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**PROFESSOR:** Are we ready? So good afternoon. Just a reminder, this week we see each other three times-- today on Wednesday and Friday in this other lecture hall for our mid-term exam.

Today we will finish the big chapter on light-atom interaction. But we're not getting rid of it, because we will be transitioning to an important aspect of light-atom interaction. And these are line shifts and line broadening. So today we start the next big chapter-- line shifts and line broadening.

But before I do that, we have to finish light-atom interaction. And I want to come back to the rotating wave approximation revisited. So I'll revisit the revisit of the rotating wave approximation.

And sometimes when I have discussions with students after class, I realize that something which I sort of casually mentioned is either confusing or interesting for you. And there are two aspects I actually want to come back here. So several people reacted to that, but some felt it was maybe a little bit too complicated. Or others asked me about some details. So let me come back to two aspects. And I hope you find them interesting.

One is when we sorted out all those terms, those [? need two ?] angular momentum selection rules. But I made sort of the innocent comment-- well, if you have  $\omega$  minus  $\omega$  in a time-dependent Hamiltonian, one term is responsible for absorption, one is for emission.

And when more than one person asked me about it, I think many more than one person in class would like to know more about it. So therefore, let me spend the first few minutes in explaining why is a time-dependent term in the Hamiltonian with plus

or minus  $\omega$ , why is one of them responsible for absorption, and one is responsible for emission?

Well, we have Schrodinger's equation, which says that the change of the amplitude in state one has a term. And if it started out with population in state two-- let's say perturbation theory, we start in state two-- then it is the only term where the differential equation, through an off-diagonal matrix element puts amplitude from state two into state one.

So what I'm writing down here is just Schrodinger's equation. And the operator  $V$  is the drive field connecting state two to state one. And so if I just integrate this equation for a short time between time  $t$  and  $t + \Delta t$ , and I'm asking, did we change the population of state one, which is now our final state? Well, then you integrate over that for time interval  $\Delta t$ .

But now comes the point that the initial state has, in its time-dependent wave function, a vector which is  $e^{-i\omega_2 t}$ . The final state, which I called one, has-- because it's a complex conjugate--  $e^{i\omega_1 t}$ . And let's just assume we have here the proportionality to  $e^{i\omega t}$ . And let me just say  $\omega$  can now be positive and negative. It will be part of the answer whether it should be positive or negative.

Well, this integral here becomes an integral of  $e^{i(\omega_1 - \omega_2 + \omega)t}$  integrated with time. And this is an oscillating function, where if you integrated with over time, it will average to zero unless  $\omega$  is equal or at least close to the frequency difference between initial and excited state.

So actually, what you encounter here is-- well, what I've derived for you here is actually, you can say energy conservation. I didn't assume it. It is built into the time evolution of the Schrodinger equation, that you can only go from state one to state two or state two to one if the drive term has a Fourier component  $\omega$ , which makes up for the difference. Or I'm using different language now. If, through the drive term, you provide photons, you provide quanta of energy, where  $\omega$  fulfills the equation for energy conservation.

And you also see from this result, when  $\omega_1$  is higher than  $\omega_2$ ,  $\omega$  has to be negative. When the reverse is true,  $\omega$  has to be positive. So that's why I said, the  $e^{+i\omega t}$  term is responsible for absorption. The  $e^{-i\omega t}$  term is responsible for stimulated emission.

You also see, of course-- but I stop here, because I think you've heard it often enough. If you integrate over short time  $\Delta t$ , this equation has to be fulfilled only to within  $1/\Delta t$ . This is sort of the energy-time uncertainty. For short times, the photon energy does not need to match exactly the energy difference.

And you also realize when we think about  $\omega$  is close to resonance, then  $e^{+i\omega t}$  does absorption. But if you're in the ground state,  $e^{-i\omega t}$  leads now to a very rapid oscillation here, which is close to  $2\omega$  oscillation. And we've discussed that in the context of the AC Stark shift, that this gives rise to the Stark shift.

We've also discussed that this term is rapidly oscillating and it's nothing else than the counter-rotating term which we usually neglect when we do the rotating wave approximation. So everything we've discussed in this context-- counter-rotating term, energy conservation, Heisenberg's uncertainty, time-energy uncertainty actually comes from this kind of formalism. Any question?

Of course, if you quantize the electromagnetic field, then you don't have a drive term with  $e^{+i\omega t}$ . You just have  $a$  and  $a^\dagger$  for the photons. And the question, which term absorbs a photon or creates a photon, does not exist.

Because you know it whether it's  $a$  or  $a^\dagger$ . But you have the two choices, whether you want to use a fully quantized field with photon operators or whether you want to use the time-dependent formalism, using a semi-classical or classical field in the Schrodinger equation.

The second comment I wanted to do is using the semi-classical picture, I was sort of going with you through some examples when the rotating wave approximation is necessary, when not. When do you have counter-rotating terms. And yes,

everything I told you is, I think, is the best possible way how you can-- I assume, because I haven't found a better one-- the best possible way to present it and explain it to using the time-dependent electromagnetic field.

But I realized after class that it may be useful to quickly state what I have said in using the photon picture. If we have circular polarization, we have, for given frequency, annihilation and creation operator. But let's assume that the mode we are considering is right-handed circularly polarized, so the operator creates a photon at frequency  $\omega$  with this right-handed circular polarization.

