIBLE] that state. So this can also be-- so do you have any problems saying that the HR generated translation along this? Is this clear to you?

AUDIENCE: So the [INAUDIBLE], the time [INAUDIBLE] are opposite. Does it mean that any other physical is the opposite time and [? changing weight ?]?

HONG LIU: It doesn't matter what you mean. It doesn't matter how you interplay, physically. Right now, I'm talking about the mathematical statement. I want to first understand the mathematical statement. So this mathematical statement has two layers. Let me write explicitly. This means i HR R minus HL, HR left acting on 0N with reach any eta this is invariant. This is invariant. This, you can just see directly from that the fact this [INAUDIBLE] that state. And then that means that this thing leaves this thing invariant.

Under the action of this guy, this operator, from the point of view of the red patch because one thing, you generate the translation in the eta because HR is the [INAUDIBLE] for the eta, so this just generates the translation in eta, which leaves rho invariant.

And this slice I'm showing here is the constant, the rho slice, so that means this generates a translation along that arrow direction, moving to positive time. And this minus sign means that, in the left patch-- because [INAUDIBLE] transformation we are moving [? in the only ?] direction. And then that operation, we will leave this state invariant. Yes.

AUDIENCE: So one question. So sometimes people have this interpretation-- I'm not sure if this picture is correct-- but in the black hole picture, we interpret the bottom thing as being some sort of white hole, which sort of spews out things. So if we reverse time, then does it sort of become a black hole again because now it's--

HONG LIU: No, no. We are talking about completely different things. Here, I'm just talking about-- in the left and in the right you can choose whatever time direction you want. I'm just making a statement that said, if I make this kind of operation-- do a time translation in the positive time direction, but in the opposite time direction in the left-- that particular operation leaves the this state invariant. For physical applications you can choose whatever time direction you want. You can choose whatever time direction you want. And this is a mathematical statement saying this particular operator leaves this state invariant, and this particular operator half of the interpretation of generate opposite time translation in the left and the right patch.

So this is an algebraic statement, but this can also be seen geometrically. This can also be seen geometrically. So if you work out the relation between the Minkowski's coordinates and the Rindler coordinates, you can actually immediately just see is that, geometrically, eta translation is a boost. in x, T. So what that does, it just generates a boost. So if this is not immediately clear to you, just go back and try to look at a translation between the two coordinates. Then you will see it immediately. So in other words, this HR actually generates a boost. This Hamiltonian essentially generates a boost. This Rindler Hamiltonian generates a boost. One second. Let me finish.

So, clearly, by definition, the Minkowski vacuum is invariant on the boost. So this statement is essentially the statement that this thing is invariant on the boost.

And then you can see this negative sign. Then you can see the negative sign from the fact that if you make a Lorenz boost-- and this is the trajectory of the Lorenz boost, and it acts off the direction in the left and in the right. So that's why there's a negative sign here. So this negative sign goes [? bonds, ?] so the geometric statement that when you make a Lorentz boost in the

Minkowski plane, and the action on the left and on the right is in the opposite direction, so the same boost will take a point here to there, but we'll take a point here to here. And you can check yourself.

And this translates into an algebraic statement-- just become this statement. And this statement is the same-- that this is invariant on the boost. So this is the first remark.

And the second remark is that, if we expand the field phi R in the right patch in terms of a complete set of modes, just ask, what do you normally do when you do canonical [INAUDIBLE]? In the right patch, say-- so this defines a [INAUDIBLE] and the creation operators for the theory in the right patch. And, similarly, you can do it in the left patch.

And then you can show, just based on that relation-- just based on that relation, you can show-- so let's consider the freescale [? of ?] [? field ?] [? series ?] so you can show that this-- just as the harmonic oscillator example we discussed before-- and this Minkowski state can be written as a [? squeezed ?] state in terms of [? their ?] vacuum.

And now you have to take the product of all possible modes, and only the j is the frequency for each mode. So this is a precise analog of the harmonic oscillator example, just because each set of modes gives you a harmonic oscillator. So you just take the [INAUDIBLE] product of all these harmonic oscillators, and then you have this relation.

Also, very similarly, the usual Minkowski creation and the relation operators are related. So this ajr, ajl by Bogoliubov transformations, just as the harmonic oscillator example. So we are allowed to write this transformation explicitly. But he's saying here, just direct generalize of the harmonic oscillator example because the field series is just a bunch of harmonic oscillators. A field series is just a bunch of harmonic oscillators. Any questions regarding this point? Yes.

