PROFESSOR: So good afternoon, everybody. You see the topic for today's class on the screen-- entangled photons. Just to remind you, when we talked about single photons and Mach-Zender interferometers, we realized that when we have a nonlinear interferometer where one mode, you can say one beam, affects the phase shift in the interferometer for the other photon. Then we get a nonlinear situation, and we can create photon states which no longer factorize.

So we have a system a, a system b, and we can no longer write down the total wave function in a wave function for system a times system b. And this is something very interesting, and this is what we feature in this section.

So we finished the last class by defining entanglement. And just to remind you, we said something is entangled if it is impossible to write it in a product of two wave functions. So therefore, if you have some correlation between the two systems, then we call the two systems and the state of the system entangled.

Now this is a definition which needs explanation, so we went through some examples. And we showed that certain states which on first sight look entangled are not entangled, because if you try harder, you find the way to factorize the state.

So I want to continue now in explaining different aspects of the definition, in particular what it means to have a system which has two subsystems, a and b. Before I do that, do you have any questions up to that point?

So we want to talk about some standard entangled state. And the most basic state is a single state.

So if you have a state which is 0 1 minus 1 0 normalized, this is an Einstein-Podolsky-Rosen state. This needs some explanation. We have often encountered, in physics, states which are simply a superposition of if you interpret 0 1 as spin up or spin down, you encounter that quite often. And I want to explain to you now what is not an entangled state for the reasons of using entanglement as a resource.

So first of all, I want to point out this state is not a single photon, because we have entangled here two qubits. Let me just contrast it. If you have a single photon after beam splitter, the single photon after beam splitter is in a superposition of mode a, mode b, 0 1 minus 1 0 divided by square root 2.

But now 0 is the vacuum. So we have not a system which can be decomposed into two partial systems a and b. You may even separate the system and then manipulate your single qubits individually by putting phase shifts on and doing other operations.

If you have this state which is a superposition between having a photon, not having a vacuum, and having a vacuum state, you cannot-- the vacuum state itself is not a separate system. You cannot take the vacuum and perform operations on the vacuum. So I know it has been confusing for me when I heard about it for the first time.

We have, so to speak, here an entangled mode, but not an entangled state. It's a singlet state. It has some aspects of entanglement, but it is not the entanglement we have defined is a resource.

So let me just repeat-- this may be an entangled mode, but to be in an entangled state requires systems where you have two parts which you can separate.

The second comment is we have situations where we have two different parts, but they can't be physically separated. So part of our definition of entanglement as a resource is that the state must be of two physically distinct systems. And we want to have parts which can be separately addressed, manipulated, and measured.

In other words, if you read out one part of an EPR pair, the other part still exists. So that's sort of what we want.

Let me illustrate that with examples. For instance, in the helium atom, you have two electrons. And the ground state-- if I use spin notation, is a singlet.

So that's this. But does the ground state of helium fulfill our definition of an entangled state? Well, if you find a way to switch off the Coulomb potential, and then the electrons separate from each other, they still maintain their spin singlet character, then you can take them, measure them, take measurements, manipulate them. But since nobody has come up with a good idea how to switch off the Coulomb potential in an atom, you can never separate the states.

And it is physically impossible to address this so the two electrons see them separately and such. So this state is not entangled, because it's not-- the two systems cannot be separated. What is usually a good choice for entanglement-- and this is why we discuss it here with photons-- is you have photon states. Photons always fly away. Photons are in a certain superposition state. You can always separate them, and take them individually.

So let's assume we have two photons, two modes. We have two photons-- actually, the two photons can be in one spatial mode, but we are now playing with the polarization-- horizontal, vertical. So if we have a state with horizontal- vertical polarization, this is a nice entangled state of two photons.

So what did means is that we have photon each. One is vertical. And one is horizontal.

But you don't know which one. If you would just look at one photon, it would be 50% horizontal, 50% vertical. It would be completely unpolarized, which would be a random state, which would require and density matrix for its description. But if one photon is horizontal, the other one is vertical and vice versa.

So it's a pure state, but all the pureness of the state comes from the entanglement and not from what one photon does by itself. So these are two photons in one mode, and they are polarization entangled. Or this brings us back to our dual [? Rail ?] single photon states. We have two photons, two qubits and each is in two modes.

