

The following content is provided under a Creative Commons license. Your support will help MIT OpenCourseWare continue to offer high quality educational resources for free. To make a donation or view additional materials from hundreds of MIT courses, visit MIT OpenCourseWare at ocw.mit.edu.

PROFESSOR: All right. Then let's go back to our discussion of what happens to classical and quantum mechanical magnetic moments when they are exposed to magnetic fields. I just want to remind you of what we did last class and what we want to wrap up today. This is rapid adiabatic passage.

I mentioned to you and explained it to you that rapid adiabatic passage is a powerful way to manipulate a classical and quantum system, and what we discussed is that when a spin points in the up direction and you sweep the resonance of an oscillating magnetic field-- the frequency of an oscillating magnetic field, through the resonance, you create an effective magnetic field in the moving frame which will rotate. And the atom, when the change is done adiabatically, will follow the rotation and therefore invert the spin. So it's a perfect, very robust method to invert population in spin systems.

What I want to pick up today is the question, how slow is adiabatic. We have to fulfill an adiabatic condition and we have already an idea already of what the adiabatic condition is but now we want to derive it. That we had this picture of a spin which is rapidly precessing. It always precesses around the direction of the effective magnetic field, and the condition adiabaticity is that the rotation of the effective magnetic field has to be much slower than the precession.

Let me just make it clear by a counter example. If the atom precesses around the magnetic field and the magnetic field would suddenly jump, then the atom would now start precessing about the new direction of the magnetic field and it would have completely changed its angle relative to the magnetic field. It would have lost its alignment with the magnetic field. So you clearly see that the condition is the direction of the magnetic field must not jump, and the only other time scheme is the

frequency of the Larmor precession, so our condition for adiabaticity is the rotation of the effective magnetic field has to be slow compared to the precession frequency.

And so we want to now derive from that condition the conditions for adiabaticity. And just as an outlook to make it interesting, what I will derive for you in the classical picture is now something which you will later encounter as the Landau-Zener parameter. But Landau-Zener sweeps, we talk about it later today is a quantum mechanical version of rapid adiabatic passage, but we now get classically a result, which will feature the Landau-Zener parameter.

So with that, let us write down what we want to look at. It is adiabatic condition, and to write it down in words is that the Larmor frequency, ω_L , which is given by the effective magnetic field, has to be much larger than $\dot{\theta}$. Now things in general are rather complicated. If we are far away from resonance, you change the frequency, but the effective field is not changing a lot.

The critical moment is really when we have the real field, we add the fictitious field and that cause a rotation. The critical moment is when we are near resonance. So in other words, we have to fulfill an inequality. The left side has to be larger than the right side, but the left side is actually smallest in the vicinity of the resonance, and the angle $\dot{\theta}$ is actually largest near resonance. That's when it sort of quickly goes to 90 degrees. So therefore, if we want to find the condition of adiabaticity, we can derive it by looking at the region around the resonance.

So the effective magnetic field is the real field minus the fictitious field caused by the rotation by the transformation into the rotating frame. So we have the magnetic field at an angle θ with respect to the z-axis. I've just written down the z component for you, and the transverse component is-- so this is this component and the transverse component is the amplitude of our drive field B_1 .

So we can just read it from the diagram. The resonance data is 90 degrees, and the correction angle is whatever we have of the effective z field over B_1 , and that means that the derivative, the angle $\dot{\theta}$, the angular velocity at which the magnetic field rotates, there's a time derivative because we sweep the frequency.

So therefore, $\dot{\theta}$ is nothing else than $\dot{\omega}$, the sweep rate of the frequency divided by γB_1 .

But γB_1 is nothing else than the Rabi frequency. And on resonance, the Larmor frequency is just a Rabi frequency, because on resonance-- sorry for repeating myself-- the fictitious field has canceled the bias field, and the only field left is the rotating field, but the rotating field is the Rabi frequency with a γ factor.

So therefore, we have the adiabatic condition that $\dot{\omega} / \omega_{\text{Rabi}}$ has to be smaller than ω_{Rabi} or to say it inverts the change $\Delta\omega$ of your drive frequency, the change $\Delta\omega$ in one Rabi period has to be smaller than the Rabi frequency. So ω has units of frequency. $\dot{\omega}$ is a derivative that's in units of frequency squared, and this is to be smaller than the Rabi frequency squared.