AUDIENCE: So, just to check-- so the right only affects the right patch and its identity everywhere else.

HONG LIU: Sure.

AUDIENCE: So that the combination leaves the bottom and the top portion just fixed. So the combination leaves the bottom and the top quarter fixed?

HONG LIU: You don't get back to the-- yeah, when we-- yeah, so this is-- so this operation itself does not direct-- so this operation itself does not direct access to the top and bottom portion. And this is a [? trajectoral ?] for them. So if you have a point there, just take them to the [? hyperbolic ?]

trajectory. So we're not taking to there.

AUDIENCE: Right. And the top and the bottom are just-- it's [INAUDIBLE]. So it keeps them fixed?

HONG LIU: No. When you define the Hilbert space, you only define for the left and the right because they don't define Hilbert space. The Hilbert space defines the given time slice, so that only includes the left and the right. And those particles run into future evolution, and that evolution is not controlled by them because they only take you along the hyperbolic trajectory. Any other questions? Good?

So the third remark is that all of the discussions generalizes in complete parallel to Schwarzschild space time

In particular, we said before, the Schwarzschild time have the falling space time causal structure. You have a whole-- so this is your regional region outside the horizon, but then you can extend this part of the space time into four regions. In particular, you have two-- again, you have R and L-- two asymptotical regions. So this way the R goes to infinity, and similarly this way, also, R goes infinity.

So, again, in this [INAUDIBLE] vacuum state, which it can be defined from going to create the [? signature ?] and the [? compact ?] [INAUDIBLE] phi this tau. Again, corresponding to an entangled state between the left and the right. And if you ignore the left, again, you'll get a similar state from the right. So the story's completely in parallel. It's what we discussed before. The only different thing is the technicalities that, of course, in the specific metric are different. The specific metric are different.

And in particular, this [INAUDIBLE] vacuum can be obtained by doing Euclidean paths integral. Again, will be the half space, so this is the tau direction. Again, you do the half space, and the times S2. It's the same thing exactly as we did for the Rindler. You do the half space and when you're interpreting in terms of this tau [? foliation ?], which is angle, then you get this entangled state. You get this entangled state.

So any questions on this? Yes.

AUDIENCE: Just to clarify. So this entangled state is really nothing more than a mathematical trick to help us? Or should I think about it physically?

HONG LIU: No, this is a fact. This is not a mathematical trick. If we're grabbing that case, the Minkowski vacuum is entangled state. If you write it in terms of the Hilbert space over the left and the right patch. This is a mathematical fact. This is not a mathematical trick.

AUDIENCE: Well in the sense that I'm perfectly also allowed, I can create something which, in my patch, gives me the same description-- if I just think of a perfectly thermal state, and I don't even have to think about that being en entangled with anything. Is that also OK?

HONG LIU: Oh, sure. Yeah, but I'm just telling you-- if you are observer in the right patch, of course you will never see anything on the left patch. I'm just giving you a physical explanation. Where does that physically-- where does that thermal [? nature ?] come from?

AUDIENCE: Sure, OK. Yeah. I just wanted to clarify that.

AUDIENCE: Is this Minkowski [INAUDIBLE] defined in the upper patch?

HONG LIU: Sorry? Yeah. So you're talking about this upper patch?

AUDIENCE: Yeah.

HONG LIU: So the stories are [? falling. ?] In the standard [? QFT, ?] in the Minkowski spacetime, you define whatever your states at [INAUDIBLE] equal to 0, then you move this time. Then, of course, that will include this part. And then the standard Minkowski time evolution, in terms of capital T, will include this part. But if you do the Rindler time evolution, then you will not involve that part. Yeah. Is that what you are asking?

AUDIENCE: So it's not defined in the upper patch?

HONG LIU: Hmm?

AUDIENCE: So it's not defined in upper patch?

HONG LIU: No. It's not-- it just does not access those informations, because the time translation is always like this. The time translation will always-- will never take you there. You have to ask, what is your time translation? So in quantum mechanics, you define an initial state and then you have a [? tone ?] intake you wove into future time. And in-- then, depend on which time you use, then cover different regions of the Minkowski spacetime. If you use the standard Minkowski time, capital T, then that will cover everything. If you only use the Rindler time, then that only covers this region or this region. Any other questions? Good.