So our 0 state-- and just to make sure that you do not confuse it with no photon, our logic 0 state, so L means logic here-- is that the photon is in the second mode, and the 1 state of our logic state means that the single photon is in the other mode. So we can now have an entangled state, which is now this 0 1 1 0 state. But these are now logic states, which means that the 0 has one photon. It has a photon in one of the modes, and the one has a photon in the other mode.

So each state here has two photons, but then the two photons have-- 0 1 and 1 0 are switched in the two parts of the wave function. And we actually saw in the last unit how Kerr medium and an interferometer can generate this state.

So yes?

AUDIENCE: [INAUDIBLE]

PROFESSOR: There are two such states. I mean, I will later tell you what the four famous Bell states are. There is one which has a plus sign and one which has a minus sign. So when we talk about spins in singlet state, it's often more natural the minus sign.

Here what naturally emerged was the plus sign, but they are both sort of Bell states and therefore, they have what Einstein-Podolsky-Rosen introduced into it. So tolerate both signs. They are two different states, but for the purpose of the current discussion, they have the same property-- they're maximally entangled.

Other questions?

OK, so let me point out that the properties of entangled states always involve two qualities. One is the non-local character, because we have correlation between two subsystems which may be together when they interact, but then they can be separated. So we have correlations which happen between the two systems which are a distance apart.

And we later come back when we talk about Gauss inequalities and that we know that physics has non-local aspects. And secondly, if you can separate two parts and they interact with the environment, the environment may interact with them differently in those two different parts. And therefore, entangled states are always regarded as fragile against decoherence.

And it's a technical challenge how do you find states? How do you implement entangled states which are robust?

Just to give you one example, if you have an entangled state which is based on electron spin, you may be more than 1,000 times more sensitive to magnetic field fluctuations in your laboratory than if you have qubits which are entangled states which are based on nuclear spin. So that's a big research area to find states which are less sensitive, or even you immune against decoherence.

OK, so I've mentioned to you that entangled states are states which you cannot factorize. But now we can sort of start playing with that definition. And we say, OK, if you have an entangled state which is up-down plus down-up, but now the contribution of down-up has only a tiny, little amplitude.

So it's almost a pure state which can be factorized with a little bit of an extra configuration which prevents us from factorizing that. I mean, that doesn't look like good entanglement. It looks at the whole entanglement of the state depends on very small admixture to the wave function.

And so what we want to address now is how can we quantify that? How can we look at this data and say, hey, this is sort of not a strongly entangled state. It has only a weak amount of entanglement.

So let's not forget, entanglement is a resource. Entanglement allows you to do teleportation. Entanglement allows you to do more precise measurements.

And what I want to convey you, if a state is only weakly entangled, it doesn't help you much to achieve precision beyond the standard limit, effective teleportation, and such things.

Before I introduce several measures for entanglement, let me talk about entanglement purification. It's a very nice subject which tells you that if you have weakly entangled state, you can make them more entangled. And actually, the effort you have to spend to make a weakly entangled state more entangled can actually act as a measurement how entangled was your state in the first place. So the purification is introducing us with one measurement for entanglement, as I just said, but it also gives you an idea how quantum state can be manipulated.

Purification is also the first example we encounter in this course for new insight into quantum physics. A lot of people thought quantum physics, at least non-relativistic quantum physics that was invented in the '20s in the last century, and by now we've understood it all. But there was an aspect of quantum physics, which I think nobody understood.

And this is when you have a quantum state, which may decohere, a quantum state which may no longer be pure, that you have ways to error correction. You have ways to get the pure quantum state back. And ultimately, I mean, in the early days when I learned quantum physics, I thought if you have a quantum state, that's nice. But if the quantum-ness has decayed away, you can't get it back.

But this is something we learn now from the new methods and new approach in quantum information science that you can do quantum error correction. You can have a stage which has decohered and you can get back to it. And there is something which-- it's not yet that which we're discussing today-- but in purification, you have states which have inferior entanglement, but you don't need to get stuck with that. You can take several of the purely entangled states and create the maximally entangled state out of it.

So that's what I want to show you. So you should look at it as one example that wow, it's really cool how we can have quantum states with inferior quality. And by doing quantum operation on those states, we get something which is more entangled, and therefore, if that's our purpose, has a higher quality.

So to introduce purification, I'm simply mentioning what are-- for two qubits-- the standard, maximally entangled states. In 10 minutes or so, we talk about how do we measure entanglement, and indeed, those states will come out as being maximally entangled.

But you already get the idea. Maximally entangled means they're not factorizable and not just by a small margin. They may be two equal parts of-- you always need two equal parts of 0 1 and 1 0, and this state has maximally entangled. It's maximally non-factorizable.

