8.512 Theory of Solids II
Spring 2009

8.512 Theory of Solids II **Problem Set 10** **Due April 29, 2009**

1. (a) Using linear response theory, derive the following expression for the magnetic susceptibility $\chi_\parallel = \partial M_z / \partial H_z$.

$$\chi_\parallel = \lim_{q \to 0} \int \frac{d\omega}{2\pi} \langle S_z(q,\omega) S_z(-q,-\omega) \rangle \frac{\left(1 - e^{-\frac{\hbar\omega}{kT}}\right)}{\omega}$$

   (b) Provided that the total magnetization $M_z = \sum_i S_{iz}$ commutes with the Hamiltonian, we can start from the expression $F = -kT \ln Tr\{e^{-\beta(H - M_z H_z)}\}$ and take derivatives with respect to H_z to derive the simpler expression

$$\chi_\parallel = \frac{1}{kT} \langle M_z^2 \rangle$$

   Show that this is consistent with the more general expression obtained in 2(a). [Hint: in this special case $\lim_{q \to 0} \langle |S_z(q,\omega)|^2 \rangle \sim \delta(\omega)$.]