The fourth remark. So now, let me call this-- so this is r and l and then this f. OK? So let me call this region f. So this story, experienced perfectly in this Schwarzschild spacetime, why an observer in infinity will see, say, thermal radiation. But actually, this does not apply to the real-life black holes, because the black hole formed by gravitational collapse only have the right and the future region. You don't have the left region. You don't have the left region. OK? So this discussion does not apply. OK? This discussion does not apply.

But this is only one of the ways to derive that the black hole have a finite temperature. In fact, the Hawkings original derivation, just by considering scalar field, just by considering quantizing scalar field alone, in the right patch, and he already did deduce the thermal nature. So even though this particular discussion does not apply, all our conclusion, all our conclusion does apply. OK? Our conclusion does apply, including the finite temperature, et cetera. OK?

So now, to interpret, where does this temperature come from? And later we will say, actually, the black hole not only have a temperature, it can satisfy all the thermodynamics. So in this case, to interpret where this temperature come from is physically more intricate. OK? So I will not try to do it now. But later, when we talk about the duality, and then that will be a better place to go back to this question. And then we can ask the precise difference between these two cases. Between the case which you have all patches, and the case which you only have two pat-- only have two regions. And that they are, actually, physically fundamentally different. Physically fundamentally different.

But the reason those conclusion applies is because the temperature, in fact, is a state that can be made by the local observer. For local observer, outside the horizon, he's not going to tell the difference between this metric and the exac-- and the almost identical metric in outside horizon. Outside the horizon, Schwarzschild metric is a perfect one. So locally, he will not tell any difference. So that's why the local observer should always see the temperature. If you see it in one case, you will see it in the other case. OK? But the underlying physics will turn out to be very different. Yes?

AUDIENCE: How are we defining temperature here? Are we defining it by means of energy, like we do [INAUDIBLE]?

HONG LIU: Yeah. Yeah. Yeah. Yeah, here, we define it in terms of the density matrix.

AUDIENCE: Oh, it's defined by that equation?

HONG LIU: Right. Yeah. So the density matrix-- so if you have a density matrix like this, which z is traced explain [? to ?] [? matters ?] beta h, then you say the temperature is one over beta. So that's how we define temperature in quantum statistical physics. Yeah. Yeah. Yeah. So this defines a canonical example for you. And this beta is the temperature, the [INAUDIBLE] temperature. OK. So any questions on this? I hope not, because I want to discuss this later, not now.

AUDIENCE: Sir?

HONG LIU: Yes?

AUDIENCE: If we hold up a thermometer like this, should we get some different temperature than if we let it fall? It's accelerated right now. Can this has been measured?

HONG LIU: No. Because it's too low, the temperature. Right. Yeah. Yeah, because our temperature would be much bigger than this temperature you are able to-- yeah. Just the fluctuation of air in this room will create fluctuations in temperature which much higher than that kind of temperature.

AUDIENCE: In space, can be a vacuum.

HONG LIU: In the space, you also have to do very precise measurement. It's-- yeah, you have to calculate it. h bar is very small.

AUDIENCE: Divide by the mass.

HONG LIU: Yes?

AUDIENCE: To-- constructing on that same question. So the same thermometer that is being held in position. So if I observe it while sitting down here, so I'm accelerating with it, I'm like the Rindler observer, and I would see it at a certain temperature, right? Although it's very small, so we haven't been able to measure everything. But if I am, instead, free-falling while the thermometer is being held in place, would I then not see-- would I then not measure a temperature? Would I measure temperature?

HONG LIU: Yeah. Yeah. You wouldn't be able to see a temperature. Yeah. Free-falling. Although won't see--

AUDIENCE: So the thermometer is being held-- so the thermometer is accelerating?

HONG LIU: No. Thermometer is also free-falling. No, if you free-fall, the thermometer also free-fall.

AUDIENCE: No, no, I mean-- he holds the thermometer, and he is sitting in his chair, but I'm the one who measures the reading on the thermometer, and I'm free-falling.

HONG LIU: Yeah, that's an intricate experiment. Then we have to analyze that. So the photon from his thermometer then will somehow go into your eyes, and you will analyze it. Then whatever is the reading on his thermometer, you will see it. Yeah. No, you're not doing any measurement. You just see the reading on his thermometer. If his thermometer has a temperature, then you will see a temperature.

AUDIENCE: Right.

HONG LIU: Yeah, you are not doing a measurement yourself.

AUDIENCE: So, OK. So now, bringing it back to non-gravitational physics, flat spacetime. I'm a [? neutral ?] observer, and I see a thermometer accelerating by me. I would therefore see it as reading a certain temperature?