So the four states which are actually the Bell states-- the famous Bell basis-- are 0 1 plus minus 1 0 and 0 0 plus minus 1 1. Just to remind you if if you have two systems, each of them has two states. You can think of spin-- spin up, down for particle one, spin up down for particle two. That Hilbert space is four-dimensional. So you need a four-dimensional basis. And the trivial basis is up up, down down, up down, down up.

But what is relevant for entangled states, we often use the basis of Bell states. And this is a new basis which spans the four-dimensional Hilbert space, but each of those bases function is maximally entangled.

OK we should correctly normalize them. And that state is often called psi plus minus. And this here is phi plus minus.

OK, so now let's get a purely entangled state. And let this state 0 0 plus 1 1. But we will have an example where b may be very, very small. So those states, this state is entangled for all choices of a and b.

So the question is now how can we take such an arbitrary state where a and b-- one of them may be small, and create a standard Bell state?

So what we have to assume for that is that we have a large supply of such states.

And so let's assume we have large supply, identical copies. And now we want to take two such copies.

And what I want to outline you is the following-- you take sort of two of your copies, and you do a measurement. And I will tell you what kind of qubit operation we need, what kind of measurement we perform. And then when the outcome of the measurement is such and such, you say-- OK let me be specific.

So if you take two copies, we have a total of two states with two photons each. And now we perform a quantum operation on two of the photons, and the other two photons we leave untouched. So now it depends when the measurement of the two photons has a good outcome, we know those other two photons are maximally entangled. They're in a Bell state.

If the outcome of the measurement is bad, it tells us the two photons are not entangled, and we throw them away. So therefore, we have a finite probability by performing measurements that a pair of our sample state with the coefficient a and b will result into a maximally entangled state-- in a Bell state.

And I want to describe now what is the protocol, what is the procedure to implement that. And what we will find-- and this is what I think you should expect here-- if the initial state has very bad entanglement in the sense that b is very small, we will need many, many attempts doing many measurements on our pair of states before we produce a Bell state. It's probabilistic, but the probability to succeed in preparing a Bell state will depend on-- we will see-- the product of a and b. Any questions? Yes.

AUDIENCE: Does the assumption of large supply of the state violate no-cloning in any sense?

PROFESSOR: The question is, does it violate the no-cloning theorem? No, it doesn't, otherwise I would not say we have a large supply, because the no-cloning theorem is absolute. But it simply means we cannot have one state, and then clone and clone more and more copies. Let me be specific. If you have an experiment which produces a certain superposition state, you can just push the button on your experiment many times, and produce, in the identical create, as many states as you want. If you have spin-up state-- well, this is now for two photons, but if you have a spin-up state, you can make as many copies as you want of the state which has been rotated by a certain angle. So therefore, in state preparation, by going through the exact procedure, we can just create as many copies of a state we want.

The no-cloning [INAUDIBLE] meant the following-- I give you one state which may be a spin which has been rotated at a certain angle. You know nothing about the state. And now you should try to make a copy out of it.

And the answer is you can't, because any measurement you do is-- if the particle were spin up and you would measure spin up or speed down, you could say I got spin up. Now I produce 10 spin up particles. But you don't know along which axis the spin has been prepared.

So unless you know which axis the spin has been oriented, and if you choose another axis, you have irreversibly lost information which cannot be retrieved.

I don't know if it helps you, but if you have a certain state, and you're going to measure it without destroying it, you need a quantum non-demolition measurement. If you're in energy eigenstate, you can measure energy. If you're in a spin eigenstate which points along Z, you can measure the direction of the spin in the Z direction without destroying it.

So if you can do a quantum non-demolition measurement on a state, you could clone it. But that violates the assumption that if I give you an arbitrary state, you do not know by definition what kind of measurement is a non-demolition measurement. You just take your chance, you try to take a Stern-Gerlach experiment, separate the spins in the Z component, and then it turns out I gave you a state which is polarized along x.

So that's a subtle, but important difference. Other questions?

So we take two copies and let's bring in Alice and Bob. So the first photon we associate with Alice. And the second photon is associated with Bob.

So if you take two copies-- we have now is an Hilbert space, a four-dimensional Hilbert space, a direct product. And if you just take the state and calculate the direct product, you get four terms-- 0 0 1 1, 0 0 1 1, 1 1 0 0, and 1 1 1 1. And the coefficients are given here.