So that means now if we start from a level  $m$  and we have now light-atom interaction, the operator which annihilates a photon with circular polarization because of angular momentum conservation can only take us to a level where the magnetic quantum number is  $n$  plus 1.

Well, the operator  $A$  [? dega ?] creates a photon through stimulated emission. And so this is now our two-level system. And now we should ask the question, in terms of rotating wave approximation, is necessary or not, are there counter-rotating terms?

Well, the counter-rotating terms are the non-intuitive terms, where you start out in the lowest state. But now instead of absorbing a photon, you emit a photon. And the operator for emission is this one.

So I can now ask, is there another term? Why don't we stick to the blue color for the photons? That there is a term which is driven by the operator  $A$  [? dega ?] circularly polarized. Well, the answer is, there may be such a term, but the state we need has now a magnetic quantum number of  $m$  minus 1 because of angular momentum selection rules. So this here is the counter-rotating term, which you may or may not neglect, depending whether you want to make the rotating wave approximation or not.

So therefore, if you got a little bit confused about the different cases I considered at the end of the last lecture, then you may just summarize the many examples I gave

you in the lecture just as a note which you should keep in the back of your head, namely-- let me first phrase it in words and then write it down.

If you have circular polarization and angular momentum selection rules, then the counter-rotating term may require a third level and is not part of two-level physics. So if you have a situation where the third level does not exist, you do not have a counter-rotating term. However, in all situations I've encountered in the lab, this third level does exist. OK.

So let me just write that down-- counter-rotating term for circularly polarized radiation requires a third level which may not exist. And then you don't have this term. But it does exist in most cases. Anyway, just an additional clarification for the topics we had on Wednesday. Any questions about that? Yes.

**STUDENT:** Are there cases other than a spin-1/2 system where it doesn't exist?

**PROFESSOR:** Well, I mentioned the example last class-- if you do spectroscopy of an S<sub>2</sub>P transition in the magnetic field of a neutron star. Then the same one splitting is so huge that, well, you can assume that this has been shifted so far away that it has been completely suppressed.

Other than that, well, we have the trivial situation which we discussed in NMR. If you simply have spin one half, then the total number of levels is only two, because we're talking about spin up and spin down. Or I constructed, in the last class, the forbidden transition, a doublet S to doublet S state. So that's two pairs of S equals one half. And then we are missing this state to couple any counter-rotating term into the system. OK.

The next subject is saturation. Now, in this chapter, I want to talk about saturation in general. I want to discuss monochromatic light but also broadband light. And I want to introduce concepts of saturation intensity of absorption cross section, certain things which I find extremely useful if I want to understand what happens when light interacts with a system.

Just sort of to whet your appetite, I will sort of show you that the absorption cross

section of a two-level system is independent whether you have a strong or weak transition. Some people think the cross section should be-- but there is a difference which is important between monochromatic and broadband light. But in the end, the concepts are very simple.

I should say, sometimes I feel it's almost too simple to present it in class. On the other hand, if I don't present it, I can't make a few comments and guide you through. So my conclusion now is, at least for now, I show you some prepared slides, and I sort of step you through, and make a few annotations, and point out certain things.

We have already partially transitioned to teach you this material through homework assignment. This week's homework assignment, which is due on Wednesday, is almost completely on saturation. And I will make a few comments where what I present you today is an extension or different from what you're learning in the homework and vice versa.

Yes. If I wanted to present you saturation, power broadening, and all that in the purest form, I would just preset you with the Optical Bloch equations. We can solve them. And then we have everything we want-- a result which explains saturation and a result which explains power broadening. And you do some of it in your homework.

However, what I want to show here is that saturation is actually a general feature of a two-level system if you have sort of three rates, which I will explain to you in a moment-- very similar to Einstein's A and B coefficient-- that all such systems have saturation.

And then you may immediately solve the Optical Bloch creation for monochromatic radiation. But for broadband radiation, we usually don't use the Optical Bloch equations, because for infinitely broad light, there is no coherence for which we need Optical Bloch equations. If you only have the Optical Bloch equations, you have solved for saturation in one limiting case, and you don't see that the concept of saturation is much broader.

So therefore, let us assume that we have a two-level system. And we have the couple two levels with a rate-- which you can think of the rate of absorption, the rate of stimulated emission. And I call the rate the unsaturated rate. In addition, there is some dissipation, some spontaneous decay described by gamma.

So  $R_u$  is the unsaturated rate for absorption and for stimulated emission. Of course, you know even before you solve those equations that there must be some saturation built in. If you would look at the fraction of atoms in the excited state and you change the laser power, which means changing the unsaturated rate, things cannot shoot up forever, because you cannot put more than 100% of the population into the excited state. However, the effect that when we increase the laser power, we do upwards absorption and downward stimulation-- means you won't even get 100%. The maximum you can get is 50%.

And what I'm just drawing for you is this is a phenomenon of saturation, and now we want to understand the details. So using this rate equation, we are defining-- this is now a definition-- the saturated rate is the net transfer from A to B.

Because we have an absorption and stimulated emission, the net transfer is the [? unsaturated ?] rate times the population difference. And this is our saturated rate. But of course, we normalize everything for atom. So therefore, our saturated rate has a rate coefficient  $S$  times the total number of atoms, or the total population in both states.

