8.421 Homework Assignment #1

Spring 2012, Prof. Wolfgang Ketterle

Due Wednesday, February 22, 2012

1. [**4 points**] When driven far from resonance the power dissipated in a mechanical (classical) damped oscillator increases linearly with the damping γ, but on resonance varies as γ^{-1}. Why does reducing the damping increase the power dissipated on resonance?

2. [**8 points**] In this problem we want to study the time evolution of a system with a Hamiltonian

$$H = -\vec{\mu} \cdot \vec{B} = \frac{\hbar}{2} \begin{pmatrix} \omega_0 & \omega_R \, e^{-i\omega t} \\ \omega_R \, e^{i\omega t} & -\omega_0 \end{pmatrix}. \tag{1}$$

This Hamiltonian corresponds to a magnetic moment $\vec{\mu}$ in a combination of static and rotating fields $\vec{B}(t) = -(B_1 \cos\omega t,\ B_1 \sin\omega t,\ B_0)$. Here $\omega_0 = \gamma B_0$ and $\omega_R = \gamma B_1$ are the Larmor and Rabi frequency associated with the static field B_0 and the rotating field of magnitude B_1, respectively, and γ is the gyromagnetic ratio. The basis is $\begin{pmatrix} 1 \\ 0 \end{pmatrix} = |\downarrow\rangle \equiv |e\rangle$, $\begin{pmatrix} 0 \\ 1 \end{pmatrix} = |\uparrow\rangle \equiv |g\rangle$, where $|\uparrow\rangle, |\downarrow\rangle$ are the states where $\vec{\mu}$ is oriented along the $\mp z$ axis. The time evolution of any state $|\psi(t)\rangle = a_g(t)|g\rangle + a_e(t)|e\rangle$ is determined by the two coefficients $a_g(t), a_e(t)$.

a.) Find the equations of motion for the system, i.e. derive explicit expressions for $\dot{a}_g(t)$ and $\dot{a}_e(t)$.

b.) Solve the equations of motion and find $a_g(t)$ and $a_e(t)$ in terms of $a_g(0)$ and $a_e(0)$.

c.) Given the initial conditions $a_g(0) = 1$ and $a_e(0) = 0$ show that the probability to find the system in the state $|e\rangle$ agrees with the classical result.

3. [**8 points**] Now we want to analyze the Hamiltonian of problem 2 in the density matrix formalism. Parameterize H as $H = \frac{\hbar}{2}[V_1\widehat{\sigma}_x + V_2\widehat{\sigma}_y + \omega_0\widehat{\sigma}_z]$, and the density matrix as $\rho = \frac{1}{2}[r_0\widehat{1} + r_1\widehat{\sigma}_x + r_2\widehat{\sigma}_y + r_3\widehat{\sigma}_z]$, where $\widehat{1}$ is the unity matrix, and

$$\widehat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \widehat{\sigma}_x = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \widehat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{2}$$

are the Pauli spin matrices.

Employing the von Neumann equation $i\hbar\dot{\rho} = [H, \rho]$ show that $\vec{r} = r_1\hat{x} + r_2\hat{y} + r_3\hat{z}$ obeys the relation $\frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$ with $\vec{\omega} = V_1\hat{x} + V_2\hat{y} + \omega_0\hat{z}$. Can you interpret this result?

8.421 Atomic and Optical Physics I
Spring 2014