VII Biological Oscillators

During class we consider the following two coupled differential equations:

$$\begin{aligned}\dot{x} &= -x + ay + x^2 y \\ \dot{y} &= b - ay - x^2 y\end{aligned} \qquad \textbf{[VII.1]}$$

From the phase plane analysis (see L9_notes.pdf) it was clear that for certain values of a and b this system exhibits periodic oscillations as a function of time. Let us analyze [VII.1] in more detail. The nullclines are:

$$\begin{aligned}y &= \frac{x}{a + x^2} \\ y &= \frac{b}{a + x^2}\end{aligned} \qquad \textbf{[VII.2]}$$

There is only one fixed point (x^*,y^*):

$$\begin{aligned}x^* &= b \\ y^* &= \frac{b}{a + b^2}\end{aligned} \qquad \textbf{[VII.3]}$$

The matrix A is (using [V.4] and [V.5]):

$$A = \begin{bmatrix} -1 + 2x^* y^* & a + (x^*)^2 \\ -2x^* y^* & -(a + (x^*)^2) \end{bmatrix} \qquad \textbf{[VII.4]}$$

The determinant and trace are:

$$\begin{aligned}\Delta &= a + b^2 > 0 \\ \tau &= -\frac{b^4 + (2a-1)b^2 + (a + a^2)}{a + b^2}\end{aligned} \qquad \textbf{[VII.5]}$$

The fixed point is stable when $\tau < 0$. The region in a-b-parameter space where the system is oscillating (stable limit cycle) and is not oscillating (stable fixed point) is illustrated in Fig. 10.

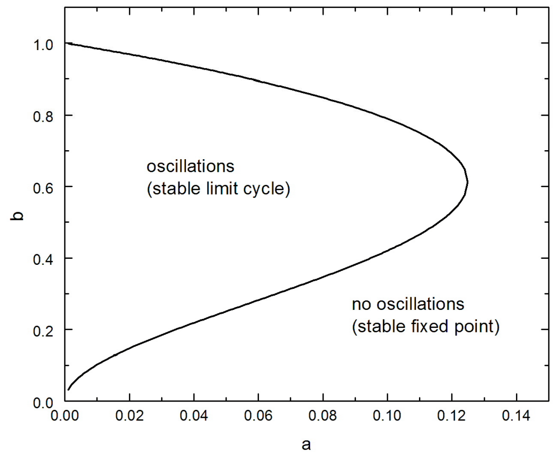

Figure 11. a-b-parameter space indicating for which values of a and b the system exhibits stable oscillations and a stable fixed point

MATLAB code 5: Limit cycle

```
% filename: cyclefunc.m

function dydt = f(t,y,flag,a,b)

dydt = [-y(1)+a*y(2)+y(1)*y(1)*y(2);
         b-a*y(2)-y(1)*y(1)*y(2)];

plot(y(1),y(2),'.');
drawnow;
hold on;
axis([0 2 0 2]);
```

```
% filename: limitcycle.m
close;
clear;

a=0.1;
b=0.5;

options=[];

[t y]=ode23('cyclefunc',[0 50],[0.6 1.4],options,a,b);

plot(y(:,1),y(:,2));
```