Eventually, we are interested in steady state. We can immediately solve the rate equation in terms of steady state, which is done there. And we find that for those, we can now eliminate one of the states from the equation, because we have the steady state ratio. And then we find that the saturated rate is  $\gamma/2$  times an expression which involves a saturation parameter.

So in other words, it's just almost trivial solutions of very simple equations, which describe the saturation phenomenon I outlined for you at the beginning. This solution has the two limiting cases which we want to see-- that at a very low unsaturated rate, the saturated rate is the unsaturated rate because there is no

saturation.

And secondly, if we would go to infinite power, the saturated rate becomes  $\gamma/2$ , because we have equalized the population between ground and excited state. One half of the atoms are in the excited state, and they dissipate or scatter light with the rate  $\gamma$ . All right. Any questions?

We now want to specialize it to a situation which we often encounter, namely monochromatic radiation. And for monochromatic radiation, the unsaturated rate follows-- well, I factored out something here. But it follows the normalized line shape, which is a Lorentzian. And therefore, our unsaturated rate is proportion to the laser power. But I usually like to express the laser power through a Rabi frequency or the Rabi frequency squared. So our unsaturated rate follows this Lorentzian.

And on resonance, this part is one. Our rate is  $\omega \text{ Rabi}^2 / \gamma$ . And the definition for the saturation parameter of one, or for the saturation intensity is that the unsaturated rate has to be  $\gamma/2$ .

So therefore, by  $\omega \text{ Rabi}^2 / \gamma$  is the unsaturated rate, it should be  $\gamma/2$  for saturation, for saturation parameter of one. So therefore, our saturation parameter resonance is given by this expression. And if you use the previous result and apply it to this unsaturated rate, we find a saturated rate which shows now the new phenomenon of power broadening.

Let me illustrate it in two ways. The saturated rate involves a saturation parameter, and the unsaturated rate is a Lorentzian. But this Lorentzian appears now in the numerator and the denominator. So it appears twice. But with a one-step manipulation, you can transform it into a single Lorentzian. But this single Lorentzian is now power-broadened. It no longer has [? width ?] of  $\gamma/2$  of the natural line with  $\gamma$ . It has an additional term, and this is power broadening.

**STUDENT:** [INAUDIBLE]?

**PROFESSOR:** No. It's still-- the resonance is at [? 0 ?] [INAUDIBLE].

The equations are trivial. It's really just substituting one and getting from an expression, simplifying it to simple Lorentzian. I just want to emphasize the result. If you drive a transition, we have now-- the [? width ?] of the Lorentzian is now  $\gamma/2$  if we have no saturation. But then if we crank up the saturation parameter, the [? width ?] increases with a square root of the power. That's an important result. The square root of the power leads to broadening.

Now let me give you a pictorial description of what we have done here. If we start with the Lorentzian and we increase the power, you sort of want to drive the system with the stronger Lorentzian. But we know we have a ceiling, which is saturation. And of course, when you drive it stronger, you reach the ceiling on resonance earlier than you reach the ceiling when you transfer it away from resonance.

So therefore, if you start with the red curve, crank up the power, you will get more of a factor, more of a result in the wings because you are not yet saturated there. And this graphical construction, which I have just sort of indicated to you, lead now to a curve which is broadened, broader than the original Lorentzian. And this is the reason behind power broadening.

I want to mention one thing here. For the classroom discussion, I have assumed that the light-atom interaction can be described by Fermi's golden rule, which we know is a limitation. When the system is, in effect, incoherent or no longer coherent, we had a long discussion about Rabi oscillation, Fermi's golden rule in the last two weeks.

But what I'm doing is mathematically correct. The Optical Bloch equation, which you'll use in your homework assignment, will include the transition from Rabi oscillation towards Fermi's golden rule. And I'm just considering this [? fundamental ?] case.

OK. I've talked about saturation of a transition. I've mentioned that we have defined the saturation parameter such that when we have saturation parameter of one, we sort of get into the non-linear regime where saturation happens. And of course, for

an experimentalist, the next question is, at what intensity does that happen? This is summarized in those equations. It's as simple as possible algebra. You just combine two equations. I don't want to do it here.

And we have a result for the saturation intensity, which has two features, which I want to point out. One is [? its case ?] with omega cube. So the higher the frequency of your transition is, the harder it is to saturate. Of course, it has something to do with that in saturation, you have an unsaturated rate, which is one half of the spontaneous emission rate.

And you remember that the spontaneous emission rate was proportional to omega cube. So that's why we have, again, the omega cube factor. And in addition, the larger-- actually, it depends. Sorry. I made a mistake.

Well, you can write the results in several ways. If you have an intensity and you go back to photons, you get factors of omega. So when I said omega cubed comes from the natural line widths, yes, it does, but it's not the only omega factor. You can write the result actually that you have a gamma squared dependence, because one gamma comes from the matrix element squared and one comes because you need to compete in your excitation with spontaneous emission.