You find that actually quite often if you do an adiabatic change of your trap frequency, things are adiabatic as long as the change of the trap frequency in one period of the trap frequency is smaller than the trap frequency, and you find something else that the derivative of your trap frequency has to be smaller than the trap frequency squared. So these are adiabatic conditions, when you tighten up magnetic or optical confinement for atoms. So this is very, very generic. The small rate of the frequency you change has to be smaller than the relevant frequency squared.

As I said, we come back to that when we do the quantized treatment of rapid adiabatic passage and we encounter that combination in the Landau-Zener parameter, which takes us to our next topic. We want to now talk about quantized spin in a magnetic field. And one of the first things I will be telling you is that everything we have learned about the classical magnetic moment you don't have to unlearn or re-learn, it exactly applies to the quantum spin.

Before we look at the Hamiltonian and the standard Hamiltonian for two-level system, let us first look at something more general, which is Heisenberg equations

of motion. So we want to write down the differential equation, the equation of motion for expectation values. So for an atom in a magnetic field, our Hamiltonian is simply the same Hamiltonian, which involves the gyromagnetic ratio, the angular momentum operator, and the magnetic field.

And you know from quantum mechanics that Heisenberg's equation of motion for any operator are simply that the time derivative of the operator equals the commutator with the Hamiltonian. In some cases, if you have an explicit time derivative of the operator, you have to edit, but we are talking here about the angular momentum operator which has no explicit time dependents.

So we are interested in the operator which describes the magnetic moment, but the magnetic moment is nothing else than the gyromagnetic ratio times the angular momentum. So therefore, the relevant commutator is the commutator of the Hamiltonian with the angular momentum operator.

Just to remind you, the Hamiltonian was proportional to z . So what we are talking about is commutators, not surprisingly, between the angular momentum operators and those commutators are just this cyclic commutation involving the epsilon tensor. So if you just put that in component by component, you realize immediately that the operator equation for the time derivative of the magnetic moment is nothing else than the cross product, the vector product, of the operator of the magnetic moment times the magnetic field. To commence, this is exact, but also it looks exactly like the classical result.

Well, you would say it's an operator equation. Usually operator equation some of them are pretty useless because you can't calculate the operators, but in this case, we can immediately take the expectation value, so we can get some immediately meaningful equation namely that the expectation value follows the same equation, and this tells us that whenever we have a quantum system, it has a magnetic moment. In an applied external magnetic field, the result is simply rotation, precession, and this is exact. It's not a classical approximation. It's an exact result for quantum mechanics.

So the way how we derived it makes it obvious that it's an exact result, which is valid not only for spin-1/2, but it is valid for any spin. If you have a magnetic moment corresponding to a spin of $10 \hbar$, this spin follows the same equation of motion as spin-1/2. Of course, a special case is valid for spin-1/2, but spin-1/2 is isomorphous to a two-level system. Any two-level system can be regarded as spin-1/2 system, therefore, this geometric interpretation that the dynamics of the quantum system is just a precession rigorously, exactly applies to any two-level system.

It's also valid, and this will be relevant for atoms, if we have composite angular momentum. For instance, we will encounter the total angular momentum F of an atom which has components from the orbital motion of the electron, the spin of the electron, and the spin of the nucleus. But if we have such an angular momentum, F , the creation of motion is it will precess around a magnetic field.

Well the small print here is, unless the B field is so strong that it de-couples the components or that it breaks up the coupling of the different parts of the angular momentum. In other words, we have simply assumed here that magnetic moment is γ times angular momentum, and that requires that the angular momentum are coupled in a certain way. If we don't fully understand what coupling of angular momentum is, we really talk about that when we talk about atomic structure. So as long as the spin state coupled to one total spin, this total spin will just precess.

This picture of precession will also be valid for a system of N two-level systems coupled to an external field, and this will be the example of Dicke superradiance, which we will discuss towards the end of the course. So very simple result, but very powerful, and this is your permission whenever you encounter any of the systems to see a vector precessing in your head. This is exact.

So we've talked about Heisenberg equation of motion for general spin, but-- you have a question, Nancy?

AUDIENCE: What did you mean by N two-level systems here? Are we talking about coherent systems or non-coherent systems?

PROFESSOR: We talk about in two-level systems and to be more specific the coupling comes because they all talk to the same magnetic field. So we have in two-level systems connected to the modes of the electromagnetic field. We start with the symmetric state. The coupling is symmetric and that preserves the symmetry of the atomic state. In other words, we will have a situation where the angular momentum is the maximum angular momentum we can get in two-level system, and the dynamics of this two-level system, the description of Dicke superradiance has the geometric visualization of this precessing motion.

