PROFESSOR: So, good afternoon. Our subject for all of today and some part of Monday is coherence in free-level systems. And what I want to show you are there are certain fundamentally new and, to some extent, surprising processes which are possible when you have three levels and not only two.

Because when you have three levels, you have one excited state which is coupled to two ground states. And that gives us the possibility of interference. And actually, all the special things I will tell you about three-level system are related to interference. Certain phenomena will constructively interfere. Other will destructively interfere.

And that leads to a number of important effects. The first effect you want to talk about is the possibility of dark states, that there are superposition states which are bombarded with two laser beams, and they're not excited. Because the amplitudes to go to the excited state destructively interfere.

When you have a state which is dark, which cannot absorb the light, then you already understand the possibility of doing lasing without inversion. Because if a lot of the population is in a dark state, you don't have to invert excited and ground state. Because the dark state doesn't count.

Lasing without inversion is a little bit more intricate than this, but just kind of the idea, the notion of a dark state helps you already to anticipate why something like lasing with inversion may be possible. The dark state is only dark in a very narrow frequency interval of the two lasers.

So if you're a very narrow resonance, then you have a very narrow resonance where something goes dark. But if something goes dark, it's not absorbing. And therefore you have electromagnetically induced transparency, which is a very, very sharp feature in frequency space.

And when you have very, very sharp features in frequency, you can change the group velocity of light, because if transmission and absorption index of refraction changes a functional frequency very, very sharply, you have huge effects on propagation lights.

So kind of all of these different subjects which I want to go through are different perspectives, but you will find similar physics. It's just we look at interference. We look at dark states in various ways. And each of them has created a cottage industry and a subfield of itself.

So I started to remind you that you're very familiar with the dark state in the situation where we have optical pumping. If you have two ground states in an excited state, and you switch only one laser, sooner or later you will just pump all the atoms into the state f and the state f is dark, and vice versa.

So what I want to show you now is that even if both lasers are on, then no matter in what kind of state or superposition state you have the atoms, the atoms see light, which is ready to excite them. But there is-- and this is what I want to show you now-- a coherent superposition state, which is still the dark state.

So the last thing I did on Monday, I wrote down for you our Hamiltonian, which is nothing else in the well-known dipole Hamiltonian in the rotating wave approximation. And what is important now is that we have pathways with Rabi frequency omega 1 and omega 2. And we have one laser fields.

The photons are created, annihilated by the a operator, and the second laser field, where the operator is C and C dega. So there are two ways how we go to the excited state. We can couple from the ground state f to the excited state and from the ground state to g to the excited state.

So when you shine on the light, atoms may be in some initial state. They will sort of spontaneously emit but scatter light. But then after some transient, the state which is stable against further illumination, is a dark state. That's like optical pumping. So you will optically pump into the following state, which I write down for you.

And then we can inspect that indeed it is a dark state. It is a coherent superposition of the two ground states. And the coefficient involves the Rabi frequency, omega 2 of laser 2, omega 1 of laser 1. And we just normalize it by taking the quadrature some of the two Rabi frequencies.

So this state is indeed a dark state. So let me show that to you by looking what happens when we expose it to this operator. This operator v in the Hamiltonian is the operator which couples the atoms to light, which couples the atom to the excited state and this causes spontaneous emission.

So what I want to show you that is a dark state, since if you take this operator, apply it to the dark state and ask, is there any possibility how you can excite, how you can excite to the excited state? And the answer is, it will be zero. The dark state has a component on-- the dark state has one component in f, and it has one component in g.

How did I want to do that? Sometimes it's easier to say it in words than to write down a long equation. The light atom interaction couples the component of the ground state to the excited state with Rabi frequency omega 1 and the state f with omega 2.

However, in the construction of the dark state, I've made sure that the amplitude of the component g is omega 2. So therefore when we apply this operator, we have omega 1 from the operator and omega 2 here. And here we also get omega 1 and omega 2. And this minus sign means the two effects cancel.

So therefore, I've shown you that even though in its full frame with second quantization, photon operators and all this, this coupling is identically 0 and therefore we have a dark state. So this phenomenon of having a dark state and populating the dark state by light scattering, you have spontaneous emission into the dark state until the scattering light staggering stops and no matter what the initial state of the atom bar where the dark state is now 100% populated. This is called coherent population trapping.

Let me mention that this coherent population trapping can be some good thing if you want to pump atoms into a certain state and then they stop scattering light. They're not heated up anymore. But sometimes you want the atoms to scatter light.

For instance, in current laser cooling experiments with molecules, there is a problem that the multiplicity of the angular momentum states, which are suitable for laser cooling in molecules is such that you always have a dark state. So if you irradiate the atoms with laser light, they will not be on a cycling transition. The cycling transition will stop.

So you have to play additional tricks. One is to apply a transverse magnetic field that the axis of the laser and the quantization axis given by the magnetic field are in two different directions. And then once the atom is pumped into a dark state, it will actually precess, Zeeman precess out of the dark state.