So anyway, this is sort of the result. And you can calculate it for your favorite atom. And for alkaline atoms, we usually find that the saturation intensity is a few milliwatt per square centimeter

Well, we can now repeat some of this exercise for the broadband case. In the broadband case, the unsaturated rate, which is the rate for absorption in stimulated emission, following Einstein's treatment of the AB coefficient is used by using Einstein's B coefficient times the spectral intensity. And now we want the same situation as before. We want to reach saturation. And saturation happens when this is comparable with gamma. And it's purely a definition that we say it should be gamma over 2. But we are consistent with what we did before.

And if you just take this equation and calculate what the saturation intensity is, well,

gamma is nothing else than the Einstein A coefficient. Here we have the Einstein B coefficient. And if we take the ratio between the Einstein A and B coefficient, the matrix element, everything which is specific to the atom, cancels out. And the saturation intensity, or the spectral density-- it's the spectral density now for broadband-- only depends on speed of light and the transition frequency cube. And it doesn't make a difference whether you have a two-level system which has a strong matrix element or weak matrix element.

I could explain it to you now at this point, but we want to hold the idea that there is a difference between single mode monochromatic and broadband excitation until I have discussed one more concept. And this is the cross section. Just to check, are there any questions? Yes, Nancy.

**AUDIENCE:** So in the broadband case, the line shift doesn't matter at all? Because in the monochromatic case, we had a line shift [INAUDIBLE].

Well, hold your question. The line shape matters. I will now discuss what is the line shape of the atom. And the quick answer is, if the atom has a line shape, we have to take the atomic line shape and do a convolution with the line of the radiation.

And we have the two situations where in one case, the monochromatic light is narrower than the line shape of the atoms. In the other case, it's broader. And this difference, in the end, will be responsible for the effect that the line widths of the atom, which is the natural line widths, will cancel out in one case and not in the other. But that's actually the result of the next five minutes. Other questions?

I know this topic can get confusing, because we go from one definition to the next. So let me just summarize. What I've said so far is we [? derive ?] an atom. We have absorption, we have stimulated emission. And we want to understand the phenomenon of saturation.

And based on the effect how we define saturation, namely that the unsaturated rate is gamma over 2, we got some nice results for the saturation intensity and for power broadening of a Lorentzian. So it's pretty much having a definition and running with

it.

And now we want to express the same physics by using the concept of a cross section for the following reason. You can do physics, you can do atomic physics without ever thinking about a cross section. You can just say, I have a laser beam of a certain intensity, and I scatter light.

But often, when we scatter something-- and you may be familiar, from atomic collisions-- you often want to write the scattering rate as a density times cross section times relative velocity. And this sort of has this intuitive feeling. If you have a stream of particles in your accelerator or a stream of photons in your laser beam, you can now hold onto the picture that each atom in your target is a little disk.

If the particle of photons hits the disk, something happens. If it misses the disk, nothing happens. And the area of the disk is this cross section. So in other words, we want to now understand how big is the disk of the atom which will, so to speak, cast the shadow, which is synonymous with absorption, when we illuminate those atoms with laser light. So for me, a very intuitive quantity.

Anyway, so all we do is we have already discussed the rate of excitation, which is now the unsaturated rate. But now we express the unsaturated rate by the density of photons times the cross section. And the relative velocity is the speed of light.

And from this equation, we find-- because everything is known, we have talked about that on the last few pages-- we find that the cross section is-- and this is the result.  $6\pi \lambda^2$ .  $\lambda$  is the wavelengths of light divided by  $2\pi$ .

So we find that for monochromatic radiation, the cross sectional of a two-level system is independent of the strength of the transition, independent of the matrix element. It just depends on the resonant wavelengths.

Now you would say, well, but what is now the difference between a strong and a weak transition? And this is shown here. If you take your monochromatic laser and you scan it, you scan it through the cross section. When you are on resonance, you

have  $6\pi\lambda^2$ . And the difference between a narrow transition with a small Einstein A coefficient and a strong transition with a large Einstein A coefficient simply means that in one case, it's narrower. In the other case, it's wider.

We talked about the phenomenon of saturation.  $6\pi\lambda^2$  is the cross section in the perturbative limit, or the unsaturated cross section. Of course, if you increase the laser power, you saturate the transition. The atom will have a smaller and smaller cross section. Actually, that's something important you should consider. When you have an atom and you increase the laser power, you scatter light. And the scattered light, or the absorbed light, saturates.

But with the cross section, we want to know what fraction of the laser light is scattered. And the fraction of the laser light scattered goes to zero, because you make your laser light stronger and stronger. And the total amount of laser light which is scattered saturates.

So in other words, you have a saturation of the scattered light. You have a saturation of the net transfer of atoms through the excited state in the limit of infinite laser power. But since the cross section is sort of normalized by the laser power, the cross section has this dependence,  $1/(1 + \text{saturation parameter})$ , and goes to 0.

And that means-- and this is sort of the language we use-- that the transition bleaches out. If you saturate the transition, the cross section becomes smaller. So when you saturate the transition in an absorption imaging experiment, which many of you do, the shadow is less and less black exactly because the cross section is bleaching out. But the amount of light you would scatter you would observe in fluorescence is not getting less, it saturates. This is sort of just the two flip sides of the coin. If anybody is confused, please ask a question. OK.

So now in this picture, we can immediately understand why we have differences between monochromatic radiation and broadband radiation. If you want to saturate a transition with monochromatic radiation, we have our narrow laser. We absorb with a cross section  $6\pi\lambda^2$ . And we have to increase the

intensity of the laser until the excitation rate equals  $\gamma/2$ . That's our definition for saturation.