Let me just underline that it if you think we have some state with some entanglement, and we separate the system, one goes to Alice and one goes to Bob. So the one which Alice has is the first part of it those states. And Bob has the other part.

So the protocol is now that Alice and Bob first, they're not doing a measurement. They're not reading out the system. They perform a unitary operation.

And what Alice and Bob have formed is the controlled NOT operation on the two qubits. Let me just write it down and then explain it. Perform what is called the controlled NOT or CNOT gate. And I've explained before in the last section how the controlled NOT can be performed using a non-linear Mach-Zehnder interferometer.

So Bob and Alice both run their two photons with the non-linear Mach-Zehnder interferometer. So what is a controlled NOT? Let me remind you. If you have two qubits, the first one is the control. If the control is 1, you flip the second one. If the control is 0, you do nothing to the second one.

So the controlled CNOT transforms. So since the first qubit is the control qubit, out of those four combinations, the controlled NOT only does something if Alice's controls and Bob's controls is 1, and then the second bit is flipped. If Alice's and Bob's controls are 0, they do nothing to the second bit.

So therefore, the 1 1 0 is transformed into 1 1 1, and the 1 1 1 1 is transformed into 1 1 0 0.

So that's the first operation. Let me just indicate it. So what has happened here, the 0 0 has been flipped into 1 1 and the 1 1 has been flipped into 0 0.

That's the first step. The second step is that now Alice and Bob measure their target qubit.

What does that mean? We have a controlled NOT operation. In the controlled NOT operation, we have the first one is the control qubit. The second one is the target of the operation. So in our, the way how we write it, Alice's target is the third in line, and Bob is the fourth in line.

So now Alice and Bob measure the target qubits. Let me just be specific. So Alice has number one and number three. Bob has number two and four, and the target qubits are the third and fourth.

So what is the probability that Alice and Bob measure both 1 1, that they both find the target qubit of 1? Well, it is this one and it is this one where the controlled NOT has flipped it. So one has the probability a squared b squared.

The other one has probability a squared b squared. So with probability 2 a squared b squared-- well, let's allow a and b to be complex. They obtain 1 and 1.

So in this case, what is left is here 0 0. What is here left is 1 1. You may just need an intermediate line to write down what the state is after the measurement. But if you read it off here, you find that when they obtain 1 1 1, that implies the post measurement state is then 0 0 plus 1 1 divided by square of 2, and this is one of our Bell states.

So what we had is we had a system of four photons after a qubit operation, the controlled NOT. Alice and Bob do a measurement together on two of the photons. Collins. And if the outcome of this measurement is 1 1, the rest of the system is in the Bell state.

So we assume that Alice and Bob have a large supply of those copies. So let's assume that they start with n copies of psi. And then, because the probability is m over n, they successfully obtain m copies of the Bell state.

And the question is, what is the probability in the limit of a large ensemble. And we will see in a few moments that this is actually a measurement of how entangled the original space is.

Any questions about purification?

Well, then let's measure entanglement.

The basic idea here is that if you have a state up/down plus down/up, it's a pure state. But this pure state has a correlation between the two subsystems. And the idea is now entanglement is that there is a correlation between the two subsystems.

And you can say well, it would be a good way to characterize entanglement if I only look at one subsystem. In this case, you look at one subsystem, and you would just randomly see spin up, spin down. Spin up spin down is described by the unity density matrix, which is, therefore, the most random state on earth.

So if you have a pure state and you only look at one subsystem, the more random the subsystem is, the more the pureness of the initial state comes from correlations, comes what is entanglement. So therefore, what I want to introduce now as a measure of entanglement is that we take the total system and then we perform the partial trace. We only look at one subsystem.

And the purer the subsystem is, the less entangled it is, because in the ultimate limit that our system factorizes into two pure states, when we look at the subsystem, we still have a pure state. So the purity of the subsystem in terms of pure state is now a measure of entanglement. The purer the subsystem is, the less entangled it was.

So the basic idea here is entanglement is related to correlations. And if you take half of a Bell state-- so if two particles which are an EPR pair and we take half of it-- then half of it is completely random. So let me illustrate that. So if you take one half of-- let's just take one of the Bell states, phi plus, which was the superposition of 0 0 plus 1 1.

So let me be specific, because we need it for the definition. We describe this ensemble of this system in a pure EPR state by a density matrix. The density matrix is nothing else than you take your total system.