Or magneto-optical traps for molecules use a magnetic field gradient, which is rapidly switched, because when you are in a dark state for one value of the magnetic field, you just change the sign quickly. And then the atoms are no longer in the dark state for the new situation you've created.

So in other words, to have these wonderful dark state, which is a major intellectual accomplishment and eventually was exploited in many experiments, for a number of experiments it's a real nuisance. And you have to find ways around it to make sure that the atoms are not stopping scattering light and don't stop scattering light.

I gave you the simple example that the dark state exists for resonant excitation. But as you can immediately show, the dark state also exists for detuning of resonance. The only thing which is important as long as the two lasers fulfill the two photon resonance condition.

So you want the difference between the two lasers is the energy difference between the two states. Also remember if you have two lasers which are very different in power-- so let's say one Rabi frequency is much smaller than the other one.

So then the dark state, the superposition which is the dark state, the dark state is now predominantly which one? The state with the stronger coupling or with the weaker coupling? Well, it's the state with the weaker coupling.

The dark state has always a larger amplitude in the state for which the coupling is weaker. And that of course immediately makes a connection to optical pumping. For instance, if omega 1 the Rabi frequency is 0, then the dark state is what we started out with a trivial example, is a state g. So you see that actually what I taught you about coherent population trapping is nothing else than extension of the concept of optical pumping.

OK, so we've talked about the dark state as a superposition of state g and f. But if you have a two-dimensional Hilbert space given by two ground states g and f, we can now find a new basis. We have defined the dark state.

But now we can get the orthogonal state, which is the bright state. It has a plus sign here and it has the omega 1 and omega 2, the Rabi frequencies reversed. So this state is now orthogonal. And you can now visualize the preparation of the dark state as follows.

We have the dark state. We have the bright state. The bright state, as you can immediately verify, is strongly coupled by the laser field to the excited state. So we have an excited state. The laser fields-- well, its two laser fields strongly couple the bright state to the excited state.

And then spontaneous emission-- two photon lights scattering-- can take you back to either of the two states. But now you see that the concept of optical pumping, which I started out with a trivial example, is now applying in the new basis to the bright and to the dark state. And what happens is you would, by optical pumping, populate the dark state and completely pump out the bright state.

Just as a side remark, there are lots of subtleties. Most of them you will probably see by yourself by just inspecting the results. But if you may ask yourself, what happens when the Raman resonance is not made, when the difference between the two lasers is not exactly the difference between the two?

Well, then what you have is you have a superposition of ground and excited state. So we have the two ground states g and f. But the two laser fields have a phase which evolves different from the phase between g and f.

And suddenly, what you have is relative-- I mean, everything is relative to the laser field, so to speak. This plus sign turns into a minus sign. So therefore, if you have pumped into the dark state, but now the frequency difference of the two lasers is slightly different from the frequency difference between g and f, this mean in essence that as time goes by, with the frequency, with the detuning away from the two photon resonance, the dark state will now precess into the bright state. And it is only for the Raman resonance, when it is exactly met, that you have a long-lived dark state, a dark state which is a 2 eigenstate of this Hamiltonian. Yes?

AUDIENCE: Would the precession also change omega 1 into omega 2?

PROFESSOR: No, we have defined omega 1 and omega 2 as the Rabi frequencies of the two lasers. Omega 1 is a Rabi frequency of the laser which talks to, I think, the state g. And omega two is the Rabi frequency of the laser which talks to the state f. So this doesn't change.

I'm just sort of telling you how you may want to think about it, that if you put in all of the temporal phase factors, on resonance, at least in some dressed atom picture, all the phase factors are 0. Because the e to the i omega t of the atomic wave function is compensated by the light atom coupling.

And therefore phase factors just disappear, and the state is stationary. But if you write down what happens as a function of time to the state, and you couple it as a function of time to the laser field, you will find out that there is an evolving relative phase between the light field and the atomic state. And the essence of that is that with these, there will be a beat node at which the dark state becomes bright and the bright state becomes dark.

OK. One important application of the dark state is the STIRAP technique. STIRAP is adiabetic population transfer. So STIRAP stands for stimulated-- what is the r? Stimulated Rapid Adiabetic Passage?

AUDIENCE: I've seen both rapid and Raman.

PROFESSOR: So we have two choices. It's both Raman and rapid. A is adiabetic, p is passage. And STI is stimulated, because it's a coherent stimulated process. The idea is the following. With this concept of a dark state, I can immediately explain to you how, by changing the intensities of the laser beams, you can make an adiabetic transfer from one state g to the other state f.

And this is important because in many atomic physics experiments, you start with one state and then you want to prepare another state. You always have the option of having some suitable pi pulse and going from one state to the next. But the pi pulse has to be exactly pi. You have to be careful what the intensity and duration of your pulse is.

But if you can adiabetically go from one stage to the other one, this is terribly robust against pretty much everything. And I want to now use the concept of the dark state to tell you how this population transfer works. But before I even get into any explanation, the picture is the following.

