8.512 Theory of Solids II
Spring 2009

8.512 Theory of Solids

Problem Set 5

Due March 18, 2004

1. Consider a two dimensional superconductor with a d-wave energy gap given by

$$\Delta(\phi) = \Delta_0 \cos 2\phi \ .$$

Assume an isotropic energy band with Fermi velocity ν_F in the normal state. The quasiparticle spectrum is given by

$$E(\boldsymbol{k}) = \sqrt{\nu_F^2 \left(|\boldsymbol{k}| - k_F\right)^2 + \Delta^2(\phi)} \ .$$

(a) Show that the energy gap vanishes at 4 points on the Fermi surface. In the vicinity of these nodal points, show that the quasiparticle dispersion is given by

$$E(\boldsymbol{k}) = \sqrt{\nu_F^2 k_1^2 + \nu_2^2 k_2^2} \ ,$$

where k_1 and k_2 are momentum components perpendicular and parallel to the Fermi surface measured from the nodal points. What is ν_2 in terms of Δ_0 and k_F ? Show that the density of states at energy E per node per spin is $\frac{1}{2\pi\nu_F\nu_2}E$.

(b) Show that at low T, thermal excitation of the quasiparticles leads to a linear T reduction of the superfluid density

$$\frac{\rho_s}{m}(T) = \frac{\rho_s}{m}(T=0) - \frac{2\ln 2}{\pi}\frac{\nu_F}{\nu_2}T \ .$$

The integral you encounter can be done by a change of variable $y = e^{-x}$.

(c) In the presence of $\boldsymbol{A}$ and $\nabla\theta$ where θ is the phase of the order parameter, the quasiparticle spectrum is changed by

$$E(\boldsymbol{k}, \boldsymbol{A}) = E(\boldsymbol{k}) + \boldsymbol{\nu}_F \cdot \frac{1}{2}\left(\boldsymbol{\nabla}\theta + \frac{2e}{c}\boldsymbol{A}\right)$$

The last term is the gauge invariant generalization of the term we discussed in class. Consider a single vortex and assume the superconductor is extreme type II.

At a distance R away from the vortex core in the $\hat{x}$ direction, calculate the density of states which is generated at the Fermi level. (Assume $\xi_0 \ll R \ll \lambda_L$.) How is your answer different if you approach the vortex core in the (1,1) direction?

(d) In an external field H, a triangular vortex lattice is formed. Show that the density of states found in (c) gives rise to the following unusual contribution to the specific heat

$$c_\nu = \alpha\sqrt{H}T \ ,$$

Make a crude estimate of the coefficient α.

For an experimental confirmation of the prediction first made by G. Volovik, *JETP Lett.* **58**, 469 (1993), see K. Moler *et al.*, *Phys. Rev. Lett.* **73**, 2744 (1994).

2. Make a table for the real part of the transverse and longitudinal response functions $K_\perp$ and $K_{||}$. Give the limits $\omega = 0, q \to 0$, and $q = 0, \omega \to 0$ for a perfect metal, a disordered metal, and a superconductor with or without disorder (16 quantities in all!). Write the leading nonvanishing contributions in terms of physical quantities such as Landau diamagnetism, conductivity and scattering lifetime.