I know I'm not explaining it exactly. I want to sort of whet your appetite for what comes later and also sort of prep you that some of the simple pictures will really carry through the course.

OK, so this is for very general spin. Let's now talk about features of, yes, the most important spin for us, namely the two-level system, which is spin-1/2. Well, the most generic system is an electron in a magnetic field B times e_z . And well, why don't we start with a clicker question.

So the question is, what is the level structure of the electron in the magnetic field? It's a two-level system, and you have two options A and B. One option is the upper state is spin up. The ground state is spin down or the opposite. So in other words, and tell me whether an electron is in the lower state when the speed is up or when the spin is down. You would say it's a stupid definition, but we talk all the time about an electron is in the spin down state or the spin up state, which is the lower energy state of the electron.

So I think by exchanging the better recent number of responses has considerably gone up. OK. The answer is the electron is in the ground state. Spin down is the lowest state for the electron. To say it in words, of course, a compass needle wants to be aligned with the magnetic field. So you want the magnetic moment to be aligned with the magnetic field, and that means the magnetic moment has to point in the plus e direction, which up but the electron has negative charge, the gamma factor is negative, and that's why for the electron, the vector of the spin and the

vector of the magnetic moment are opposite.

So just try to find some main-mode technical thing. The electron lives in the basement. It wants to be down, spin down. This is the lowest state for the electron. However, if you have a system which has a positive gyromagnetic ratio, which would correspond to, well, positive charge, nucleus if it has somewhat normal magnetic moment, then in that case, the spin up state is less energetic than the spin down state.

So let me just write that down. So the correct answer is this one, and it involves that gamma is negative. The gamma is a gyromagnetic ratio, the ratio between magnetic moment and angular momentum, and for negative charges, it's negative. For positive gamma, the situation is inverted.

So let's just use for a moment the result we got from Heisenberg's equation of motion. We know the classical result. We have already derived for the classic spin, the classical result for the expectation value of the magnetic moment. But now I want to sort of relate it to something quantum mechanical because we know that the classical solution equals the quantum mechanical solution.

So if you have a two-level system, the z component of the magnetic moment is the difference between spin down and spin up. And because of conservation probability, p_{up} and p_{down} is unity, we can also write that as 2 times-- let me now introduce e for excited state, just I know it's hard to keep track of spin up, spin down. I want to make sure that I mean now the excited state, so the excited state for the electron is spin up.

So we have this condition, so therefore, the excited state fraction of a two-level system is related to the expectation value of the magnetic moment in that wave, and now we want to use the classical result we derived. We derived the classical result when for t equals 0 all the spins were in the ground state, and by using the result we derived previously, we have the $1/2$ from the previous line and then the magnetic moment, μ_z . We found an expression which involved the Rabi frequency, and the off resonant Rabi frequency are sine square generalized Rabi frequency times time

over 2.

So the two factors of $1/2$ and $1/2$ cancel, and what we find now for the quantum mechanical system using Heisenberg's equation of motion is that if you prepare a system initially in the ground state, the excited state probability, the fraction, the excited state oscillates with a Rabi frequency, and this is, I think, the second time in this course and not the last time that we see the Rabi, that we obtain, the Rabi transition probability.

But let's go further. We have now discussed the classical spin. We have sort of done classical quantum correspondence with Heisenberg's equation of motion. We know that this in general, it implies Rabi oscillations, but now we want to go deeper into the quantum domain by talking about the spin- $1/2$ Hamiltonian. So in other words, we want to go beyond expectation values. We want to talk about the wave function itself.

So the Hamiltonian, which we will use for major parts in this course, it's one of the fundamental Hamiltonians in physics. Of course, there's the harmonic oscillator. There is a hydrogen atom, but then there's this Hamiltonian, which is a two-level system with splitting ω . And then we make, which is often a simplification, where we use a pure exponential, so a single frequency in complex notation where the complex exponential $e^{i\omega t}$ is the drive term.

So this is one of the simplest Hamiltonian for this kind of system. It's a two-level system with a splitting, and now it is driven and the simplest drive term is not cosine ωt or sine ωt as we will see. The simplest drive term is $e^{i\omega t}$. So this is now our Hamiltonian. and since it is so important, let me ask you a clicker question whether this Hamiltonian can be exactly realized in nature or it is an approximation.