The dark state-- if you change laser parameters, you can change which state is the dark state. I gave you the simple example if one laser beam is on, the state g is the dark state. If the other laser beam is on, the state f is the dark state. So if you switch one laser beam off and the other one on, you've changed the definition of the dark state.

And as I want to show you is the dark state, if we can set it up in the situation that the dark state is the absolute ground state of the system, and you know when you change parameters of your Hamiltonian, adiabeticity tells you that you always stay in the ground state. See now, even without any equations you understand what adiabetic population transfer is. But let's work it out.

So we want to understand-- and this is our adiabetic process-- what happens when the laser parameters, omega 1 and omega 2, change slowly. So for pedagogical reasons, I will drop all the assumptions I'm making now, but let me make them.

So let's discuss degenerate states first. So g and f are degenerate. And let me also discuss blue-detuned light. And you will see in a moment why. So that would mean that-- and let's set up the situation. We have the state g, f, and e.

We have a green laser beam and an orange laser beam. And we know already, from what we have just learned, that this should be transformed into-- and let me just be more specific here. I said I want to use blue-detuned light.

So now we do what we just learned. We express things as the dark state and the bright state. And here we have the excited state. And the beauty of degeneracy is that dark state and bright state are superposition states, but since they're superposition states of two states with the same energy, I can draw them without twisting my hand or doing sort of a double line.

Anyway, but now what happens is the following. The bright state is coupled to the excited state by the two laser fields. And the question is, what is the energy of the bright state relative to the dark state?

Well, if I have a blue-detuned laser beam, I get an ac-Stark shift, which increases the energy. So therefore, the state which sees the light, which scatters photon-- scattering photon means there is an excited state admixture. An excited state admixture means there's an ac-Stark effect.

And the ac-Stark effect is positive. So therefore, the dark state is-- the uncoupled state is the lowest state in the ground state manifold. And even if you would not generalize it to multiple levels and all that, if there is one dark state, it's not upshifted by the ac-Stark effect. Whereas every state which sees the light is upshifted by the ac-Stark effect which provides a blue shift.

So the dark state is the lowest quantum state. So therefore, we can now use the general concept in quantum mechanics of adiabatic state transfer to achieve a perfect 100% transfer from one ground state to the other one by tailoring as a function of time our laser fields as follows.

We start out in a situation where the only beam which is on is-- I want to change that to orange-- is the orange laser. And when the orange laser is on, then the dark state is g. Then we ramp down the orange laser. But we ramp up the green laser. And we have the situation that initially g and eventually f is the dark state.

So this is a picture behind the rapid adiabetic passage from state g to f. I want to point out that the application of the two laser pulse, first orange and then green is called the counter-intuitive sequence. Because if you ask yourself the problem, you're starting out in state g and you want to go over to state f, which laser would you first switch on if you're in state g?

Well, if you want to get out of state g, you would say first the green laser to go up and then the orange laser to go down. So what I just said, first the green and then the orange is called the intuitive sequence. But STIRAP works with the counter-intuitive sequence.

OK. Any questions? Yes?

AUDIENCE: So what's the intuition we have with this?

PROFESSOR: Well, I gave you the intuition. I said, you want to have a dark state. The dark state is the ground state, the lowest state of the manifold. So you want to first make sure that g is the ground state. And then you turn opposite your Hamiltonian, and eventually f is the ground state.

And this is exactly the situation. f is the ground state with the green light on. I've given you the intuitive picture. Now I will actually come back to that. There is the kind of-- it's called the magic of the adiabetic state transfer that if you always stay in the dark state, if you always stay in the dark state, you never populate the excited state.

So it looks like magic that you can go from the ground state to the final state without having ever any population in the excited state. And of course, can that be true? No, because there is no coupling direct. So somehow in this picture of adiabetic transfer always staying in the dark and never populating the excited state, maybe we have over-emphasized something.

We need a little bit of excited state, otherwise we cannot go there, because our Hamiltonian does not allow the direct path from g to f. So a lot of people think that STIRAP is a method where you can go from g to f without going through the excited state. I have many, many people talking about it.

So I want to discuss it in the next few minutes and to some extent demystify it. But since you're asking me-- now in a black and white picture, STIRAP is the way. I'm correcting myself in a moment, but STIRAP is a way where you can go directly without going through the excited state.

And this is done by the counter-intuitive sequence where you just want to keep atoms in the dark state, whereas the intuitive sequence is really based on the fact you want to take population here, put population here and the stimulate it down. And this is not what STIRAP is about. Other questions? Yes?

AUDIENCE: So if there is very strong coupling to a fourth state from the excited state, because you need to-- the population needs to travel through the excited state in STIRAP, it's possible to spoil the whole process.

PROFESSOR: We will talk about it. And actually in the next minute, I want to sort of simply calculate how many photons are emitted during the passage. And if you now say the photons have a branching ratio, this could be a branching ratio to you fourth state.

But by just calculating what is the time integrated population in the excited state multiplied with gamma, this is the amount of photons scattered. And you can say if you try to do a STIRAP with the bosons then condensate, every time you scatter, you heat up, you reach momentum states outside the condensate. And those momentum states are your fourth state.