So therefore, the laser intensity is proportional to  $\gamma$ , because we have the cross section is constant, but the product of cross section and laser intensity has to be equal to  $\gamma/2$ . However, now consider the case that you use broadband radiation. The spectrum is completely broad.

Now, if an atom has a stronger transition, its cross section is wider, and the atom can sort of absorb a wider part of the incident spectrum. So therefore, if the atom has a stronger transition, it automatically takes, absorbs more of your spectral profile. And therefore, the result for the saturation and for the spectral saturation intensity is independent of the matrix element and the strengths of the transition.

In general, if you're not in either of the extreme cases of monochromatic light or broadband light, what you have to do is you have to use this cross section as a function of frequency, and convolve it to a convolution with a spectrum of the incident light.

And this is exactly done here. You take your frequency-dependent cross section. You do the convolution with the spectrum of the incident light. And if you assume the incident light is spectrally very broad, you simply integrate over the Lorentzian line shape of the cross section. And then you find exactly the same result as we had two slides ago, that the saturation intensity is independent of the strengths of the transition. OK.

Can you think of a very intuitive argument why for spectrally broad radiation, all the properties of the atoms cancel out? If you think about one physical example for, let's say, black body radiation-- this is spectrally broad. So you have an atom in a black body cavity. And the atom experiences a very broad spectrum.

For what number of photons, black body photons per mode would we find saturation? Think about it. It's a simple criterion you can formulate for black body radiation to saturation your transition in terms of the number of photons per mode.

You crank up the temperature in your cavity. How high do you have to go with the temperature in order to saturate an atom which is inside your black body cavity?

**AUDIENCE:** One photon.

**PROFESSOR:** Pretty close.

**AUDIENCE:**  $1$  over  $[\text{? degenerates. ?}]$

**PROFESSOR:**  $[\text{? Degenerates. ?}]$  OK. No  $[\text{? degenerates. ?}]$  I hate  $[\text{? degenerates. ?}]$  That's your private homework to put in  $[\text{? degenerates ?}]$  afterwards. The answer I came was  $n$  equals  $1/2$ , I think. I run the risk that I'm off by a factor of  $2$  now. But the argument was that--

**AUDIENCE:** The rate equals  $[\text{? degeneracy ?}]$  by  $n$  by  $\gamma$ . So if the rate equals  $\gamma$  over  $1/2$ , that mean that  $[\text{? degenerates ?}]$  by  $n$  equals  $1/2$ . And if  $[\text{? degenerates ?}]$  equals  $1$ ,  $n$  equals  $1/2$ .

**PROFESSOR:** Yes. OK. So spontaneous emission, we know that spontaneous emission-- from our derivation of spontaneous emission-- corresponds to one photon per mode. And our criterion now is that we want to have an absorption rate or stimulated rate which is  $\gamma$  over  $2$ . So we get sort of  $1/2$  the effect of spontaneous emission when we have  $1/2$  a photon per mode.

So therefore, spontaneous emission absorption is proportional to  $n$ . And I think if  $n$  equals  $1/2$ , then we have the unsaturated rates equal to  $\gamma$  over  $2$ .

So this is a very physical argument. When we put an atom into a black body cavity, and we have  $1/2$  a photon per mode occupation number, then we saturate any atom we put in. Because using Einstein's argument, we have now the rate coefficient for absorption emission for stimulated emission and absorption is just  $1/2$  of the rate coefficient for spontaneous emission. And that explains that all atomic properties have to cancel out.

So now question for you. We talked about the fact that if you have hyperfine

transitions, that it would take-- what was the value? 1,000 years for spontaneous emission? So that we can completely neglect spontaneous emission.

On the other hand, we've just learned that saturation only comes from spontaneous emission. Without spontaneous emission, we wouldn't have saturation. But now I'm telling you that any atom should really be saturated if we put it in a black body cavity where  $\bar{n}$  is  $1/2$ .

So what is the story now if we put an atom into a black body cavity, and we are asking about, will we saturate? The hyperfine transition. Will we eventually have-- saturation means we have [BLOWS AIR],  $1/4$  of the atoms in the excited state,  $3/4$  in the ground state. So the  $\Delta n$  has been reduced from 1, which it was initially, to  $3/4$  minus  $1/4$ , which is  $1/2$ .

What will happen? I mean, this was almost like a thermodynamic argument. Will we equilibrate and saturate hyperfine transitions in a black body cavity based on this argument that for  $\bar{n}$  equals  $1/2$ , we should really saturate everything?

**AUDIENCE:** Yes, but it's going to take a long time?

**PROFESSOR:** Yeah. So for those conditions, if your black body cavity was  $\bar{n}$  equals  $1/2$ , you should saturate any two-level system completely independent what  $\gamma$  is. And if the  $\gamma$  is 10 nanoseconds or 10,000 years, you will saturate it. The value of  $\gamma$  has completely dropped out of the argument. But of course, if you want to reach any kind of equilibrium, it will take a time scale, which is  $1/\gamma$ . And then we are back to 1,000 years. Questions? All right.

Then let's conclude this chapter and start our discussion about line shifts and line broadening. I have a problem with the tablet computer. I draw a line, but the computer draws a line somewhere else. So maybe I should just go back to this one and then copy things over. OK.