For instance, that you always have cosine ωt as a drive, and cosine ωt is $e^{i\omega t} + e^{-i\omega t}$. And then maybe with the rotating wave approximation, you throw away a term. So I sort of wanted to ask you, is this an idealization that we have a coupling which is a simple, complex

exponential, this nature always more complicated or is there a simple way to realize this Hamiltonian in nature? So what do you think?

Stop display. What's funny about that?

AUDIENCE: Equal to.

AUDIENCE: There are two [INAUDIBLE].

PROFESSOR: Oh, they couldn't make up their mind. The system should reject those votes. Anyway, it means most of you anticipate what I want to derive to you that we can actually exactly get this Hamiltonian and indeed, this is the Hamiltonian I will derive for you in the next few minutes, which is the Hamiltonian of spin-1/2 in a magnetic field coupled to a rotating magnetic field.

So we start out with fields which are real, real field, no imaginary numbers, no complex numbers. These are real fields, but when we write down how the real fields couple to spin-1/2, we get $e^{i\omega t}$ and $e^{-i\omega t}$ without any approximation.

So since at least 80% of you know the result, just regard it as an exercise to introduce how we spin matrices in a nice way, also this will help you how to do pre-set number 1. So the Hamiltonian is the spin coupled to a magnetic field, and if we express that by angular momentum operators, the gyromagnetic ratio it involves the operator for the spin in the z direction.

So the two-level system has a splitting of $\hbar\omega$, so it's plus 1/2 minus 1/2 $\hbar\omega$, and that means the diagonal part, the non-driven part is simply given by the Pauli spin matrix σ_z and ω is the energy splitting, which is proportional to the applied magnetic field. And up-down, and excited and ground are the eigenstates of this Hamiltonian with energies plus minus $\hbar\omega/2$.

So this is the same Hamiltonian, but now add a real rotating B field B_1 . So the drive Hamiltonian H_1 is the same magnetic moment but now coupled to a time-dependent

rotating field. The amplitude of the rotating field is the Rabi frequency divided by gamma, and we assume that the field is rotating in the xy plane. So it's $e_x e_y \cos(\omega t)$ $e_x e_y \sin(\omega t)$.

And now I put in two minus signs here for convenience. If you want, I've just shifted - it's just a definition. I've changed the definition of the amplitude by a minus sign. So this is the Rabi frequency, and the magnetic moment divided by gamma is nothing than the spin. Magnetic moment is gamma times the spin, so therefore, we are back to the spin operators and the spin operators, if I factor out $1/2 \hbar$, are now the Pauli spin matrices σ_x and σ_y .

And if you look at those spin matrices, then you'll realize that we go complex in our Hamiltonian, not because we have approximated a real field $\cos(\omega t)$ by some $e^{i\omega t}$, but because when we have a rotating field and we write down it in Pauli spin matrices, we get imaginary units from the σ_y spin matrix.

So that means we have now for this system rewritten the coupling term took you to the rotating field H_1 as $0 \ 0$ $e^{i\omega t}$ $e^{-i\omega t}$. And therefore, the Hamiltonian is the famous two-level Hamiltonian with ω_0 and the Rabi frequency, which I wrote down at the beginning of this chapter.

So we'll leave that here, but we will use it even more in 8.421. This is the famous dressed atom Hamiltonian. It is the starting point to calculate eigenstates and eigenvalues, not just in perturbation theory. You can go to oscillating fields at arbitrary strengths, so you can solve exactly in the dressed atom picture using this Hamiltonian, the problem of a two-level system plus one mode of the electromagnetic field no matter what the drive term, what the strengths of the electromagnetic field in the spin mode is.

So this describes the two-level system plus one mode of the electromagnetic field with arbitrary strengths. And as I said, we talk about some things here but others I explored in 8.422. Questions? Yes, Will.

AUDIENCE: We refer to the eigenstates and eigenenergies of this Hamiltonian as dressed in

states in the same way as we refer to address states in a fully quantized fiction? Do we refer to both cases as dressed states?

PROFESSOR: Yes. That's a good question. The question is now, what are the dressed states, and Will, I think you are referring that there are two ways to talk about the coupling of a two-level system to one mode of the electromagnetic field. It is this same classical picture where we introduce and let me say an analog amplitude of the electromagnetic field which drives it. And then there is a fully quantized picture where you first quantize electromagnetic field and you couple to photon number states.

