

## Assignment 6

Due: Monday, April 9, 2012

### 1. Transition Lifetimes and Blackbody Radiation

- a) Do you have to worry about blackbody radiation when you trap a Bose-Einstein condensate in a magnetic trap? Any transition to another state will spin-flip the atoms and/or give the atoms recoil energy, causing the atom to be ejected from the trap. Assume that after each absorption event, it will lose one atom in the trap, and you want to ensure a trapping time of at least one minute in a closed, cubic box. Consider sodium, which has a dominant electronic excitation at a wavelength of 590 nm and a lifetime of 16 ns.
- What is the average number of photons per mode from blackbody radiation at this transition which leads to an absorption rate of 1 photon per minute? Hint: You might find Einstein's coefficients useful for this problem.
  - What is the corresponding blackbody temperature? Do you need to shield the vacuum system from room temperature radiation or cool the vacuum system to cryogenic temperatures (*e.g.* 4 K)?
- b) Here we estimate the lifetime of the hydrogen in the  $F = 1$  hyperfine level of the  $1S$  state. The decay of  $F = 1$  to  $F = 0$  gives rise to the famous 21 cm line of radio astronomy.
- What is the dominant coupling mechanism for this process, *e.g.* is it an electric dipole transition, or a magnetic dipole transition? or it is actually an electric octupole transition?
  - What is the lifetime of the  $F = 1$  state? Assume that the matrix element is  $ea_0$  for electric dipole transition,  $\mu_B$  for magnetic dipole transition, and  $ea_0^2$  for electric quadrupole transition, where  $a_0$  is the Bohr radius, and  $\mu_B$  is the Bohr magneton.
- c) A hydrogen Bose-Einstein condensate has been created in the  $F = 1$  state at MIT in the group of Dan Kleppner and Tom Greytak (Fried *et al.*, *Phys. Rev. Lett.* **81**, 3811 (1998)).
- What is the average number of photons per mode from blackbody radiation at the 21 cm line at 300 K and 4 K?
  - For blackbody radiation at 300 K and 4 K, at what rate does it induce transitions to the  $F = 0$  state respectively, which cannot be magnetically trapped?
  - Should you be concerned about blackbody radiation from the environment limiting your experiment with hydrogen in the  $F = 1$  state, if you need a trapping time of 1 minute?
- d) You probably noticed from parts a) and c) that the average number of photons per mode at their respective relevant frequencies and temperatures are rather different. In particular, there are many more photons per mode for one of them than the other. Yet, the lifetime is much longer for the one with more photons per mode, for these two examples. Why is that?

## 2. Atom-Cavity Oscillations

In lecture the interaction of a two-level atom with a single empty mode of the radiation field was discussed for the case in which the field is in resonance with the atom:  $\hbar\omega = \hbar\omega_0 = E_a - E_b$ . This would occur if the atom were in an ideal cavity tuned to  $\omega_0$ . The problem is to find out what would happen if the cavity were tuned to some other frequency  $\omega' \neq \omega_0$ .

- a) Does this system have stationary states? Explain.
- b) The atom is placed in the cavity in its upper state,  $|a\rangle$ . Find the probability  $P_a(t)$  as a function of a later time  $t$ , that it stays in state  $|a\rangle$ .
- c) Explain whether or not the overall energy of the system is conserved in part b).

## 3. Atom-Cavity Oscillations for a Rydberg Atom

A Rydberg atom is an atom with an electron in a high principal quantum number  $n$  state. The frequency for the transition  $n \rightarrow n-1$  goes as  $n^{-3}$ , and the relevant electric dipole matrix element for this transition goes as  $n^2$ . (The corresponding units are atomic units, and the relations approximate.)

- a) Find the frequency, in Hz, of the transition  $n = 50 \rightarrow n = 49$ . Figure out the dimensions of the smallest cubical cavity which can be tuned to that resonance. (Frequency in atomic units is always in radians/time.)
- b) Find the vacuum Rabi oscillation frequency for this system. Express your answer in Hz.

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