The thermometer itself isn't a measurement. You need to have something to measure yourself.

HONG LIU: Yeah, it's not [INAUDIBLE].

AUDIENCE: Sorry?

HONG LIU: No. No, the thermometer-- whatever thermometer is doing, what you are doing-- you just read the thermometer. And it has nothing to do whether the thermometer has a temperature. Thermometer maybe have a temperature due to some other reason. Whether you have-- what you could do is just read that thing. Yeah.

HONG LIU: OK. So let me continue.

OK. So now, we found a black hole has a temperature. So then, you only need to take a very small leap of faith. Saying, if this guy has a temperature, then it should satisfy thermodynamics. OK? Then we say, this must obey thermodynamics. OK? Also obeys thermodynamics. Black hole.

And then, you immediately deduce there should be entropy, because now you can just apply the first law. For example, we have the thermodynamical relation dSdE should be 1 over t. OK? Say, if we think of t as a function of e, then by integrating this equation, we should deduce what is the entropy of the black hole. So remember the t of the black hole is 1 over-- remember, the t of the black hole, the t of the black hole. Yeah, let me do it here. t b h of the black hole is h bar kappa divided by 2 pi, which is the h bar divided by 8 pi g n times m. So this would be just 8 pi g n times m h bar. Of course, you identify the mass of the black hole with its [? image. ?] So you identify them.

So now, you can just integrate this. You can just now become a trivial exercise. Then you find s e is equal to 4 pi g n e square divided by h bar. And the plus integration constant. OK? And this integration constant we can say to be zero, because if the black hole have zero mass, of course there's nothing there. And so then, we just have this formula.

So this formula can be written a little bit more geometrically. So also remember, the black hole-- the Schwarzschild radius is 2 g n m. OK? So this can be written a little bit in the geometric way. So this can be written in terms of s 4 pi r s square divided by 4 h bar g n. OK? Now g n goes to the downstairs, because the r s contain two powers of g n. So this is given by the horizon area of the black hole, divided by 4 h bar g n. OK?

So now we conclude-- we conclude just a bit-- let's connect these two formulas together. The temperature of the black hole is h bar times the surface gravity divided by 2 pi. And the entropy of a black hole is the area of the black hole, area of the horizon of the black hole, divided by 4 h bar g m. So as I said before, for the black hole, there are two very important geometric quantities. One is kappa, surface gravity. The other is the horizon area. And then they enter in a very-- in a nice and simple way, into the temperature and the horizon-- the entropy of a black hole. And let me call this equation one. This is an important equation.

So let me just note one thing. This temperature is rather funny, if you look at that formula, because the mass is in the downstairs. Say, if you increase the mass, then the temperature actually decrease. This is actually opposite to your everyday experience. OK? Because when we increase the mass, the black hole temperature decreases. OK? If you calculate the, say, specific heat, the specific heat is smaller than zero. OK.

So we will later see this is actually an artifact of a black hole in asymptotically flat spacetime. So here, we're considering black holes in asymptotically flat spacetime. If you think about black holes, say, in the spacetime like [INAUDIBLE] space, and then actually, the temperature will go up with the mass. As in the ordinary story. OK? So this is just the third remark.

So another third remark is that this equation, these equations tend not to be universal. So we derive it to the simplest Schwarzschild black hole, but actually, those relations apply to all black holes have been discovered. Just apply to every black hole. OK?

So now let me talk about general black holes. So we are mostly just making some statements. Because most of the statement I make here, they are highly nontrivial. And each statement may take one lecture to prove, or even more, so I will not really prove them. I just [? coat ?] them. And I can also not prove it on the spot.

So first is something called the no-hair theorem. So no-hair theorem says that stationary-- stationary's a key word-- and asymptotically flat-- this is also a key word-- black hole is fully characterized by the first, mass. Second, angular momentum. Third, conserved gauge charges. OK?

So the Schwarzschild black hole we talked about corresponding to a special case, which angular momentum is zero, and the conserved charge-- yeah, conserved charge-- for example, the electric charge. For example, electric charge. OK? And so-- yeah, so let me just give some-- so typically we denote mass by m, and angular momentum by j, and the electric charge by q. So the Schwarzschild black hole goes one into the j equal to zero, and the q equal to zero, but more general black holes you can have both j and the q.

So for our-- in string theory, there can be many, many different gauge fields. So in string theory, actually there are many, many different charges. So black holes, in string theory, can have many, many more charges than, say, just in the standard model. Just in the standard model. Yeah. Just for all those black holes, this equation one still holds. OK? Yes?

