8.512 Theory of Solids II
Spring 2009

**8.512 Theory of Solids**

**Problem Set #2** **Due: February 24, 2004**

1.) Estimate the mean free path for plasmon production by a fast electron through a metal by the following steps:

(a) for small $q$ it is a good approximation to assume that $\varepsilon^{-1}(q,\omega)$ is dominated by the plasmon hole, i.e.

$$\mathrm{Im} - \frac{1}{\varepsilon(q,\omega)} \approx A(q)\delta(\omega - \omega_{pl}) .$$

Determine the constant $A(q)$, using the f-sum rule.

(b) Using (a), write down an expression for the probability of scattering into a solid angle $\hat{Q}$ by emitting a plasmon.

(c) Estimate the mean free path for plasmon emission. Put in some typical numbers (electron energy = 100 KeV, etc.)

2.) Dielectric constant of a semiconductor.

(a) In a periodic solid, show that the dielectric response function is given within the random phase approximation by

$$\varepsilon(q,\omega) = 1 + \frac{4\pi e^{2}}{q^{2}} \sum_{k,G} \frac{\left|\left\langle \vec{k} + \vec{q} + \vec{G} \right| e^{i\vec{q}\cdot\vec{r}} \left| \vec{k} \right\rangle\right|^2 \left( f(\varepsilon_k) - f\left(\varepsilon_{\vec{k}+\vec{q}+\vec{G}}\right)\right)}{\varepsilon_{\vec{k}+\vec{q}+\vec{G}} - \varepsilon_{\vec{k}} - \omega - i\eta} \qquad (1)$$

where $\vec{G}$ are the reciprocal lattice vectors.

(b) We will evaluate Eq. (1) in an approximate way for a semiconductor in the limit $\omega = 0$ and $q \to 0$. We will work in the reduced zone scheme. Argue that the energy denominator in Eq. (1) may be replaced by the energy gap $\Delta$. To estimate the numerator, derive the following theorem:

$$\sum_b (\varepsilon_b - \varepsilon_a) \left| \langle b | e^{i\vec{q}\cdot\vec{r}} | a \rangle \right|^2 = \frac{\hbar^2 q^2}{2m} \qquad (2)$$

where $|a\rangle$ and $|b\rangle$ are the eigenstate of a Hamiltonian H with a kinetic energy term $-\hbar\nabla^2/2m$. This is a generalization of the f-sum rule in atomic physics. It is proven by evaluating the expectation value of

$$\left[\left[H, e^{i\vec{q}\cdot\vec{r}}\right], e^{-i\vec{q}\cdot\vec{r}}\right]$$

in the state $|a\rangle$.

(c) By making the further approximation that the energy difference in Eq. (2) may be replaced by the energy gap Δ, show that for a semiconductor

$$\varepsilon(q \to 0,\ \omega = 0) = 1 + \left(\frac{\hbar\omega_{pl}}{\Delta}\right)^2$$

where $\omega_{pl}$ is the plasma frequency. Estimate $\varepsilon$ for Si and Ge and compare with experiment.