Actually the beauty of it that the two solutions are exactly the same. So in other words, how to say, if you couple an atom to one mode of the electromagnetic field. We have two ways how we can solve it. One is, we introduce a coherent electromagnetic field, and there is an exact unitary transformation which tells us if we have the quantized field in a coherent state, we can do unitary transformation, and what we get is exactly this Hamiltonian.

So therefore, this is also-- you may not recognize it-- this is actually the quantum description of the electromagnetic field when it is in a coherent state. The other option is, we use the dressed atom picture maybe following some work of [INAUDIBLE] and others where we assume this single mode of electromagnetic field has in photons, and then we solve it for this photon number state.

So in other words, these are the two ways how we can relatively, easily treat the problem. Either we assume the quantum field is in a coherence state or it's in a flux state. But since the dressed atom picture in the standard way assumes that the photon number, N , is large, there is a correspondence that in the limit of N of N being large, the flux state description and the coherent state description fully agree.

And you pick what you want. If you introduce the electromagnetic field explicitly with it's quantum state, you get the dressed atom picture as a solution of a time independent problem, whereas here with a coherent state description, the coherent state oscillates, cosine ωt , with a time-dependent problem. And actually I

should say whenever I get confused in one picture, I look in the other picture and it becomes clear. I generally prefer where we have N photons, it's because we can discuss everything in a time-independent way, but for certain intuitive aspects, this is also variable, so in the end, you have to learn both.

And in your homework, you will actually write down the general solution for this Hamiltonian as an exercise. Nancy.

AUDIENCE: I think I'm confused a little bit. So in the flux state picture, the dressed states can be exactly part of an independent matter of coupling between a lesser photon on any excited state. So like we can write eN as 1 and g or something like that.

PROFESSOR: Yeah. You couple a photon field with N photons and energy in $\hbar\omega$ to N minus 1 $\hbar\omega$.

AUDIENCE: But in this one, is there like a direct photon number thing, because we haven't quantized the field yet? So what do the dressed states mean at this point?

PROFESSOR: Well, the fact is that if you start out with a coherent state, your photon field is not in photons, it's a laser beam. The laser beam or the coherent state is in a quantized description, a superposition of many flux states. So therefore, the number of photons in a coherent state fluctuates or has a large Plutonian statistics, and if you take one photon out or not, it doesn't make a big difference.

For instance, for those of you who know how the coherent state is state defined, the coherent state is defined as when you act on the coherent state with an annihilation operator, you get the eigenvalue times the coherent state. So that tells you you have a fully-quantized description of your laser in terms of a coherent state. You take one photon out and what you get? the same state back.

And this may immediately justify that what we write down here is simply the coherent state with its amplitude and the amplitude of the coherent state would be B_1 , the amplitude of the drive field. And we don't really need other states because a coherent state has the property. You to take a photon out and you still have the same state. So therefore, we don't have to keep track of the coherent state. It's

there all the time.

But what I'm saying can be formulated more exactly when we use the appropriate formulas. But this is sort of the bridge. That's why we do not have to keep track of the photon state. It's because the coherent state has those wonderful properties.

Other questions?

OK. So this is the famous Hamiltonian. And of course, if it's the famous Hamiltonian, we want to solve it. As I said, the general solution is left to the homework, but I want to sort of show you parts of the solution to tell a story. And the question is, well, how do we solve this Hamiltonian? The answer is, we do exactly what we did in the classical problem. We transform to the rotating frame.

In other words, this Hamiltonian is best solved by doing-- you can actually solve it directly. You can just put in a trial wave function and solve it. But I want to sort of bring out the big idea here which is analogous to what we have done in the last few classes, namely we have involved rotating frames.

So what solves this Hamiltonian is a unitary transformation, and the unitary transformation is this one. And so this unitary transformation, let me first write it down, it transforms the Hamiltonian to the time independent one. We have now time independent diagonal matrix elements. Our diagonal matrix elements have changed. Δ is now the detuning of the trial frequency from the energy splitting of the two-level system.

In particular, when we are on resonance, the diagonal matrix elements have disappeared. This is the result of the unitary transformation, and let me just show you this transformation over here. Can be actually written as an operator involving the z component of the magnetic field. And what I just wrote down for you is actually the quantum mechanical operator, the rotation operator for performing a rotation around the z-axis.

So by selecting the rotation angle to be ωt , that's how I can generate the unitary transformation, and this unitary transformation makes the Hamiltonian time

independent. So in other words, everything is in the classical system. We just go to a frame which rotates with a [INAUDIBLE], and we find the time-independent problem. So now this Hamiltonian can be easily solved.