So actually, by discussing how much light scattering happens in this transfer, I will pretty much talk indirectly about how much population is going into a fourth state. So that's what you want to understand now-- how true is the magic of the dark state transfer where we can go from one state to the next without ever going through the excited state. Yes?

AUDIENCE: So are the two laser frequencies the same for that?

PROFESSOR: Yes. I actually assumed that-- I come to that in a second-- that the two states are degenerate. And for all this dark state busyness, we have to fulfill the Raman resonance. So if the two states are degenerate, then the two laser beams.

AUDIENCE: So then what's the meaning of increasing one of them? We are satisfied--

PROFESSOR: OK. One possibility is that this is an m equals 0 state. This is an m equals minus 1 state. This is m equals plus 1. And this is sigma plus. And this is sigma minus radiation.

So yes, and this was actually very important for the whole discussion of three level coherence that this level can only talk to the green laser, this level can only talk with the orange laser. And we have two ways to accomplish it. One is by frequency difference. But if you don't have a frequency difference, we need polarization selection rules. Yes?

AUDIENCE: It's sort of a silly question. In this particular case, if I have all my atoms in g, can I turn on the green laser? Can I optically pump everything to f? Right? And so in this particular case, is there an advantage of doing STIRAP as opposed to optical pumping?

PROFESSOR: Yes there is, and we'll talk about that. There is a difference between transferring population or only transferring an amplitude. And so it depends what you want. If all you want is to get the atoms from g to f and you don't care how they are heated up, so that's OK.

But the STIRAP process is a stimulated process, so you have momentum k and here you have the momentum k2. So you take the atoms from an initial state and put them in a very well-defined momentum state. So a PEC in one state after STIRAP transfer is in another state, maybe if you have the lasers in a counter-propagating way back to zero momentum.

Whereas if you do optical pumping, which involves spontaneous emission, then you always have dissipation. You create entropy. And because spontaneous emission goes into many modes, you have many different recoils, and eventually your final state is no longer a single state. It has many different momentum states involved.

So that's why we've been talking here about a coherent transfer, sort of assuming that optical pumping with spontaneous emission and the many modes involved is not what you want because of heating. OK, so lots of interesting questions.

First, let's now drop all the assumptions we've made. So what happens when we change from blue detuning to red detuning? Well, for blue detuning, we use the argument that adiabetically we stay in the ground state, but for red detuning it's a higher state, and we adiabetically stay in the higher state. Same thing.

What happens if f and g are not degenerate? Well, the argument is the following. It's the same result, and I can quickly show that to you by saying use the dressed state basis. You have the state g with Na and Nb photons in laser beams.

Let me be consistent. I call the operators c and Na and Nc. And if you now look at the state f, which has one photon less here and one photon more here, those two states are degenerate. So I was just showing the situation if g and f are not degenerate, here we have a laser beam with Na photons.

Here we have a laser beam with Nc photons. If we include the photons in our description, we have a degeneracy. And that's what matters. But what is relevant of course is you have this degeneracy only when the energy difference between g and f is the energy frequency difference between the two laser beams. So it is essential, and you see that again here, that the two photon Raman resonance condition has to be made.

So we've talked all the assumptions so we know you can, in a general system you can transfer between two hyperfine states. You can use red or blue detuned laser. Everything is robust.

OK. So STIRAP goes by the name dark state transfer. And what I want to address now is, what is the magic which is going on? How can we transfer from state g to state f without going to the excited state?

And the answer is-- I gave it already to you-- it's not possible, because the only couplings we have are the two couplings which I've shown. But-- and this is sort of the gist you should carry away. Since the whole transfer is a coherent process-- and I will be more exact in a moment-- we are not building up population in the excited state.

We're just building up a small amplitude. And the population is the small amplitude squared. It's, so to speak, epsilon squared. And so you win one power of epsilon, one power of the small amplitude, if you do a coherent transfer, as compared to incoherent transfer, which relies on shuffling over population. And I want to show you now how this pans out. Jenny?

AUDIENCE: But in the limit of infinitely adiabetic, infinitely slow, you still would have the transfer happen without ever going into the excited state, right?

PROFESSOR: Well, I will show you that the integrated population in the excited state goes as one over the transfer time. So if you allow an infinite amount for the transfer, you can make the population [INAUDIBLE] slow. But ultimately, what I also want to show you is that how much you put into the excited state depends-- and I think this is a very satisfying result for me-- depends on your resources. It depends on your laser power. if you have infinite laser power, you can keep the population infinitesimally small.

And if you have infinite time. But if you go to the lab and do an experiment and say, I want to be done in 24 hours, well this defines your time budget. And the strongest laser you have in the laboratory defines your laser power budget. And given those two resources, there is a small amount.

And I want to give you an estimate for that. And then we have to discuss, is that small amount, which we get in STIRAP, does that really stand out over other methods to transfer population? And the answer will be interesting. So let me just say, no. Excited state is needed as a stepping stone.

I want to give you a quantitative touch without doing too much math. So let me just write down Schrodinger's equation. We have the three amplitudes, the three states involved, e, g, f. Schrodinger's equation gives us the time derivative. c dot f.

