

8.421 Homework Assignment #8

Spring 2012, Prof. Wolfgang Ketterle

Due Wednesday, April 25, 2012

1. [4 Points] Convolution of Line Shapes

When two separate physical processes both contribute to the line shape, the resultant line shape is the convolution of the distributions. Say $D_1(\omega - \omega_0)$ and $D_2(\omega - \omega_0)$ are the normalized line shapes of the first and second type of process. The resultant line shape is then their convolution

$$D_R(\omega - \omega_0) = \int_{-\infty}^{\infty} d\omega' D_1(\omega - \omega') D_2(\omega' - \omega_0)$$

a) (1 point) Consider that D_1 and D_2 are both Lorentzian functions with FWHMs (full widths at half-maximum) Γ_1 and Γ_2 , respectively. Show that D_R is also Lorentzian and find its FWHM. (Hint: use the Fourier convolution theorem)

b) (1 point) Do the same if D_1 and D_2 are Gaussian and Γ_1 and Γ_2 are r.m.s. deviations.

c) (2 points) Find the FWHM of a Gaussian with r.m.s. deviation $\Gamma/2$. Find the r.m.s. deviation of a Lorentzian with FWHM Γ .

2. [8 points] Intensity Distribution Due to Spontaneous Emission

An atom of total angular momentum J has a spontaneous radiation rate A . It radiates to a lower level with angular momentum $J' = J - 1$. The problem is to find the rates for the various allowed transitions, i.e. the fraction of the radiation that goes into each of the possible transitions $(J, m) \rightarrow (J', m')$. The rates can be found by applying the following considerations:

- The sum of the rates out of each state (J, m) must equal A .
- The sum of the rates into each state (J', m') must equal $A \times \frac{2J+1}{2J'+1}$.
- An unpolarized mixture of radiators in level J must emit equal intensities of light with each of the three polarization components.
- The rate for a transition $(J, m) \rightarrow (J', m')$ must be the same as for $(J, -m) \rightarrow (J', -m')$.

For $J = 2$, $J' = 1$, designate the transitions by letters as follows:

$$a: m = 2 \rightarrow m' = 1$$

$$b: m = 1 \rightarrow m' = 1$$

$$c: m = 0 \rightarrow m' = 1$$

$$d: m = 1 \rightarrow m' = 0$$

$$e: m = 0 \rightarrow m' = 0$$

1. Find the rates for a through e , and present your results on a figure.

2. Find the rates for a through e , using the Wigner-Eckart theorem. (Clebsch-Gordan coefficients can either be worked out from first principles, or taken from a table in one of the quantum mechanics or spectroscopy texts.)

Note that the transition rates calculated here are important in experiments involving laser excitation. Because emission and absorption rates are proportional, the distribution of emission rates yields the relative strengths of the transitions, i.e. their relative rates of excitation.

3. [8 points] Saturation Spectroscopy

This problem guides you through the concepts of saturation spectroscopy. This is one of the techniques to perform Doppler-free spectroscopy, i.e. to extract a narrow line (with the natural linewidth) in a gas with a broad velocity distribution. It nicely illustrates the combination of homogenous and inhomogeneous line broadening. Saturation spectroscopy is frequently used to lock lasers to atomic lines. You should not get into nasty integrals for this problem. The drawn lineshapes should clearly show the basic features, but don't have to be exact.

a) Homogeneous broadening

Consider a dilute gas of density n composed of atoms with resonant frequency ω_0 and linewidth Γ . The gas is exposed to monochromatic light of frequency $\omega_L = \omega_0 + \delta$ and intensity $I = s I_{SAT}$ where I_{SAT} is the saturation intensity. Let us ignore the effects of the motion of the atoms, i.e. consider temperature $T = 0$. What are the densities of atoms in the ground state n_1 and the excited state n_2 , including the effect of saturation? What is the cross-section for absorption? The gas is in a box of length L along the direction of the incoming light. What fraction of the light is absorbed? (This is a small fraction, so don't worry about the effect of light attenuation on the saturation of the sample).

b) The Bennet hole

Now, let's endow these atoms with a mass m and a temperature T . Let the incoming light have a wavevector \mathbf{k}_L along the z -axis. What is the population density distribution in the ground state $n_1(v_z)$ as a function of v_z , the component of velocity in the z -direction? You should find that the light "burns a hole" (known as the Bennet hole) in the distribution of absorbers. What is the position of the hole? How do the width and depth (relative to the population for $s = 0$) vary with saturation parameter s ?

c) Inhomogeneous broadening

Consider that we sweep the frequency of the incident laser ω_L and measure the (small) absorption of the beam. Determine the fraction of the light absorbed as a function of s and δ and compare with its value at $s = 0$ (you don't need to solve the integral). For high temperatures ($k_L \bar{v}_z \gg \Gamma$), what is the width of the absorption line? Does saturation affect the width?

d) Saturation spectroscopy

To actually get some benefit from saturating the gas, we introduce a second laser beam. *i*) We can add a weak probe beam at frequency ω_p with wavevector \mathbf{k}_p . What is the absorption of this beam, including the effects of the saturating beam (\mathbf{k}_L, ω_L)? Again, just write the integral, and take the length of the box along \mathbf{k}_p to be L . Draw the absorption line shape, identifying the position and width of its features. *ii*) We can also just retroreflect the original beam. Draw the population distribution $n_1(v_z)$ for $\omega_L \neq \omega_0$ and $\omega_L = \omega_0$. Identifying the depth of the Bennet hole(s), draw the lineshape of absorption of the retroreflected beam (i.e. we scan ω_L). What is the width of the central feature (at $\delta = 0$)?

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