AUDIENCE: So in proving this theorem, we're going to kind of start off with a certain definition of what is a black hole and what isn't a black hole. So what is the key feature that defines it? Because there's lots of metrics. And some of them are characterized by only these three things, and some are not, and--

HONG LIU: You must have an event horizon.

AUDIENCE: Just the presence of some horizon?

HONG LIU: Yeah. So event horizon. Yeah.

AUDIENCE: OK. But in Rindler we have an event horizon.

HONG LIU: Hmm?

AUDIENCE: In Rindler we have an event horizon that's not a black hole [INAUDIBLE].

HONG LIU: That's true. You should at least have object. You should have some mass. You should have some quantum number.

AUDIENCE: I could be accelerating next to the [INAUDIBLE].

HONG LIU: So, no, no, no, no, no. Rindler is called observer horizon. It's observer-dependent horizon. And in the black hole, it's not.

AUDIENCE: So you cannot go to any frame where there is no [INAUDIBLE]. What if you are free-falling?

HONG LIU: Hmm?

AUDIENCE: What if you are free-falling?

HONG LIU: No no no. Yeah, what I'm just saying that the-- here, here the horizon, you can-- different observers have different horizons.

AUDIENCE: So you can [INAUDIBLE] the horizon out of the picture?

HONG LIU: So here, the horizon is arbitrary. It depends on your observer. Even though I draw here, but-- this does not have to cross the origin. This can be anywhere. It's anywhere. Just here, there's no [? generating ?] a horizon.

AUDIENCE: But if I'm free-falling into a black hole, I also don't have a horizon, right?

HONG LIU: No.

AUDIENCE: Because I'm causally connected [INAUDIBLE].

HONG LIU: That's true, but independent of that, there's the spacetime structure, there's a horizon there. In the spacetime structure of the Minkowski space, there's no horizon. In order to talk about [INAUDIBLE] horizon, you have to talk about specific observer to have specific motion. Yeah, I write down the Schwarzschild metric. And the different [INAUDIBLE] in the way. There is already an event horizon.

AUDIENCE: How do you write down a metric in a [INAUDIBLE] invariant way?

HONG LIU: No, I'm just saying the motion of the horizon is a [INAUDIBLE] variant.

AUDIENCE: OK. Yeah. But if I'm in a frame that's free-falling, then I don't see a horizon, right? Isn't that [INAUDIBLE] which transported me to a frame where there is no horizon?

HONG LIU: Maybe let me say this. If this can make you a little bit happier. To a [? symptotic ?] observer, there's a horizon. There's a different [INAUDIBLE] invariant horizon.

AUDIENCE: OK.

HONG LIU: Yeah. Yeah, yeah. Yeah, if you don't want to fall into the black hole, for those people, there's a horizon.

[LAUGHTER]

And the difference from here, if you fall through the horizon, nothing happens.

So this no-hair theorem is remarkable, because it says if you have a star which collapsed to form a black hole, in this process, all the features of the star were lost. Because black hole only are characterized by those numbers. OK? But star, you can characterize in many, many other different ways. But the black hole, essentially, don't have any features. So this is called no-hair theorem. All features. Yes?

AUDIENCE: What if I add something like [INAUDIBLE] scalar field or other field into that [INAUDIBLE]. [INAUDIBLE] theorem?

HONG LIU: Yeah. Yeah. The story is a little bit more complicated. Let's try to not go into that. There's something called a secondary [INAUDIBLE], et cetera. Yeah. It can. It can, but this is refers to Einstein. But if you go to the frame of Einstein plus-- Einstein plus some meta-field, and then this as a statement is true. Any other questions?

OK. So now-- so historically, of course, people did not discover the temperature first. So historically, people first have the no-hair theorem. So it seems like black hole is completely featureless. OK? It's a very boring object that don't have any feature. And then, people discovered the so-called four laws of black hole mechanics. For general black holes-- for general stationary black holes, again.

So the zeroth law-- again, we just [? coat ?] them. [INAUDIBLE] is gravity. Kappa is constant over the horizon. The first law is that if you change the mass of the black hole a little bit-- OK, imagine you put something, throw something into a black hole, you change the mass of the black hole a little bit, then you find such a relation. a is the area of the horizon

So J is the angle of momentum. And omega is the angle of where you can see the horizon. So if you have angle of momentum on the black hole, we would be rotating. So omega is the angle of frequency.