So this is now the density matrix. So in our case, psi a b is just the state. And now we describe the subsystem by performing a partial trace on rho. The partial trace is over the system b. And that means we take all eigenfunctions k b of state b, sum over all k's, and this is our partial trace.

So therefore, we would take our statistical operator from the line above, and perform the partial trace where those states are the state 0 and 1 of b. So these are the states of b. So when you do that, you just insert that. You find that what you get is 1/2. a has been traced out, so b has been traced out.

And what we obtain for a is just from the two terms above. This gives that, and this gives that. And you immediately realize that this is 1/2 times the identity matrix.

So therefore, we have shown that this is a completely random state. So now we can characterize the randomness of the partial trace of the density operator obtained by performing the partial trace with the standard von Neumann entropy.

As a reminder, the von Neumann entropy for statistical operator rho is defined as the expectation value of rho log rho, where we take the logarithm with respect to the base 2. So this is the trace of rho log rho. Or if we use the eigenvalues of rho, we multiply eigenvalues with the logarithm of the eigenvalues.

So for a pure state, the entropy is 0, because a pure state has one eigenvalue, which is 1, and the log of 1 is 0, so we get 0 for pure state. For completely mixed state, we're talking about a state which has two dimensions, so it can be up and down. A completely unique state has probabilities of 1/2 each. And then we say the entropy of this state is one, or we call it one bit. Yes.

AUDIENCE: About the volume [INAUDIBLE] expectation of rho log rho is called a trace [INAUDIBLE] expectation of log rho, expectation of [INAUDIBLE] trace of the operator times density matrix.

PROFESSOR: OK, so this is just a reminder of how we measure entropy of density matrix. And now we apply it to entanglement. We define now the entanglement for the entanglement e over state psi a b to be the entropy of the density matrix for system a after tracing out system b.

And for pure state, this is-- it doesn't matter whether we trace out a or b if you start with a pure state. The entanglement, the entropy of the statistical operator rho a and rho b are the same.

I tried for a moment to prove it. I saw it quoted somewhere. I didn't succeed in a split second, so either I overlooked something, or it's a little bit more involved to show that.

So therefore, to use the inverts, our definition says that the entropy, so the entanglement, is nothing else than the entropy of the reduced density matrix. And we immediately see if we have any of the four Bell states, by performing the partial trace over one qubit, we obtain the identity matrix. So therefore, the entropy of all the Bell states is 1.

Let me state without proof-- when we come back to the purification scheme the result is that the probability or the optimum probability-- if you do stupid measurements on your states, of course you get nothing. But the optimum strategy to create pure Bell states out of your reservoir of poorly entangled states-- so for an optimum strategy, the success probability m over n actually turns out to be not a different measure of entanglement. It is the entanglement which we have just defined through the entropy of the partial trace.

So therefore-- and I think this is nicely illustrated with the purification scheme-- entanglement is a real resource. When you have better entanglement to start with, then you can get more copies. You can get more pure Bell states out of your supply of poorly entangled states.

So therefore, you lose more if your states are not fully entangled. You lose more of them, and therefore, the success of the purification scheme makes it clear how entanglement is a resource. If you have entanglement, it's precious. You had to do something to get it.

I didn't point out entanglement is not something which one number characterize it, it's all. We introduced to you already another measurement of entanglement through the Schmidt number, which was in homework number two. And usually when you have different measures for entanglement, they're not one to one related.

It seems that, similarly when we measured non-classic light, we had a G2 function. We had [INAUDIBLE] bunching, antibunching. We have negative quasi probabilities. And it's often clear that one system which is truly non-classical, fulfills all the criteria, but how the quantitative measurements are related to each other is really subtle.

In the case of entanglement, for a long time it has even been a big question in research-- if you have an arbitrary density matrix with a complicated many-body system, how can you even characterize the entanglement? We are focusing here on pure states where things are fairly simple. But in the general situation of a many-body system, it can be quite challenging just define and measure entanglement.

Any questions? Yes.

AUDIENCE: If you have a general [INAUDIBLE] that's not necessarily a pure state, can you show that it's always the reduced trace of the bigger [INAUDIBLE]?

PROFESSOR: Say again, if I have a general state which is--

AUDIENCE: You have a subsystem-- let's say you have two spins, and then you have a density matrix for the first-- or actually just to say that you have one spin and you have a density matrix for that single spin, it's a matrix, not necessarily a pure state. Let's say it's a mixed state. Then you can introduce a fake second spin and show that this matrix is the trace of a matrix [INAUDIBLE] and maybe just put that one in a pure state and do some stuff, do some calculations more easily.