Since we assumed in the dressed state picture, we can assume everything is degenerate. But now we have couplings. And the only couplings are-- and this is what I was emphasizing-- between ground and excited state with the Rabi frequency omega 1 and between the state f and the excited state with Rabi frequency omega 2.

So the important part is-- I can write it down, but the third equation is the one I want to focus, that you can only build up population in the final state by transferring with a Rabi frequency omega two population from the excited state. So let me just assume that Rabi frequency omega 1 equals omega 2 equals omega Rabi. Let's have a symmetric situation so I can now completely focus how can you get from the excited state to the final state, but how you get from the ground state, from the state g to the excited state is fairly symmetric here.

OK, so what we want is we want that at the end of the transfer time, t transfer, the final state amplitude is on the order of 1. This would be 100% percent transfer. So therefore, just looking at the differential equation above, that means that this here has to be 1 divided by the transfer time.

So therefore, we find that for this adiabetic transfer, the excited state amplitude has to be-- and I neglect factors on the order of 2 now-- has to be 1 over the Rabi frequency times the transfer time.

So the probability to be in the excited state is the amplitude squared. And if you are asking, which I think is a practical question and I suggest to use this as a figure of merit, when we discuss how well, how perfectly we have transferred population, we can now ask, what is the probability of spontaneous emission?

P spontaneous is the probability to be in the excited state times gamma integrated over time. So what we obtain for that is yes, it's proportionate to gamma. But we have the excited state amplitude squared. We've multiplied with the transfer time. So therefore, what we have in the denominator is the laser power or the Rabi frequency squared times the transfer time.

So therefore, this probability to go to a spontaneous emission, which is also the integrated probability of having populated the excited state, it goes to 0 in the case that you have infinite laser power or that you're infinitely patient.

So one question is, how long can the transfer time be? And this is actually setting an limit to it. We're talking about a coherent transfer. So the transfer time has to be smaller than the coherence time of your system. For instance, we do that all the time in my lab. If you go from on sublevel to another sublevel, and we have magnetic field noise, if the magnetic field noise is destroying the phase relationship between the two hyperfine states, then the coherent transfer gets interrupted, and then eventually we are no longer talking about the amplitude. We are talking about population in the excited state.

OK, so we have this result, which at least demystifies what many people say-- actually I've heard truly experienced researchers in the field to say, we can do the transfer without ever populating the excited state. So the truth is what I just derived for you. But now we want to compare it to what would happen if you use an incoherent picture.

If you use an incoherent picture, let's assume we would use a pi pulse, put all the population in the excited state, and then we would stimulate it down. So we have a pi pulse. So we build our population.

And then we use a second pi pulse. Well, we would then have about unity population in the excited state. It would emit light at a rate gamma. And it would take the time of the inverse Rabi frequency, the Rabi period, to stimulate it down.

Now you can say, what happens if-- maybe that's dumb to have a pi pulse and put all the population in, take a pi over 20 pulse, put a little bit population in, stimulate it down. The next batch, the next batch, the next batch, but you get exactly the same result, because each population which you've put into the excited state had to wait for a Rabi period to be stimulated down.

So therefore in the incoherent picture, if you sort of transfer batches of population, you realize that the probability for spontaneous emission or for populating a four state is independent of the transfer time, because it cancels out. If you do small pieces of population, you have a smaller rate of emission, but you integrate for longer and the two temporal factors cancel out. And now you realize that the coherent transfer is more favorable by another power of omega Rabi times t transfer in the denominator.

So that's why people prefer STIRAP. Questions about that? Can you suggest another method how we could transfer population from one state to the next? I gave you two examples. One was this magic, or not so magic, dark state transfer. The other one is a method where we put population, stimulated it down, next batch of population, stimulate it down. What is another process which we can do to transfer population?

AUDIENCE: Landau-Zener?

PROFESSOR: Pardon?

AUDIENCE: Landau-Zener?

PROFESSOR: Landau-Zener? Actually, the STIRAP is a Landau-Zener. The dark state is always the lowest state. There is a crossing sort of in between, but it's an avoided crossing. Actually, in a way, you may want to think about it. The STIRAP is actually a Landau-Zener transfer between where the lower state is a dark state, the upper state is a bright state. Will?

AUDIENCE: I still have a question. So Landau-Zener [INAUDIBLE] But the STIRAP scheme is sensitive to the run-on resonance.

PROFESSOR: Absolutely it is.

AUDIENCE: So how robust is it against small violations [INAUDIBLE]?

PROFESSOR: Well, I gave you already the hint when I discussed that if you are detuned from the Raman resonance, this detuning, at the detuning frequency would be between dark and bright state. So this picture what is dark and what is bright is scrambled up by that. So in other words, as long as you're inverse detuning is smaller than the transfer time, you're OK.

Because the dark state is the dark state. But if during the transfer time you turn the dark state into a bright state, you've completely messed up the scheme. So in other words, I know people in this hallway, in my group want to STIRAP transfer between molecules.

