8.512 Theory of Solids II
Spring 2009

8.512 Theory of Solids

Problem Set 4

Due March 11, 2004

1. The response function $K_{\mu\nu}$ defined by

$$J_\mu = -K_{\mu\nu} A_\nu$$

can be decomposed into the transverse and longitudinal parts.

$$K_{\mu\nu}(\boldsymbol{q}, \omega) = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) K_\perp(q, \omega) + \frac{q_\mu q_\nu}{q^2} K_\parallel(q, \omega)$$

   (a) Starting from the linear response expression, calculate $K_\perp(q, \omega = 0)$ for a free Fermi gas. [It may be useful to choose $\boldsymbol{q} = q\hat{z}$ and compute K_{xx}.]

   (b) Using the results from (a), show that the Landau diamagnetic susceptibility (including spin degeneracy) is given by

$$\chi_D = -\frac{e^2 k_F}{12\pi^2 mc^2}$$

   Check that this is -1/3 of the Pauli spin susceptibility. For an alternative derivation using Landau levels, please study the discussion in Landau and Lifshitz's *Statistical Physics,* Vol. 1, p.173.

2. This problem deals with the nuclear spin relaxation rate $1/T_1$, which is measured in NMR experiments. We model the nuclear spin by a two level system with spin operator $\boldsymbol{I}$. We assume the contact interaction $H = A\boldsymbol{I} \cdot \boldsymbol{S}(0)$ where $\boldsymbol{S}(r) = \Psi^\dagger_\alpha(\boldsymbol{r})\boldsymbol{\sigma}_{\alpha\beta}\Psi_\beta(\boldsymbol{r})$ is the spin operator for the electron.

   (a) If the nuclear spin is initially polarized to be up, show that the relaxation rate is given by

$$\frac{1}{T_1} = \frac{2A^2}{\hbar^2} \int_{-\infty}^{\infty} dt \left\langle S^+(t, r = 0) S^-(0, r = 0) \right\rangle \cos \omega_n t \qquad (1)$$

   where $\omega_n = \mu_n H/\hbar$ is the nuclear precession frequency, which is much less than the typical electronic energy scale and can be set to zero.

(b) By converting Eq.(1) to Fourier space, calculate $\frac{1}{T_1}$ for a free fermion gas at temperature T. Show that $\frac{1}{T_1 T} \propto [N(0)]^2$ where $N(0)$ is the density of states at the Fermi energy. This is known as the Korringa relation.

Please read p. 79–72 in Schrieffer's book on superconductors (or p.266 of Phillips' book) to understand how $1/T_1$ is modified by the onset of superconductivity. Note the appearance of the coherence factors which we encountered in the calculation of the superfluid density.