Our next big chapter is line shifts and broadening. So the first question is motivational. Why should we be interested in line broadening? Well, the answer is almost trivial. No resonance is infinitely narrow. Whenever we want to interpret any

result we obtain spectroscopically, we are not observing a delta function, we are not observing a resonance, we are observing a line shape.

And unless we understand the line shape, we may not accurately find the resonance frequency. You could, of course, assume that your line shape is symmetric, which may be the case but is not always the case. So without understanding line broadening, you cannot interpret spectroscopic information.

And eventually, as I mentioned in the first chapter of this course, the art of analyzing line shapes and finding the line center is very well advanced. When we have caesium fountain clocks, the accuracy how you operate the clock as a frequency standard is on the order of one microhertz.

But those fountain clocks with you toss up the atoms for one second in the atomic fountain, they fall up and down, well, like a rock, which takes about a second for a rock to go up and down a meter. So therefore, the line width is on the order of one Hertz.

So therefore, people have to understand any single aspect of the line shape at the level of  $10^{-5}$ , or  $10^{-6}$  to have this kind of accuracy. OK.

So I thought I want to start this unit by collecting from you examples for phenomena which cause broadening and shifting of lines. And well, my list has about 10 of them. Let's see how many you get. So what phenomena can lead to line shifts and line broadening?

**AUDIENCE:** Phonons.

**PROFESSOR:** Phonons? In terms of-- OK, AC stark effect. Pardon?

**AUDIENCE:** Magnetic field noise.

**PROFESSOR:** Magnetic field noise. OK. I tried to-- yes, very good. OK, yes. Let me just try to group it a little bit further, because I want to discuss it. So we have external fields. And external fields can have AC stark shifts. If an external field is noisy, we have

noise fluctuations. All right. Anything else?

**AUDIENCE:** Doppler shift.

**PROFESSOR:** Doppler shift. Yes. So we have the velocity of the atoms. Doppler shift.

**AUDIENCE:** Collisions.

**PROFESSOR:** Collisions. Very good. Well, we just talked about one thing.

**AUDIENCE:** [INAUDIBLE].

**PROFESSOR:** Exactly. When we have external fields, we can have external fields like magnetic fields or electric fields which cause shift and broadening. And if there's noise, additional shifts. But when we regard those fields as drive fields, they can do power broadening.

Maybe by collisions, I should add the keyboard "pressure broadening." The higher the pressure in your gas cell is, the more collisions you have and the more you have broadening. Other suggestion?

If you don't have any of those effects, do you measure delta function? What's the line width? Will?

**STUDENT:** [INAUDIBLE].

**STUDENT:** Spontaneous emission.

**PROFESSOR:** Spontaneous emission. Yes. And if you don't have spontaneous emission, do we then measure delta function?

**STUDENT:** There's a Fourier limit.

**PROFESSOR:** The Fourier limit. You can call it observation time, or time of light broadening. If an atom flies through your laser beam and you can interrogate it only for a finite time, you have a broadening due to the Fourier theorem. And this can be called time of flight broadening and time of-- or interaction time broadening.

**STUDENT:** Rotations and vibrations?

**PROFESSOR:** Rotations and vibrations. Not really. These are more-- then the system has more energy levels, and that's what you want to find out. So maybe I'm more asking, how are those energy levels-- how do they appear spectroscopically? Well, I think that's pretty complete. Two external fields. If you want, you can add gravity. There is a gravitational red shift, which is general relativity.

But anyway, let me look over that and try to categorize it. What we had here actually all comes from a finite observation time. Either we do not have the atom long enough in our laser beam, and that sets a limit. Or if you are interested in an excited state and the excited state decays, then the atoms themselves have terminated our interrogation time.

The second class here, velocity, I would summarize that we have motion of the atom. It's a form of motional broadening. We will actually discuss, when we discuss motion, also the possibility of having atoms in a harmonic oscillator potential, ions in an ion trap. So these are now trapped particles. This will actually often give rise to a splitting of the line into side bands. So we want to discuss that.

I've already mentioned external fields, conditional [INAUDIBLE] interrogation, power broadening. Some power broadening will actually result into a splitting of line into [INAUDIBLE] triplet. So power will not only broaden the line, it can also split the line. And we want to discuss that.

And finally, we have the effect of atomic interactions. So for interactions, I think we should add something like mean field shifts, which also goes sometimes by the name of clock shift. If you're not at zero density, your transition can be shifted by the presence of other atoms. Will?

**STUDENT:** Isn't collisional broadening or pressure broadening sort of just an ensemble average of a stark effect? So that's sort of an external field?

**PROFESSOR:** That depends now. Collisions is one of the richest phenomena on the list here. You're ahead of me. But in the next few minutes, I wanted to actually see, well,

maybe we should-- those categories are not mutually exclusive, because one part of collisions is. An atom is in the excited state, it collides, it gets de-excited. So then collisions have no other effect than sort of give us a finite observation time, where there is an effective lifetime, which is just the time between two collisions. So it can be this.

There's another aspect of collisions, that every time there is a collision, an atom feels the electric field of another atom. And then we have some form of collisional broadening, because we do some statistical averaging over Stark effects, over level shifts.