And we have to stabilize our lasers. So we have automatically little bit laser fluctuations. But if you do the transfer in a millisecond, it is sufficient to have the laser frequency under control at the kilohertz level. So that's a requirement.

But back to my question. We have discussed sort of an incoherent transfer and we've discussed STIRAP. Can you suggest another method how you can go from one count state to the next?

AUDIENCE: We talked about optical pumping.

PROFESSOR: Optical pumping? Yes, but optical pumping means we really go with spontaneous emission. And then we have 100% spontaneous emission. So we want a process where we go from one momentum state to another momentum state. Well--

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yes. Optical pumping is a two-photon process. Actually, everything's a two-photon process here. But optical pumping is actually a process where we have a Raman to photon transition. The first leg is stimulated. The second one is spontaneous. But I would suggest now what [INAUDIBLE] said, let's have a two photon Raman transition.

So we want to go from state g to state f with two photon Raman transition. Let's assume we have a detuning delta. So now, do you have any expectations whether these-- we can do a pi pulse.

Two photon Raman transition is a two photon Rabi frequency. I'll remind you in a second of it. And if you do a pi pulse in that, we have half a Rabi oscillation, which takes us from cg to f and we have 100% transfer. So do you have an expectation if this Raman process how will it turn out to be in our figure of merit, how many photons are emitted from the excited state during the transfer?

Will it be the same as STIRAP? Will it be the same as in population transfer? Will it be something else?

AUDIENCE: You have to scale.

PROFESSOR: It will scale with the detuning, but-- OK, what I will do is actually-- I will in a moment say, OK. We have a certain laser. What I'm going to say? We have to compare apples with apples. And here we have the detuning as a parameter. And what I want to replace is I will-- what will replace the detuning by the time it takes to do the pi pulse.

So the detuning is determined by the condition that we want to do the transfer in the same time as the STIRAP. So this will eliminate the detuning. So we want to use the same resources. Our resources are time and laser power. And we want to use our resources of time and laser power in the same way for the two photon transition.

So the question is what happens to-- what is the answer to the question, how much excited state is involved in the two photon Raman process as compared to STIRAP? Do you have an expectation? Who of you is doing STIRAP in the laboratory?

A few people, yeah. Actually, I know that 99-- my observation is that 99% of the people say STIRAP is special because it's a dark state transfer. And when I started to confront famous people in the field and say the two photon Raman process does exactly the same, they were completely surprised.

Anyway I want to show you in one equation that the two photon Raman process has exactly the same integrated population in the excited state, and therefore it is as good-- the two photon Raman process performs exactly as well as the so-called dark state transfer, where you never go through the excited state.

And this for me completely demystifies it, because the two photon Raman process has in common with the STIRAP that everything is coherent. And if everything is coherent, it means we are not building a population. We are only building up amplitude in the excited state.

So let me just show you in one equation that indeed it works out in that way. It's also a nice way to quickly recapitulate what we learned in the previous chapter about two photon transitions. So the two photon Rabi frequency is nothing else than the product of the two Rabi frequencies divided by the detuning.

And of course as-- if you use studied larger and larger detuning as you said, everything will slow down. But we know that the transfer time will be the inverse of the two photon Rabi frequency. And this is delta over omega square. So therefore, we will eliminate delta from our equations by replacing it by the time it will take to perform a pi pulse between the two ground states.

OK, so what is the probability to be in the excited state? Well, we've done the theory of the ac-Stark effect. We have beaten perturbation theory to death.

You know the first order admixture is omega Rabi over delta and the probability is omega Rabi squared over delta squared. So therefore, the probability to spontaneously emit a photon is omega Rabi squared over delta squared times gamma times t transfer. And this is nothing else than gamma divided by omega

Rabi squared times t transfer.

So if you use a Raman pulse to transfer the population coherently, the integrated population of the excited state, which is a measure for heating and measure for winding up in the wrong state, emission to a false state is inversely proportional to your laser power and inversely proportional to the time you can afford to do the transfer. Exactly the same as for STIRAP. And the limit to the transfer time in both cases is set by the coherence time.

Try to tell people who do STIRAP that a two photon Raman pulse would do exactly the same as STIRAP, at least in populating the excited state. They will be surprised. Because here, you explicitly go through the excited state.

We don't have a counter-intuitive sequence. We switch on both lasers simultaneously. But I think this for me illustrates or completely demystifies what people have said about the dark state transfer and the STIRAP process. Any question? Cody?

AUDIENCE: Why do people use STIRAP? What's the advantage?

PROFESSOR: Well, why are you using Landau-Zener sweeps when you go from one hyperfine state to the next, and why don't you use a pi pulse with your f generator, because it's more robust.

AUDIENCE: [INAUDIBLE]. As you said [INAUDIBLE].

PROFESSOR: But similar, if you detune-- if you detune your-- the condition is the same. If you have your two Raman laser and you detune them, you no longer have resonant Rabi oscillations. You have off-resonant Rabi oscillations.

