8.512 Theory of Solids II
Spring 2009

8.512 Theory of Solids

Problem Set 8

Due April 22, 2004

1. (a) Using linear response theory, derive the following expression for the magnetic susceptibility $\chi_{\parallel} = \partial M_z / \partial H_z$.

$$\chi_{\parallel} = \lim_{q \to 0} \int \frac{d\omega}{2\pi} \langle S_z(q,\omega) S_z(-q,-\omega) \rangle \frac{\left(1 - e^{-\frac{\hbar\omega}{kT}}\right)}{\omega}$$

   (b) Provided that the total magnetization $M_z = \sum_i S_{iz}$ commutes with the Hamiltonian, we can start from the expression $F = -kT \ln Tr\{e^{-\beta(H - M_z H_z)}\}$ and take derivatives with respect to H_z to derive the simpler expression

$$\chi_{\parallel} = \frac{1}{kT} \langle M_z^2 \rangle$$

   Show that this is consistent with the more general expression obtained in 1(a). [Hint: in this special case $\lim_{q \to 0} \langle |S_z(q,\omega)|^2 \rangle \sim \delta(\omega)$.]

2. Using the results of Problem 1,

   (a) Calculate the low temperature $\chi_{\parallel}(T)$ for a Heisenberg antiferromagnet. Show that it is proportional to T^2.

   (b) For an antiferromagnet with an Ising anisotropy, argue that $\chi_{\parallel} \sim e^{-\Delta/T}$. What is the value of Δ?