Now, there is a third aspect of collisions, which is maybe surprising to many of you. And this is actually-- I put it here under motion. It is collisional narrowing, or [? diche ?] narrowing. There's one limiting case when you have collisions, that collisions lead to a narrower line and not only to a broader line.

the reason is a little bit-- if you put an atom in a buffer gas and it collides with all the buffer gas atom, it cannot fly away. So buffer gas and collisions can sort of help to increase your observation time. But only if the other effects of collisions are absent.

So anyway, I thought this is a number of really interesting effects. And you already see from my presentation and discussion that it makes perfect sense to discuss them not one by one, as they appear in other chapters, but try to have comprehensive discussions of those.

Let me talk about one classification of those shifts and broadening. And one is the distinction between homogeneous and inhomogeneous broadening. So the picture here is that if you have-- let me just give you the cartoon picture.

If you have different atoms, atom 1, atom 2, atom 3. A homogeneous broadening situation is if the line has been broadened for each atom in the same way. An inhomogeneous broadening situation is that atom 1 has a line here, atom 2 has a line here, atom 3 has a line there. And if you look at the statistical ensemble, you may find the same line widths as on the left-hand side, but the situation and the

mechanism is a very different one.

So the different characteristics are that here, we have a mechanism which broadens or widens the line for each atom. Whereas here, there is maybe not even any line broadening for the atom. It's more a random shift to individual atoms. And the widening happens for the ensemble.

Another very important distinction is in the left case, if you have one powerful laser, it can talk to all the atoms. Whereas in the right-hand side, you may have a laser with a certain frequency, and it may only excite one group of atoms in your ensemble. So this is the opposite here.

In many situations do we have a physical picture where, in homogeneous broadening, we can understand it as random interruptions of the phase's evolution of the atom. For instance, through spontaneous emission, or you can see certain collisions-- just mean the phase of the excited state is suddenly perturbed. And therefore, the phase is randomized.

So if the physical picture is a random interruption of the phase evolution, well, a random interruption of a phase evolution means that there is an exponential decay of coherence. And the line shape, the Fourier transform of an exponential decay is a Lorentzian. Whereas the physical picture behind inhomogeneous broadening is that you have random perturbations. And if you have many random or small perturbations, they often follow a normal distribution, which is a Gaussian.

There's one other aspect of an inhomogeneous broadening. If it's an inhomogeneous broadening, it is as if the individual atom is not broadened, the individual atom is actually sharp, it has a longer coherence. And you can-- there are techniques to make that visible. And one famous technique, for those of you who have heard about it, are an echo technique.

So having explained to you in a general way the difference between inhomogeneous and homogeneous broadening, how would you classify the line broadening mechanisms we have collected before? Which one are inhomogeneous

broadening?

**STUDENT:** Doppler broadening.

**PROFESSOR:** Doppler broadening. We exploit that when we do saturation spectroscopy in the lab, when we just talk to one component of the velocity distribution. What else?

**STUDENT:** [INAUDIBLE].

**STUDENT:** Collisions.

**PROFESSOR:** Collisions. That's actually a good one. Usually, collisions are classified as homogeneous broadening, because the simplest model for collisions is collisions are sort of just hard-core collisions which just de-excite the atom, completely change the coherent phase evolution. And therefore, collisions would broaden the transition for all atoms to a line widths which is 1 over the collision rates.

However-- and this shows that the distinction cannot always be made-- you can actually have collision rate which depends on the velocity. The faster atoms may have a smaller collision cross section than the slower atoms. And now you have an inhomogeneous aspect of the collision rate. And therefore, collision rate becomes inhomogeneous.

I mean, the standard example for inhomogeneous fields would-- if you have an inhomogeneous magnetic field, you have stationary atoms-- well, not in an atomic gas, but maybe in [INAUDIBLE] or in a solid. And you have an inhomogeneous magnetic field. This is actually the standard case of nuclear magnetic resonance, that each atom possesses at its local magnetic field. And the line shape is inhomogeneously broadened. Colin?

**STUDENT:** [INAUDIBLE] clock shift sometimes, in some circumstances.

**PROFESSOR:** If the density is constant, you would actually say the mean field is the same for all atoms in the ensemble. But if you have a trapped atom sample where the density drops at the edge, you may actually have a sharper line, and less broadening, or less shift at the edge of the cloud.

Anyway, so I think you have all the tools to classify it. And you see from the discussion that sometimes it's not so obvious. Or you may have a mechanism which has [? both ?] inhomogeneous that it does something to all atoms. So for instance, collisions broaden all the atoms, but then different atoms are more broadened than others. So there may be also an inhomogeneous aspect.

But finally, let me ask you the following. It seems the first items on our list had sort of a very natural explanation in terms of the Fourier theorem, that, well, we only talk to the atoms for a finite time. Or the atoms decide not to talk to us for longer, because they spontaneously decay. Now, maybe you want to give me some arguments why some of the other mechanisms are actually also due to some form of finite time of interrogation.

Well, if I would say, can we regard collisions as an effect of finite observation time?

Well, if I rephrase "observation time" to "finite coherence time," that something interrupts the coherent evolution of the wave function, I think we would say the collision time sets a time limit to the coherence time and therefore, should also be regarded as due to the finite time, we can drive the atom in a coherent fashion.