And fie is the electric potential. So if you have a charge, than the back hole, all the current electric fields, this is electric potential at the horizon. And you always, in this old notation, you normalize the electric potential to infinity to be 0.

So this is the first law. It just say if you change mass of a black hole, and change some angle of momentum and change some charge then to the first order they satisfy this relation. They satisfy this relation.

So this is just purely mechanics, a classical of gr. This is pure, classical gr. And then there's a second law, which is also classical gr, is that horizon area never decreases. And the third law, says the surface gravity-- let me just call it kappa. This kappa surface gravity over black hole cannot be reduced to 0 in the finite number of steps.

AUDIENCE: What do you mean by number of steps?

HONG LIU: Find the number of procedures.

AUDIENCE: Like?

HONG LIU: Say if each time you throw a particle to a black hole, this is called a step.

AUDIENCE: Sure. OK. Thermometer, like [INAUDIBLE].

HONG LIU: That's right. Yeah. This is called a step. Yeah. Right. OK. So all of these are classical statement. And so this tells the second law.

For example, if you throw something to a black hole, the black area will increase. So if you collide the two black holes, and then if you collide two black holes, they will merge into a bigger black hole. And this bigger black hole, the area will be larger than the sum of the area of two black holes-- than the area of two black holes. Yeah.

So of course this law just-- these four laws then become immediately just like the four laws of thermodynamics. You make this identification of one. This identification of one-- so if it's one, this just becomes the four laws of thermodynamics. Yeah. Thermodynamics. The four laws of thermodynamics.

In particular, the first law, if you substitute the copper and A by the temperature under the entropy, than this just becomes the standard, the first law, in particular. The first law, it's just dm, sdt, tds plus omega dj plus 5 dq. So this is really the first law of thermodynamics.

So historically, these four laws of mechanics actually was discovered before Hawkings radiation. So first they discovered this black hole law theorem. And then they discovered these four laws of black hole mechanics.

Then they say, ah, this is really look like thermodynamics. And they even patterned these four laws to precisely like the four laws of thermodynamics. But they could not imagine the black hole was a thermodynamic object. They could not imagine the black hole was thermodynamic object.

So they were saying, if you look at the old paper-- there was a very famous paper by Bardeen, Carter, and Hawking, which discuss these four laws of black hole mechanics. And they said, this four laws of black hole mechanics should actually transcend the standard of thermodynamics. The black hole actually should transcend all of these, our traditional physics.

But in 1971 or 1972, a young graduate student called Bekenstein, so he was a graduate student at Princeton. So he was studying under a guy called Wheeler, studied under Wheeler. And so he was very uncomfortable with the fact that if you throw something into a black hole, than that thing is gone.

So he was a very uncomfortable with that. Because if you throw something to a black hole, it's gone, then he concluded that's really the second law of thermodynamics. Because if you throw to a black hole, that thing is gone, then the entropy associated with that thing is gone. And the black hole is just black hole, have this no-hair theorem. And then you violate the second law of thermodynamics.

People like Wheeler or Hawking, they say, ah, this is great. Black hole transcend the thermodynamics. But Bekenstein was uncomfortable. He thinks thermodynamics should transcend black hole.

And then based on the second law of the black hole, then he said, so maybe-- so he wrote a series of papers, a few papers. I don't remember. He say, if we think black hole has entropy proportional to the area, then the second law of thermodynamics can be saved. Because this area of level decrease, and if you throw something to a black hole, even thought that, the entropy associated to that guy's lost. But that area also increased. The area also increased. And then you can say it's the second law of thermodynamics.

Now, he actually proposed to generalize the second law of thermodynamics. So he proposed a generalized second law. He said, you take the total, if you take the total entropy of the system to be that of the black hole, and then matter field outside the black hole, then this ds total must be non-decreasing.

Of course, now if we accept, if we take this leap of faith to really think black hole as a sum or object, then of course this generalized second law has to be true because the thermodymaics-- some object. But when Bekenstein proposed it, it was really bizarre to say in a nice way because it just sounded crazy, just outright crazy.

Because how can black hole have entropy? Black hole absorb everything. Just how can it have entropy? And it just completely was disregarded by people. And it was discarded by people. And anyway, but then now that he was right, then of course after Hawking's discovery of a Hawking radiation, they become very natural for the black hole to have entropy.