And you will find as a special case when you start with an amplitude, initially you start in the ground state, then the excited state amplitude square is the Rabi oscillation, something we discussed 40 minutes ago, but before, we got it from the classical quantum mechanical correspondence using the Heisenberg equation of motion and here it comes out by explicitly solving for the wave function for the dressed Hamiltonian. Questions?

I want to say a few words now about rapid adiabatic passage, but this time by emphasizing the quantum mechanical aspects. In other words, we have a clear understanding what happens classically. We have a clear understanding what happens in the adiabatic limit, but I just want to sort of in the next 10 minutes use what we have already learned, combine it with the quantum mechanical Hamiltonian and tell you that, well, when you are not fully adiabatic, you actually have transition probabilities between the two states. So I want to sort of bring in the concept of transition probabilities to the case of-- what I want to say is rapid adiabatic passage when it's no longer adiabatic, but what this just means when we sweep the frequency and we're not in the adiabatic limit.

So how do we describe it quantum mechanically? We start out with a Hamiltonian, which has-- we use our rotating framework for convenience that allows to write down exactly the same Hamiltonian in time-independent picture. So the Hamiltonian has two parts, a diagonal part and an off-diagonal part.

So if I show the energy as a function of detuning δ -- well, maybe I should times 2 over \hbar , just normalize it so then it becomes just the straight line at 45 degree y equals x . So the unperturbed-- the Hamiltonian without drive has a level crossing at detuning 0 . Then we add to it the drive term. Well, let me just write down, not the drive term but the full Hamiltonian.

So the full Hamiltonian with the addition of the drive term has δ minus δ and

now it has the coupling with the Rabi frequency. That means that on resonance, the degeneracy between the two levels is split by the Rabi frequency, and if I now show you the energy eigen levels of this two-by-two Hamiltonian, it will asymptotically coincide with a dashed line through this and through that.

So in other words, I'm just reminding you that a non-diagonal matrix element turns a crossing into an avoided crossing. So when we take the frequency ω and we sweep the detuning, so we change δ and do a sweep of the frequency ω at a rate $\dot{\omega}$, then we sweep through the resonance and in one limit, we have rapid adiabatic passage or in general, we realize the Landau-Zener problem of a sweep through an avoided crossing.

So what I'm formulating here is it's the so-called Landau-Zener crossing or the Landau-Zener problem, which is the quantum mechanical description of you take a system by changing an external parameter. Here, we sweep the frequency of the rotating field, but by changing the external parameter, we sweep the system through the avoided crossing.

And it has the two limiting cases that when we go through this crossing very, very slowly, the adiabatic field then tells us we stay on one of these adiabatic solid curves, and this is the case of rapid adiabatic passage, which we discuss in the classical limit. But it is also the other solution if you would sweep through it very, very fast, you're in the diabatic limit, you follow the dashed line and you start up here and you wind up there.

The Landau-Zener problem is actually a problem which you find it in all its text books, but to the best of my knowledge, there is no simple, elementary derivation which I could give you in a few minutes. And if the mathematical problem is a nice, mathematical demonstration of an exact, solvable model, but to my knowledge explicitly deriving it is not providing additional insight. It's one of the cases where the result is more insightful and much simpler than the derivation.

So what I want to give you is, therefore, simply the textbook result. So in the adiabatic limit, you stay on the solid line. If you do not cross the avoided crossing

very, very slowly, you'll have a non-adiabatic probability to jump from one level to the other one. And this non-adiabatic probability is expressed as an exponential function which involves the Landau-Zener parameter. And the Landau-Zener parameter in this exact solution is ω_{Rabi}^2 times this new rate, $d\omega/dt$ or $d\delta/dt$ minus 1.

This square should go outside the brackets, so therefore, what we find is from the exact solution that the Landau-Zener parameter is a quarter times-- and this should now look familiar, the ω_{Rabi} frequencies squared over $\dot{\omega}$. And when we discussed the limit of adiabaticity classically, I hope you remember I gave you the argument by looking at the adiabatic condition that the adiabatic case requires $\dot{\omega}$ to be much smaller than ω_{Rabi}^2 . So here very naturally what appears in the quantum mechanical problem is just the ratio of the two quantities we compared when we looked for the limit of adiabaticity.

So therefore, the probability for a non-adiabatic transition is simply involving this ratio ω_{Rabi}^2 over $\dot{\omega}$. So in other words, we know already from the classical argument, but here we confirm it, adiabaticity requires that this inequality is met.