If I take power broadening, we just discussed power broadening. Well, what is the rate-- or 1 over the rate of power broadening? We just discussed that that's maybe nice to take it out of the context. We discussed before that power-broadened line widths is  $\gamma$  over  $2$  times  $S$  plus  $1$  square root of it-- the saturation parameter. But when does power broadening happening? And what is the real time scale for what is the physical--

**STUDENT:** Spontaneous emission.

**PROFESSOR:** Spontaneous-- so we had a criterion that the unsaturated rate has to be comparable to  $\gamma$ . Let's forget about factors of  $2$  now. But that means that the Rabi frequency has to be comparable to  $\gamma$ . The Rabi frequency tells us a time of Rabi flopping. So actually, power broadening can be understood as a finite observation time broadening, but the atom is leaving the excited state not by

spontaneous emission, but by stimulated emission.

So in other words, stimulated emission interrupts our ability to observe the atoms in the excited state. And so again, we see that there is a process coming in which interrupts our observation time of the unperturbed atomic levels.

Well, let me go one step further. Let me ask you, do you have any idea how we could discuss the Doppler shift as due to some finite time scale? You would say, well, yeah, that's dimensional analysis. If you have a broadening, a broadening is a frequency,  $1$  over the frequency is the time, and there is a time scale associated with Doppler broadening. Sure.

But now my question is, what is the physical time scale with Doppler broadening? Yes?

**STUDENT:** [? Collisions. ?]

**PROFESSOR:** No. We have an ideal gas without any collisions-- just a [INAUDIBLE] distribution. You're right. In practice, yes. But I try to create an idealized situation.

So what is the time scale of Doppler broadening? You may have never heard the question. But this is for me, what I want to really teach you when I teach all these different line shifts and line broadening. There is a common denominator.

**STUDENT:** You could think of the atom [INAUDIBLE] emission. And then you would have velocity [INAUDIBLE] emission.

**PROFESSOR:** You're talking about recoil shifts, and the atom is changing its velocity due to recoil. This would something in addition, but it's not necessarily the case here. I give you a physical argument. If I make the atom heavier and heavier and heavier, the effect of the recoil vanish. But then I can heat up the heavier atom, that it moves with the same velocity as the slow atom.

So there is an effect which you can associate just to the velocity and to the velocity distribution. And that's what I want to discuss now. But there is another effect with the recoil. But I can say the recoil is a finite mass effect, for that purpose. The mass

is sort of my handle, whether the recoil of a singular photon is important or not.

Yes?

**STUDENT:** [INAUDIBLE]?

**PROFESSOR:** Yes, but this is really a more trivial finite observation time. When you heat the wall of the chamber, it's a collision with the chamber. It means we have only a finite interaction size.

Now, let me sort of guide you to that. The secret here is when we say, you have a finite lifetime, a finite observation time, what matters when we do spectroscopy is the time we can observe the atoms coherently. If the atoms de-phase, if the atoms get out of coherence-- for instance, if you have collisions-- if collision de-excite the atom-- we'll talk about it later-- it's like spontaneous emission.

But then there are collisions which just create a phase hiccup, that the excited state gets a random phase. So an interruption of the phase, an interruption of the coherent evolution is, in effect, an interruption of us probing the atoms in a coherent way. And then the Fourier transform just tells us, this time, or  $1$  over this time, is the line which we observe.

And you would say, but how does it come into play with atoms with a velocity distribution? In the following way. If you line up several atoms and they interact with a laser beam, some atoms are faster, some atoms are slower. If some of the atoms have moved compared to the slower atoms, one additional wavelength, then your ensemble of atoms is no longer interacting with the laser beam in a phase-coherent way. Because of the different velocities, they are now talking to random phases of the laser.

So therefore, Doppler broadening is nothing else as a loss of the atoms to coherently interact with a laser, because some of them have moved an additional wavelength in the laser beam.

Well, if that is true-- but what happens if the laser beam is like this, with the

wavelengths, and the atoms go perpendicular? What happens then?

**STUDENT:** There's no Doppler.

**PROFESSOR:** Then there is no Doppler effect. So what I'm saying is fully consistent with every single thing you know about the Doppler effect. OK.

So I think there's not much more we can do today. But let me give you the summary of this discussion. To the best of my knowledge, all line broadening mechanisms can be described by using the concept of coherence time.

And it's a coherence time of a correlation function. It's pretty much the correlation function of the phase which the atom experiences. At  $t$  equals 0, it experiences one phase of your drive field. And a later time, how long does it stay coherent with the coherent evolution of the phase of your drive field of a correlation function?

However, in the case of inhomogeneous broadening-- and this is what I just discussed with the different atoms starting together and having different velocities. In the case of inhomogeneous broadening, I have to include in the description of the correlation function ensemble averaging.

So this is our agenda. On Wednesday, I will start to discuss with you very simple cases. I sort of like, before I introduce correlation function, we have the generalized discussion to summarize for you the phenomenological description of just Rabi resonance, Ramsey resonance, exponential decay, simple Doppler broadening, the recoil effect, that you have a clear physical picture of what the different phenomena are. And then we describe them with a common language, with a common formalism, which is a formalism of correlation functions.

Any questions? One obvious question-- the chapter on line shifts and broadening will not be on the mid-term. OK. See you on Wednesday.