And in particular, this formula, after you determine that the black hold have this temperature, after you fix this pre-factor, then you can also, just from the first law, just from here, just from here, to fix that pre-factor of a black hole. So Bekenstein could not decide what's the proportional constant. But once you get the temperature, then this proportional constant just uniquely fixed from this equation, so without using that. Just using the first law of mechanics, then you can immediately integrate that as entropy. Anyway--

AUDIENCE: How did Bekenstein find--

HONG LIU: It's a postulate. It's a gas. Yeah. He just postulate, if we imagine--

AUDIENCE: He didn't derive it?

HONG LIU: No there's no way to derive it. He was just saying, if you imagine black hole has entropy proportion to the area, than the second law of thermodynamics can be saved. And he wants to save the second law of thermodynamics. Yes.

AUDIENCE: So one question. So this makes sense classically. If I have a system of entropy, I throw them to a black hole. Entropy is like ignorance. If I throw ignorance into the black hole, maybe there's somehow the black hole also becomes more ignorant or something.

But what does this mean in terms of quantum mechanics when I have a pure state? From quantum statistical mechanics, substance the entropy, you just don't know something about your state. And it's your fault. And it's not like the black hole should car if it's your fault or not or something. So why does this makes sense quantum mechanically? Maybe it doesn't.

HONG LIU: You mean, why does black law have entropy make sense quantum mechanically?

AUDIENCE: Yes.

HONG LIU: It's the same thing as-- this room has entropy. This room we use quantum statistical physics. If you believe black hole is an ordinary object, then--

AUDIENCE: We only quantum statistical physics because we're ignorant, the full state of--

HONG LIU: Yeah. For a black hole, we are also ignorant. Yeah. This is actually something I'm going to talk about now. Any other questions?

AUDIENCE: I have something [INAUDIBLE]. So basically, I think it can be [? a healing ?] experiment. So where we hear things of [INAUDIBLE]. There is one unit of entropy, and it enters black hole. It increased the energy of black hole, which increased the mass of black hole, which increased the [INAUDIBLE] of the black hole. So in this way you can actually derive the semi-qualitatively derived out of proportionality.

HONG LIU: Yeah. To derive with these precise [? cohortions-- ?] no, I don't think they derive the [? cohortion ?], but I need to check. I don't think there's any way to derive the-- you can, say, put some bond on the [? cohortions ?]. But you cannot derive the [? cohortions ?]. I think that argument would not enable you to derive the [? cohortions. ?]

Anyway, so let me just mention a few more things about black hole. This is just pure-- actually, I'm running out of time. So let me just mention some paradox or paradoxes for the black hole.

So we have shown that the black hole is some object. So Jordan just asked. But we know the ordinary thermodynamics has statistical physics behind it. So the immediate question is actually, does the black hole entropy, for example, have a statistical interpretation?

So this is one question. And another question is that, does black hole actually respect quantum mechanics? Does black hole respect quantum mechanics?

So if black hole entropy have a statistical interpretation, then this give you a very-- so it black hole have a statistical interpretation that means-- so A, if affirmative, that implies that each black hole, even though black hole at a macroscopic level only is characterized by these three things, But macroscopically, must have internal states. Or, maybe I should call it macrostate of order to the entropy, which is the A of the black hole area 4h bar e.

So hidden behind this no-hair theorem is actually a huge number of macrostates. Just like the air in this room, even though we describe the unit temperature, pressure, and the energy stature, but given that macroscopic data, they can be huge number of macro states. And then a similar thing should happen to the black hole.

And in order to see that the black hole does have, a statistical interpretation, then you have to find so many states for a black hole and in order to answer the question A. So this question has to be answered in the affirmative for many different type of black holes. Say in string theory and also [INAUDIBLE] spacetime, and they see the spacetime. Using string theory method or using this holographic duality, we will see examples later. We will see examples later.

So these really confirms that the black hole is really a quantum statistical system. So regarding this question B, then this long time paradox-- so this A has also been a paradox for many years, and was only resolved-- the basic calculations were able to do only in 1996, when the [INAUDIBLE], they did some very special [? Schwarzschild ?] black hole, which they counted this exact number of states. They counted the exact, this number of states, and about for a very specific type of black hole.

So B is rated to the so-called Hawkins information paradox, information loss paradox. So I will not have time to go into detail here. Let me just give you a very rough version of it. So you can see the pure state, could see the star, a big star in the pure state. So we know from gr that a sufficiently massive star will eventually always collapse to form a black hole.

So if we imagine you have a pure star in the pure state collapse to form a black hole, and if quantum mechanics is preserved throughout the process, then this black hole should also be a pure state. So that means the black hole should be just one of all those max number of possible states. There should be a pure state, but only one of those all possible states.