PROFESSOR: Yes. If you have a density matrix, you can always regard it as a partial trace of a bigger system. That means you always represent your state as a pure state, but it is entangled with a bigger system. However, what the big system is, is by no means unique.

I'm missing the technical word-- it's called unraveling the density matrix. You can always represent the density matrix written down in forms of pure state. But this unraveling of the density matrix-- no, actually, its related.

When I said yes, I thought about the unraveling of the density matrix. You can always write down a density matrix as a mixture of pure states, but which are the pure states is not unique. So when you say my density matrix is half of the atoms are spin up and half of the atoms are spin down, somebody else would say, no, that's not true. Half of the atoms are spin [INAUDIBLE] and spin x and some are in spin minus x, and those representations are equivalent.

So I was just thinking of that as to write down the density matrix in the pure state basis. But you are asking about--

AUDIENCE: [INAUDIBLE] what he's asking about [INAUDIBLE]

PROFESSOR: Somehow, but I was thinking of that, but I think the answer to your question is yes. So you said, you confirmed that.

AUDIENCE: Yes, but I am forgetting the name of the theorem here.

AUDIENCE: Isn't it just purification again?

AUDIENCE: Yeah, it's related to purification. You can prove that you're using purification.

AUDIENCE: [INAUDIBLE]

AUDIENCE: Using purification [INAUDIBLE] but I'm completely forgetting the names of [INAUDIBLE]

PROFESSOR: But wait. We've talked here about-- just to be clear, we've talked here about purification of a pure state. We started with a pure state and we purified it to be a Bell state by doing certain measurements. So we've not talked here about density matrices, but it sounds very plausible that you can always construct a bigger system.

But I should look it up and see if there is an exact proof. It's-- intuitively it sounds correct. Other questions?

OK, so we have discussed the definition of entangled state. We've talked about purification of entangled states and how to measure it. I want to talk now about how we can create entangled states for atoms.

Maybe let me say the following-- so by now we are convinced. Entangled states are great, and we want to create them. And for photons, I showed you that some simple element-- beam splitters, Kerr medium-- can create entanglement. It's much harder to do that with atoms.

Now, we want to do it with atoms, because atoms-- in contrast to light-- they're pretty much staying still, whereas photons always move at the speed of light. And the only way to make photons stand still is you put them in a cavity and then they bounce back and forth. But even in super cavities with the highest reflectivity mirror, you get-- what are the longest ring-down times you get-- fraction of a second, milliseconds, depending kind of in which domain you work-- microwave domain, optical domain.

Whereas atoms, you can hold onto your qubits for a long time. So therefore, if you want to use entanglement as a resource for certain protocols, you want to have entangled atoms where the entanglement would like for a long time. So the question is now for atoms, we do not have perfect beam splitters and perfect Kerr mediums. Also, we can control interactions between atoms for [INAUDIBLE] using VSEPR resonances, but that's another story.

But now let me ask a question, how do we entangle atoms?

And I want to first show you that if you had the right system, things can be fairly simple. This is a suggestion which was made almost 20 years ago, and it goes like follows-- if you have a diatomic molecule of two identical atoms, in this case mercury, and mercury doesn't have-- this molecule doesn't have any electron spin, but mercury has a nuclear spin.

And if you now photo-dissociate mercury and two mercury atoms fly away, then you have separated a spin singlet into two parts. And now you have created the Bell state up/down minus down/up. So you could say this is sort of the example-- this realizes very closely the example I gave you with the helium atom, where I said you have an electron in spin up and spin down, but there was no way to separate the electrons. Therefore, you can say it is a state which has entanglement, but it's not entanglement as a resource.

But right now, here it becomes a resource once you have found a method to separate the two parts of the wave function that you can give one to Alice, give one to Bob, and they can perform the operations on it. Well, if it looks so simple, why don't we have entangled atoms everywhere? This experiment has been suggested 20 years ago, but nobody has done it, or some several groups have worked on it.

You need a molecule with suitable states. You don't want any electron spin which interferes with that. All you want to have is two nuclear spins. You want a singlet state here. So those requirements are not easily fulfilled with real atoms.

Of course we liked-- and also who wants to work with mercury? Mercury has transitions in the ultraviolet. I think there's only one group in the world who has operated a magneto optic trap with mercury. So it's not your tabletop atom.