And you will find if you want to do the transfer in half a period, you have to make sure that you don't get one cycle of the beat node or one cycle of the detuning in your transfer time. So the condition for the frequency stability for the two photon Raman and the STIRAP I think are pretty exactly identical. So I think it's pretty much the robustness.

And you have to decide what you need and what you want. For instance, if you have laser beam and the laser beam has an inhomogeneous profile, you cannot meet the pi pulse condition for the atoms in the middle and at the edge of your laser beam. But in STIRAP you just provide plenty of extra power and then everything is robust against the laser beam profile.

So there are clearly advantages like that. But all what I point out is the advantage is not the population of the excited state. It's the same. Questions?

OK. So this is dark state and STIRAP. Our next topic is lasing or gain without inversion. So in other words, I want to show you what was actually rather recent accomplishment that this common belief that you need population in the excited state, which is larger in the ground state, for optical gain and lasing is not true.

Well, it's true for two level system, but it's not true when you have more than two levels. And I discuss it here for the case of a three-level system. And this is really insight which came as recent as in the '80s. One reference is the person who pioneered a lot of work on three-level systems and provided a lot of insight was Steve Harris at Stanford.

And this was a paper in the late '80s which enunciated the possibility of having lasing without inversion. And when you are a graduate student like me, who was educated earlier, you say, gee whiz. That's not what I learned in class. But of course it's often the small sprint. It's the assumption for two-level system.

So the secret behind lasing without inversion is that you can have cancellation of absorption by interference. In other words, this belief that you need inversion comes that you want that the stimulated emission is stronger than the absorption. But if you have a three-level system, you have to be careful. We have interference effect.

And what I want to show you is that we can create a situation where we have destructive interference for absorption, but we don't have destructive interference for stimulated emission. So therefore, we can now afford to have more population in the ground state, because the absorption out of the ground state does not have to be suppressed by minimizing the population in the ground state. We have an interference effect, which suppresses absorption from the ground state population. And therefore, the ground state population can be larger than in a two-level system.

I could simply tell you you can hide population in the dark state, end of discussion, and you get the basic idea. But the concept of lasing without inversion is richer than that. It's connected to the dark state, but there is more to it. So that's why I want to spend the rest of today's class in discussing this concept.

So I want to discuss it in the simplest possible scheme. So I want to discuss first the simple system. In the end, this is not the system in which it has been accomplished. But I think what you realize is I draw you up a very simple scheme. I make all assumptions, and there is maybe no atomic, no suitable atom where you can realize it.

But then I will do what I've done so often in this class. I turn around and say, but if you go to a tri-state basis, we can exactly engineer that. So in other words, if you need a certain level scheme and you don't get it, you can just take another level, put it there with a strong laser as a virtual level, and you get the level scheme you want.

And in a dress state description, you have exactly the idealized level scheme I'm presenting to you. So therefore with that justification, let me just try to give you the simple scheme where you can understand this absorption cancellation by interference. I mean, later in the end, we want to use lambda systems with two stable ground state, two hyperfine states.

That's how all the research is done. But now I assume we have a V system. So we have two excited states and we have one ground state. And I want to assume that by coupling through radiation, spontaneous emission, those excited states couple to the same continuum.

I want to say that this is an important assumption. If you have two states, hyperfine states, n equals plus 1, n equals minus 1, which would emit photons of different circular polarization, no. They are not coupling to the same continuum. They would be distinguishable, and then certain interference effects will not happen.

So eventually, the magic of quantum interference comes if it is absolutely indistinguishable which excited state has emitted the photon. So we have to really be-- set up something like this. So it must be fundamentally impossible to distinguish photons emitted through one state or the other.

So we know already if we treat the system, we couple the atoms to the continuum. That means in the end, that those states acquire a certain width. And yes, this is a situation we want to discuss. So what we have set up now is a situation where we have, with these two states, two indistinguishable paths to scatter photons.

So let's say we take the ground state and we shine laser light on it. It's a two photon process. A photon goes in, a photon comes out. And we can now go through excited state 1, through excited state out then a photon comes out and eventually we have now the coupled through this two photon process the ground state to continuum of modes.

But those two processes to go through e1 and e2 are indistinguishable. So I will give you immediately an expression where in perturbation theory, we are not adding intensities. We are adding the two amplitudes. It's like a double-slit experiment.

Or in other words, the situation I want to discuss now is the following. We have all of the population in the ground state. We have the two excited states. And our laser is tuned in between. And now we can scatter light. We've talked about really scattering and such.

And you know that for an infinitely heavy atom, the emitted radiation is a delta function at the incident light. And you will not know, you can never know which excited state was involved in the scattering. So we assume that we have excited state-- which one is the lower one? I think e1. This is e2.

And our laser beam is detuned. We have a detuning of delta 1 and delta 2 respectively. So now we have to add amplitudes when we do perturbation theory. So in second order perturbation theory, for the light scattering, if you want to derive

Fermi's golden rule as we have done a number of times here, you remember, the critical part-- I'm not writing down the whole expression-- the critical part is sort of a product of two matrix element or two photon matrix element which takes us from the ground state through the light atom coupling to the excited state.