But then Hawkins had an argument saying this is impossible. Because if black hole is a pure state, then when black hole evaporates-- so the black hole evaporates, eventually the black hole will be gone. So the one funny thing about black hole is that the temperature is inverse proportionate to the mass. So when the black hole is big, then the temperature is low, then the radiation is small.

But when you start radiate, then the black hole mass will decrease. And then the temperature will be higher than we radiate mole. So it will be acceleration process. And eventually-- presumably black hole will be gone. So this kind of semi-classical argument that we are given applies only for a massive black hole much greater than the planck mass, only much larger than a planck mass.

So below planck mass, what happens? Nobody knows. But at least this radiation statement should be robust for the mass much, much larger than the planck mass. Now Hawking then say this is a paradox because according to his calculation, the radiation is simple. And we know that the sum of radiation does not contain any information. It cannot contain information but it's pure state, because sum or radiation is information free.

And so the sum of radiation which come out, come out until you reach say the mass of all the planck mass. And here we reach the planck mass. And then before you reach that mass, because the radiation is simple, there can be no information can come out. And when you reach that planck mass, it just becoming possible for such a huge amount of internal state to be encoded in the planck mass object. So he concluded that the information must be lost, and the black hole must violate mechanics.

So this is a very heuristic argument. But I highly suggest, if you are interested, to go read his original paper, which is very beautiful. Because he was really trying to think of black hole as a ordinary quantum mechanical object.

And the way he was thinking about it is really very nice. And actually, it's not very different from we are thinking about black hole right now, from the holographic duality. He was really thinking that's it's a quantum mechanical object. But then he reached this paradox.

Anyway, so this paradox had bothered the people for more than 30 years. So he discovered the Hawking radiation in 1974. I think he proposed this paradox in 1976. So for more than 30 years, people argue with each other what is going to happen. It's typically divided into two camps.

So the gr people, they think black hole is everything, quantum mechanics nothing. And the black hole must be able to violate quantum mechanics and will bring us to a new frontier we never see before. And the particle, these people are saying, a black hole-- oh, we can even creating the accelerator-- must obey quantum mechanics. So people would just argue with each other and without really setting the question in a very convincing way to either camp.

But this holographic duality, in the context of holographic duality, then the black hole [INAUDIBLE] spacetime, then you can actually rephrase this question about the black hole information laws. And the holographic duality strongly suggests-- I think it's maybe not really completely proved-- strongly suggests at least that the black hole is just a ordinary quantum mechanical object. We are not transcend the quantum mechanics. We are not transcend the quantum mechanics. Yeah. I am really out of time. Yes?

AUDIENCE: So you said the star is in a pure state. But how can that be, because it has a temperature and it's also thermal system? So how can you put it in a pure state?

HONG LIU: No. Black hole does not have to be in the thermal state. No. I can certainly imagine a star, which is in the pure state. In real life, maybe it's hard to construct them. But on the paper, I can do it.

[LAUGHTER]

In principle, so how many atoms in the star? I don't know. Maybe 10 to 100? Now let's imagine there's 10 to the 100 atoms.

AUDIENCE: But it's radiating all the time though. Doesn't it entangle with things and make it not pure?

HONG LIU: Don't worry. Don't worry. I can certainly write down a wave function for 10 to the 100 particles, which is in the pure state. And this will be a big object. And according to the rule of gr, this thing will collapse to form a black hole.

AUDIENCE: What about the light that it's emitting, which it has--

HONG LIU: No. There's no light. No. We work with zero temperature, just pure state. There's nothing.

There's nothing.

AUDIENCE: No temperature? Zero temperature?

HONG LIU: Yeah. In principle, I can do that.

AUDIENCE: In that case, will the black hole have the temperature, Hawking temperature?

HONG LIU: Yeah. The black hole will have Hawking temperature.

AUDIENCE: But it's still in it's first state?

HONG LIU: The black hole will have a Hawking temperature, will have-- similar, have entropy, but will be a pure state. So this is the essence of the information paradox. So this is the essence of the information paradox. And then we will be able to explain it, why this is so, using the holographic duality.

AUDIENCE: So as the star collapses, it gains a non-zero temperature. It starts at zero temperature, and then collapses, and becomes [INAUDIBLE].

HONG LIU: Yeah. It can. Yeah. It seemingly have a long zero temperature. Yeah. Yeah. Maybe let's stop here.