So therefore, let's now talk about a method how we can entangle atoms by using light. So if you take atoms, maybe we can use the light which has been emitted from the atoms, performing measurement on the light, and then depending on the outcome of the measurement, we know the atoms are entangled.

So Professor [? Swann ?] calls this the poor man's entangler. I don't know why poor man's. You still need quite a bit of equipment to do it. But at least it seems the poor man's solution if you can't make the above experiment work.

So this addresses a question that, just for technical reasons matter is more difficult to entangle. Whereas photons are easy.

Yeah so you can say the idea is related to the purification scheme. We can't take a system of two atoms, atom one and atom two which are unentangled, and we shine some laser light on them, excite them. Then they emit photons. So all we can do is-- we can only talk to the atoms with the photons. So the only thing we can do now is we can measure the two photons. And then the situation will be similar as in the purification scheme, where Alice and Bob did a measurement.

If Alice and Bob said both of our target qubits are one, then what was left behind was in a pure, entangled Bell state. And similarly, what you want to do here is we have two atoms. There was nothing special about them but they scatter light. And if you now perform a measurement on the photons and the outcome of the measurement is positive, then we know for sure what has been left behind is entangled. That's the idea.

So it shares with the purification scheme that it is a probabilistic entanglement. You run your experiment many times. You do a measurement. If the measurement is positive, you say now I have an entangled state.

And maybe then you can move on to measure the entanglement. You can move on to do teleportation, other things you want to do with entangled states. But if your measurements says no, bad luck. The probability hasn't worked out this time. You just press a button again, scatter light again of your two atoms and/or your two ions and hope that the next outcome is positive.

So the idea is we want to introduce now a probabilistic method. It's based on two atoms emitting light. And the result is with a certain probability that we get entangled atoms.

Let me just scroll down and show you one thing I want to--

When I prepare the notes and everything is clear to me, but then I want to explain it to you and say hey, I have to motivate you. If I just go through a few lines, you wonder what it leads to. So let me maybe first give you the explanation I would have given you a little bit later.

What is an entangled state is if the atoms are in maybe one of two ground states-- one atom is in one ground state, the other one is in the other one, or it is flipped. So we know one atom is in the ground state one. One atom is in the ground state two, but we don't know which one is in which. It is in the superposition state.

So what you need now in this scheme is the following-- if the atoms scatter light, they can go to two different ground states. And we know to which ground state they have gone, because due to selection rules, they reach one ground state with polarization one. They reach one ground state with polarization two.

So therefore, when we had two atoms, they emit light and we would measure the polarization of the light, we would know in which state they are. But if you know atom 1 is in state 1, and atom 2 is in state 2, this is not entangled.

So what we have to do is when the two atoms have emitted photons, we have to mix the photon at the beam splitter. And after the beam splitter, when we measure that the photon is polarized, we know that one of the atoms is in one of the ground states, but we don't know which. So therefore, we can now use-- the photon carries through the polarization the information in which ground state the atom is. But now we have to perform operations to the photons that for fundamental reasons, fundamental quantum measurement, we know there is one photon in one state, but we have no way to ever figure out which atom has emitted the photon.

So that's the idea, and the protocol I want to show you now is what do we have to do to the two photons to make sure that we never know, that we can't find out which atom has emitted the photon. And we have to do little bit more tricks also to make sure that when we do a measurement on the photon, we know the atoms are in the Bell state. Questions about that?

So therefore, we need a beam splitter. So we can come back to what we have already introduced in the last section.

So each atom will emit a photon. And I will actually show you at the end of the class that people have entangled with that scheme two ions which were in two different ion traps. So you have two distance atoms. They emit light, and after the measurement process, you know they are entangled. And that's pretty cool.

So the situation how we do it is we have two photons which come. But the first thing we have to make sure is we have to scramble the photons. We have to make sure that we can't find out from which atom the photon has come, and this is done with the beam splitter.

So there is one aspect of beam splitters and two photons which have to explain to you now. And this is this famous HUM, Hong-Ou-Mandel. This is the Hong-Ou-Mandel interference. It's a very famous effect, and it's actually very, very special. So let me explain what happens when we have two photons in the same state-- two identical photons, same frequency, same polarization-- coming to a beam splitter.

So we have already all the tools. So we have a beam splitter characterized by this angle theta, which through cosine theta, sine theta determines what the beam splitter is doing. And all we want to do is we apply it now to the state 1 1.