OK, I could stop here, but since we are using sort of diabatic sweeps in the laboratory, as long as I've been involved in doing quote "atom science," I want to sort of go one step further and teach you a little bit more about this formula and try to provide insight, and often insight is also provided when you apply perturbation theory. So I know the adiabatic case is very simple, but I want to look at the diabatic case and then look at transition probabilities in a perturbative way.

This is actually the way how we often transfer population in the laboratory. So I want to understand better the way how we transfer population from one curve to the other one. So if we do a fast sweep, we call it diabatic. So in other words, if we have this crossing and we go really fast, well, what happens is this is the crossing between spin up and spin down.

If you go much, much faster than the Rabi frequency, the spin has no opportunity to

change its orientation. So therefore, the wave function, the spin has to stay up or down and that means the system just goes straight through the crossing. Because spin up has positive slopes, spin down has negative slope. Being adiabatic, staying on this lower adiabatic curve would actually require the system to go from spin up in this part of the adiabatic curve to spin down in the other part. And to flip a spin cannot be done faster than the Rabi frequency, so if you sweep fast, that's what's happening.

So we have two trivial limits, one is the adiabatic limit or just the adiabatic curve and nothing happens. The other limit is the infinitely-fast limit and nothing happens again when we look at the diabatic basis, which is spin up and spin down. But now let's be almost adiabatic, and this is a problem which we really want to understand physically and intuitively because that means the system spins the small timelier resonance, and there is a small probability to make a transition.

So if you had one of your hyperfine states, pick your favorite hyperfine state, you'll rapidly sweep the frequency. You will find, unless you sweep it infinitely fast that there's a small probability in the other hyperfine state. And that's what you want to calculate now, and I want you to understand how would you estimate and calculate the small probability.

So let's now estimate the result, namely for the small probability in perturbation theory. And actually what I'm calculating for you here is, if you've used evaporation-- I know half of the class is doing that-- if you apply an $i F$ field, you don't have to make it so strong that you and the adiabatic limit. You are exactly in this limit. The atom will slosh several times through the resonance in an almost diabatic way, but there is a finite spin flip probability and that's how you evaporate atoms.

And I want you now to fully understand the derivation, what is the probability of ejecting atoms in the almost diabatic limit with $i F$ spin flips. That's a limit where 90% of the BEC experiments operate. So I hope everyone realize it's an important question and also I hope everybody understands the question because now I have bigger questions for you.

The first question which I will ask you, should we calculate that transition probability by using perturbation theory for an incoherent transition or for coherent transition? Let me just explain you what I mean and then I ask you for your opinion.

Coherently, we simply say in perturbation theory, we start with our population in state 1. We have to do the coupling Hamiltonian time dependence of the population in state 2 and that means if we integrate this equation for a short time, we find an amplitude a_2 . And the probability to be in the state 2, which is the amplitude squared, is proportional to the Rabi frequency squared times the effective time squared, the effective time of driving the system.

Coherent processes are always quadratic in time. If we do it incoherently, well, the way how we describe incoherent processes are Fermi's golden rule, which we've all seen. And the probability in Fermi's golden rule is very different. Well, it is proportional to the Rabi frequency squared, to the matrix element squared, but Fermi's golden rule gives us a constant rate, and for constant rate, the probability is rate times time. So now it is linear in time and then because of the delta function in Fermi's golden rule-- I'm missing a symbol so I use gamma here. It has nothing to do with the Landau-Zener probability. This is just the density of states.

So I hope you know now what is the difference between coherent or incoherent. The most important part is that things are linear in time for an incoherent pulses rate equation and at least for small times quadratic in time for coherent pulses.

So now we come to this process where we take atoms from spin up to spin down. We evaporate with a weak, course, almost adiabatic with weaker f drive, so we are closer to the diabatic limit. And so if you think about this problem, I want you to tell me if this process, the perturbative transition close to the diabatic case is that should we use when we apply perturbation theory, the coherent picture or the incoherent picture.

In other words, is the dynamics of the quantum system, when we go relatively quickly for the Landau-Zener crossing, is that a coherent or an incoherent process? I could see where it doesn't matter, but it does matter. So I think it's an open

question.

Let me give you the answer. It is coherent, and you can see it in the following way. What is the source of incoherent here? We have a Hamiltonian.

The Hilbert space is by two-by-two. There is no coherence which can be lost. There is no spontaneous emission to other states. There is no reservoir. We don't have a small system which couples to a bigger system, and then the small system-- we do that on Wednesday-- has to-- tomorrow, Wednesday-- has to be described by density matrix. We have the none of the physics which would give arise to incoherent physics. It is coherent.