And then from the excited state why are the coupling eventually back to the ground state, which is now a continuum of modes. And we sum over all possible modes. And what we have here is the detuning denominator. Well, if you want to do a little bit better, I'm just mentioning here for completeness, we want to add the imaginary part.

That's more placeholder for a command I want to make later. It's not really essential. But the new thing I want to discuss now is that since we have two excited states and not one, we have to sum over the two excited states, and it's a coherent sum.

So if I detune between-- if my laser detuning is tuned in between, then delta 1 is positive and delta 2 is negative. So therefore, we do a sum here with opposite signs. And if I can neglect the gamma, assuming the gamma is small or much smaller than the detuning, then I'm just adding up two numbers with opposite sign.

And depending now on the matrix element, if they're identical I get a cancellation if I'm detuning half between. But even if the matrix element are different, I will always find a detuning where the sum is 0.

So to the extent that I can neglect the imaginary part, the gamma in the denominator, I have now this situation that this is 0. It vanishes for a certain laser frequency, omega 0, tuned between the levels e1 and e2. And what I've assumed here is that the detunings are larger than the decay widths, and therefore I can neglect that. Any questions about that?

So OK. Very trivially, two excited states which are indistinguishable. It's the modern version of Feynman's double-slit experiment. We add up the amplitudes and by necessity, we get 0. But that sounds so trivial, but it took a Steve Harris, some genius person to invent it, to realize that when we have light at omega 0, it would be absorbed by e1. It would be absorbed by e2.

But it is not absorbed in the situation we have with e1 and e2. But now we have to make the connection that the statement that we need inversion for lasing was related to absorption, but now we have cancelled absorption. So now what we want to do is we want to create a situation where we have lasing.

We want to put population into the excited state. But the population in this excited state will be much, much smaller than the population in the ground state. So you're not even close to inversion.

So let me draw the diagram. We have lots and lots and lots and lots of population in the ground state. We have 0 population in e1. And in e2, we just have a little bit. And now you would say, well, you don't have inversion.

How can you get lasing? But if we have now a cavity at frequency omega 0, then you can say the Lorentzian profile of the excited state e2 overlaps with omega 0. So at least you have some possibility for the excited state e2 to decay into the cavity mode.

And therefore, a very weak field in the cavity mode will be amplified by stimulated emission without suffering any absorption. So therefore it will be amplified and we have gain. So if you have a cavity at omega 0, we have gain because we have some stimulated emission at omega 0 but no absorption.

Of course, if you would tune your cavity to be in a resonance with e2, you would have more stimulated emission, but you would have much more absorption and you would not be able to get a net gain for a small probe field. So you would not have lasing.

So you can say that what we have achieved here in a three-level system, we have accomplished destructive interference for absorption. But the way how we've put population just in e2, we have not any destructive interference for the stimulated gain. So we have tweaked on the lasing equation, on the laser equation the absorption part, but not the stimulated emission part by interference in a three-level system.

So how can this be realized? Let me just give you the first possible realization and then I think we should stop. We could, in principle, realize it with hydrogen and a DC electric field. Remember, hydrogen has the 1s level, has the 2s level, and the 2p state.

And you all had a nice, little nice homework assignment that when you have a DC field, you actually couple 2s and 2p. And if your DC field is stronger than the Lamb shift, you actually create a superposition of 2s and 2p states. And now, if you tune your laser right in between, you have a dark resonance.

So the situation is if I would now plot the absorption versus frequency, you would find that if you're in resonance with the upper state you absorb, but then even in between, there is this notch where you have zero absorption. So for this simple case with hydrogen in a DC electric field, the dark resonance occurs when you have equal detuning.

You have an equal detuning for the atom with respect to the two states. And I like to sort of connect it to classical physics. You know that absorption is related. When you absorb, you have light scattering.

The photons are not disappearing. They're just scattered into other modes. That's what absorption is about. And classically, light scattering comes because you have an oscillating dipole moment. But what happens is if you have this coherent superposition and you're right in between, you have a positive detuning with one harmonic oscillator and a negative detuning with the other harmonic oscillator.

So if you drive an harmonic oscillator-- and this is one and this is the other-- with positive detuning, with red detuning, the dipole moment is in phase with the electric field. In the other case, it's out of phase. So now you have the two excited states and you're driving two equal but opposite dipole moments in those two states.

And therefore, the two dipole moments add up to 0. Therefore the laser, just using our understanding of a simple harmonic oscillator, is not creating any dipole moment. Therefore, we're not scattering any light and therefore we're not absorbing any light.

So this is how you can now realize in the hydrogen atom using DC magnetic field and creating this level structure, you can create lasing without inversion. But I will tell you on Wednesday-- no, today is Wednesday-- next Monday, next week that we don't have to deal with the difficulties of hydrogen atoms in the laboratory.

We can just use our favorite alkalize and dress the alkalize up with laser beams and eventually create the same situation with an atom which is experimentally much simpler to address. Any questions? OK, one more week to go. See you on Monday.