So what we will find is that there is a probability to get one photon each. Then there is a probability to get two photons in one output. And it will actually be the same probability with a minus sign in the amplitude to get 0 2.

And the matrix which acts on each of the photons has cosines and sines. So what we get is products of trigonometric functions. And here for getting two photons in one arm, it's square root 2, cosine theta, sine theta.

And the spectacular thing here is what happens when we have a balanced beam splitter.

If you have the 50-50 beam splitter-- and this is called the Hong-Ou-Mandel interference, you have the situation where you have a beam splitter, you have one photon, you have two photons. And it is absolutely impossible that afterwards, you have one photon in each arm. So you send two photons on a beam splitter, and after the beam splitter is, you have either two photons coming out here, or two photons coming out there.

You can say it's Poissonic stimulation photons and bosons. That all plays a role. If one photon goes one path, the other photons follow suit.

This is now very powerful, because it already happens, of course, when the two photons are identical. We had to use in this formalism the photon is in the same mode on each part. But that means now the following-- if you set up photo detectors here, and each photon detector makes click, you know you had one photon each. And that tells you that the two photons were not identical-- for instance, because they have different polarizations.

So if you want to now, with the atoms, get a Bell state, where atoms decay-- one atom decays to state one, one atom decays to state two-- the signature of that would be that we have one atom each. And if you do it right, one atom each is an ingredient for 1 2 plus 2 1 for Bell state. We can detect that we have one photon each because at such a beam splitter, it's only in this situation that we can get one photon after each beam splitter if we start with two non-identical photons.

Of course, by the way, experimentally there are quite some challenges, even if you have identical photons of the same polarization, if they arrive as a nanosecond pulse, and they don't arrive exactly at the same time at the beam splitter, then you first split one photon, and then the next photon, and the two photons cannot influence each other. So there are a lot of experimental requirements to realize this ideal experiment.

So as I was preparing for today's class, I actually saw a paper which just came out in "Science."

So in this month's "Science" they discussed the Hong-Ou-Mandel interference experiment with fermions. So let me just explain what happens.

OK, now it fits.

So if you have two identical photons, if you have two particles which impinge on the beam splitter-- they are the two input beams for the beam splitter-- you would say you have four different outcomes. One is both come out here, both come out here. Or they are both reflected, or they are both transmitted through.

And what happens is bosons-- as I just pointed out-- bosons characterized by photons can only do that. Identical photons want two bunch up. They appear in pairs-- 50% left output, 50% right output.

Well, for fermions, they just do the opposite. For fermions, you will always get one particle each. You may immediately, of course, explain it with the Pauli Exclusion Principle, which does not allow two particles to be in the same state after the beam splitter.

And you should contrast it with classical particles. When you have two classical particles, you will actually find that all of those four possibilities, each of them has a 25% probability. So it was a major experiment which was featured in "Science" when people realized that with electrons, they created electrons in a semiconductor structure, and showed through some statistical measurement that this was the physics which happened.

The measurement they did is, I forgot details-- if you have bosons and you get either two here or two there, you have more fluctuations in your system than for fermions. And they conclusively showed that they had realized the Hong-Ou-Mandel interference for fermions.

I think we have to stop. Let me just add one more thing and then we are done. So this Hong-Ou-Mandel interference is at the heart of how perform the measurement which is ultimately entangling the atoms. But we need one more element. We need sort of to scramble the photons in one more way, and this is by adding circular polarizers at the input.

So we assume for now that we start out with linear polarization in those states. And here is our beam splitter. Here is mode a and mode be, which we detect. And before we measure this Hong interference, we put in quarter-wave plates which provide circular polarization.

If you start with linearly polarized light-- we put in a polarizer-- after the circular polarizer, we have this linear superposition of horizontal and vertical at mode one, and we have linear polarization-- superposition of linear polarization in mode two. So this is a situation at the input of the beam splitter. And if we expand it, we have probabilities where the polarization is different, and where the polarization is the same.

So we have two detectors here. And if both detectors click, then we know that the input to the interferometer was not H H nor V V, because in that case, the Hong-Ou-Mandel interference would have directed both photons to one output, and we would not have obtained clicks from both.

So therefore, when both detectors click, we know that the quantum state before the interferometer-- before the beam splitter-- was H V or V H-- or, of course, in another basis, one of those. And this is sort of the ingredient, which I will show you on Wednesday, which can lead to probabilistic entanglement of atoms. Any questions?

OK, one announcement-- we post this week's homework assignment later today. It will be due in a week. See you on Wednesday.