But maybe I'm oversimplifying. Is somebody who said it's incoherent who wants to maybe press me harder and tell me why you think it is incoherent. Well, one possibility is-- and this is why we often use Landau-Zener sweeps in the lab. We have fluctuations of the resonance frequency.

And when we go and sweep through it, we don't know exactly when we hit the resonance. And if you would take an ensemble of systems and you go through the resonance at different times, you will get an ensemble of wave functions, which has different phase factors in it, and in the end, you will actually need a density matrix to describe it.

But this is now an experimental imperfection which I haven't assumed here. So in other words, what you should do is the following. Whenever you sweep through Landau-Zener crossing, you start with the ground state and what you get out is a superposition of ground and excited state. And the Hamiltonian determines absolutely every aspect of the amplitude in the ground and the excited state including all phase factors.

In other words, if you do a Landau-Zener crossing in a way that you prepare 50-50% of the atoms, they are always face coherent and you can use this process as a beam splitter in an atomic thermometer. Fully coherent.

We have five minutes, so if we assume now, I hope we all agree that the amplitude

is obtained in this coherent wave, then, of course, the question is, but what is the effective time? When we sweep the resonance, we are far away. Nothing happens. Nothing happens. Nothing happens. We go through the resonance, everything happens. Nothing happens. Nothing happens. Nothing happens.

So this is the effective time when the wave function really changes and we create the coherent and mixture in the second state. And it is this effective times squared which determines what happens, what our transmission amplitude is. So therefore, the question is, what is the effective time in the Landau-Zener crossing?

I can give you three choices. One is, the effective time is, well, we change ω and the effective time is, how long, what is the time until we have a detuning, which is equal to the Rabi frequency. Another possibility is that the effective time where we coherently drive the system is 1 over the Rabi frequency, just the Rabi period. Or another choice, how I can construct time out of the two frequencies where it's ω Rabi and $\dot{\omega}$. These are our two elements.

So another possibility how I can get time is, $\dot{\omega}$ is frequency squared, and the square root of 1 over $\dot{\omega}$ at least fulfills the dimensional criterion that this is time. So my question for you is, what do you think is the effective time during which we drive the system coherently? I have to tell you before I made up the problem, I do not know the answer. But I can also tell you that there is only one answer which is correct.

All right. I have to say I expected A to be the correct answer, but I convinced myself it's only B. And the answer is the following, and I know I have to stop, but I only need three more lines. The effective time is not the time until you are detuned, the effective time is which I can call the dephasing time, the time during which everything is coherent.

What happens is, we change the frequency $\delta\omega$, and the $\delta\omega$ is, of course, $\dot{\omega}$ times δt . So if we change the frequency by sweeping it in such a way that we are detuning and now with that detuning, if we let the system evolve, we would get a phase shift of π . That's sort of the maximum where

everything adds up coherently.

If we would go longer in time, the frequency has changed to the point that what we add to the amplitude of the other state is no longer in phase to what we have added before. You can sort of look at it like this. You have a little bit of phase a_2 , You build up. You build up. You build up by adding amplitude with the same phase. But now you are sweeping, and this is the criterion where you start to pile up things with the wrong phase and then the phase eventually becomes randomized and you're not effectively contributing.

This equation defines the window Δt during which we effectively add substantial amplitude in the second state, and it involves Δt squared, so neglecting factors of unity, the result of this is that, indeed, Δt is $1/\Omega$. And indeed, if we say the probability for coherent drive is Rabi frequency squared times Δt squared, what we now obtain is the Rabi frequency squared over Ω , and this is our Landau-Zener parameter.

So in other words, if we check with the exact result, the Landau-Zener probability, $1 - P_{\text{non-adiabatic}}$ the Landau-Zener problem is $e^{-2\pi/\gamma}$ or $\gamma\pi/2$ over the Landau-Zener parameter, if I do an expansion of the Landau-Zener parameter for a small value of the exponent, the exact result is $2\pi\gamma$, and this is indeed proportional to Ω Rabi squared over Ω .

So in other words, what I've shown you is that coherent time evolution with this weird effective time, what I motivated physically, exactly that produce the limit of small γ from the exact result for the Landau-Zener crossing. OK. I know time is over. Any questions? OK. So today was officially our Monday class, so we meet again tomorrow, the same place, the same time.