8.512 Theory of Solids II
Spring 2009

8.512 Theory of Solids II **Problem Set 8** **Due April 13, 2009**

Type II Superconductor.

We begin with the Ginzburg-Landau free energy

$$F = \Delta f \int d\boldsymbol{r} \left\{ \frac{T - T_c}{T_c} |\psi|^2 + \frac{1}{2}|\psi|^4 + \xi^2 \left| \left(\frac{\nabla}{i} + \frac{2eA}{c} \right) \psi \right|^2 \right\} \tag{1}$$

and consider T slightly below T_c.

1. Calculate the free energy difference between the superconducting state and the normal state and show that the thermodynamic critical field H_c is given by

$$\frac{H_c^2(T)}{8\pi} = \Delta f \frac{1}{2} \left(\frac{T_c - T}{T_c} \right)^2 \tag{2}$$

2. Now turn on a uniform magnetic field H such that we are in the normal state ($\psi = 0$) and gradually reduce H. We look for an instability towards $\psi \neq 0$. The field at which the instability happens is defined as $H_{c2}(T)$. We calculate $H_{c2}(T)$ by the following steps.

   (a) Using the condition $\delta F = 0$ under a variation of ψ, show that ψ satisfies

$$\xi^2 \left(\frac{\nabla}{i} + \frac{2eA}{c} \right)^2 \psi + \frac{T - T_c}{T_c} \psi + |\psi|^2 \psi = 0 \quad . \tag{3}$$

   (b) Near the critical point, the last term in Eq. (2) can be ignored and we have a linearized equation. Instability ($\psi \neq 0$) occurs when the linearized equation has a negative eigenvalue. Notice that this equation has the same form as that of a single electron in a magnetic field, where the solution is known to be Landau levels. Use this fact to show that

$$H_{c2} = \frac{(T_c - T)}{T_c} \frac{(\phi_0 / 2\pi)}{\xi^2} \tag{4}$$

   where $\phi_0 = hc/2e$.

(c) Calculate the London penetration depth $\lambda_{\rm L}(T)$. Express H_c in Eq.(2) in terms of the temperature dependent coherence length $\xi(T) = \xi\left(\frac{T_c-T}{T_c}\right)^{1/2}$ and the London penetration depth $\lambda_{\rm L}(T)$. Show that the condition $H_{c2} > H_c$ implies

$$\kappa = \frac{\lambda_{\rm L}(T)}{\xi(T)} > \frac{1}{\sqrt{2}} \quad . \tag{5}$$

Equation (5) is the condition for type II superconductivity.