Chapter 7

Chiral Symmetry

Chiral symmetry in the strong interaction (and specifically in QCD). Exploiting an approximate, hidden symmetry to simply description of π_i's and their interaction weak processes involving hadrons (recently) some modern processes.

Leading ideas (predicting QCD) :

1. There is a good approximate symmetry of the strong interaction under algebra $SU(2)_L \times SU(2)_R$ (or $SU(3)_L \times SU(3)_R$) with $SU(2)_{L+R} = isospin$. Small instincs breaking.

2. This symmetry is spontaneously violated in the ground state. Pseudoscalar mesons (π, κ, ν) are collective modes (Nambu-Goldstone bosons) associated with broken symmetry direction.

3. The generators of these symmetries appear in the electroweak interactions. In the standard model, these hypotheses are consequences of

$$m_u, m_d \ll \Lambda_{QCD} \tag{7.1}$$

$$m_s \leq \Lambda_{QCD} \tag{7.2}$$

$$< \bar{u}u >=< \bar{d}d >=< \bar{s}s >\neq 0 \tag{7.3}$$

in massless limit.

N.B. : It is very important that turning of the masses is a soft perturbation so that we can do perturbation theory around the massless limit.

$$L_{QCD} = L_{m=0} + m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s \tag{7.4}$$

Note: tricky PT due to massless particles.

The specific realization in QCD is more powerful:

(a) Link to PQCD – corrections

(b) Concrete realization of breaking

  i. L_1 transforms as $(3, \bar{3}) + (\bar{3}, 3)$ under $SU(3)_L \times SU(3)_R$

  ii. Contributions from anomalies as mentions above.

7.1 History and Sketch of Example Application

Goldberger-Treiman formula (1957)

$$g_{\pi NN} = \frac{g_A M_N}{f_\pi} \tag{7.5}$$

This works well, but their derivation was cheesy.

Nambu (1960-62) relate to approximate symmetry and correlate with lightness of π mesons.

$$< O|j^s_\mu|\pi > \sim f_\pi p_\mu \; (measured\ in\ \pi \to \mu\nu) \tag{7.6}$$

$$< O|\partial j|\pi > \sim f_\pi p^2 = f_\pi m_\pi^2 \approx 0 \tag{7.7}$$

$$< n|j^s_\mu|\pi > = g_A \bar{u}(n)\gamma_1\gamma_\mu u(p) \; (+P.S.) \tag{7.8}$$

$$0 = < n|j^s_\mu|\pi > \tag{7.9}$$

$$= \underbrace{g_A M_N}_{direct} - \underbrace{f_\pi g_{\pi NN}}_{\pi\ pole} \tag{7.10}$$

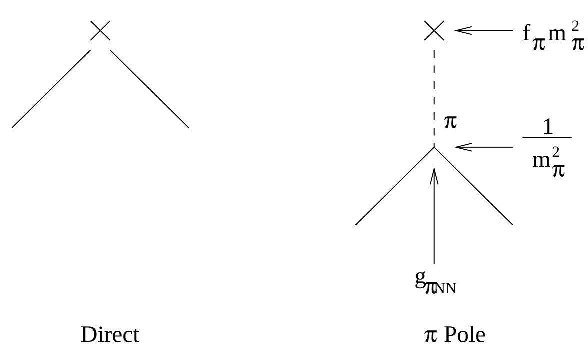

Figure 7.1: Goldberger-Treiman.

⇒ Goldberger-Treiman.

This is the tip of an iceberg of applications, as alluded to above.

7.2 Meson Masses, $U_A(1)$ Problem

Standard (Gellmann – Oakes – Ronne) GM-O-R → (Gellmann – Okulbo) GM-O will discuss this using effective Lagrangian with

$$< \bar{g}_{Li}, g_R^j >= \nu\delta_i^j \tag{7.11}$$

there are low-energy states associated with slow motion in the vacuum manifold

$$< \bar{g}_{Li}(x), g_R^j(x) >= \nu\Sigma_i^j(x)\ (\Sigma \in SU(3)) \tag{7.12}$$

Write

$$\Sigma = exp(\frac{ZiM}{f}) \tag{7.13}$$

$$M = \begin{pmatrix} \frac{\pi^\circ}{\sqrt{2}} + \frac{n}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^\circ}{\sqrt{2}} + \frac{n}{\sqrt{6}} & K^\circ \\ K^- & \bar{K}^\circ & -\frac{2n}{\sqrt{6}} \end{pmatrix} \tag{7.14}$$

$$L = \frac{f_{\bar{u}}^2}{8} tr\partial^\mu\Sigma^+\partial_\mu\Sigma \tag{7.15}$$

gives the properly normalized axial current.
Note

$$\begin{matrix} SU(3)_{L+R} & flavor\ symmetry \Sigma \to U^+\Sigma U \\ SU(3)_L \times SU(3)_R & \Sigma \to U^+\Sigma U \end{matrix} \tag{7.16}$$

No potential is allowed, since $\Sigma\Sigma^+ = 1, det\Sigma = 1$.
Quark masses likewise transform as $U^+\mathcal{M}\mathcal{V}$ (since they go with $\bar{g}_{Li}g_R^j$).
So

$$\Delta L \equiv \upsilon Tr(m_q^+\Sigma + m_q^+\Sigma^+) + highness\ in\ \partial, m \tag{7.17}$$

with

$$\mathcal{M} = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix} \tag{7.18}$$

$$\Delta L = -\frac{4\upsilon^2}{f^2} tr \begin{pmatrix} \frac{\pi^{\circ 2}}{2} + \frac{n^2}{6} + \pi^+\pi^- + K^+K^- + \frac{\pi^\circ n^\circ}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{\pi^{\circ 2}}{2} + \frac{n^2}{6} + \pi^+\pi^- + K^\circ\bar{K}^\circ + \frac{\pi^\circ n^\circ}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{2}{3}n^2 + K^+K^- + K^\circ\bar{K}^\circ \end{pmatrix}$$

$$\times \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix} \tag{7.19}$$

$$m^2_{\pi^\circ\pi^\circ} = \frac{4v^2}{f^2}(m_u + m_d) \tag{7.20}$$

$$m^2_{\eta\eta} = \frac{4v^2}{f^2}(\frac{m_u + m_d}{3} + \frac{4m_s}{3}) \tag{7.21}$$

$$m^2_{\pi^\circ\eta} = \frac{4v^2}{f^2}\frac{m_u - m_d}{\sqrt{3}} \tag{7.22}$$

$$m^2_{K^+K^-} = \frac{4v^2}{f^2}(m_u + m_s) \tag{7.23}$$

$$m^2_{K^\circ\bar{K^\circ}} = \frac{4v^2}{f^2}(m_d + m_s) \tag{7.24}$$

Phenomenologically:

$$m_u, m_d \ll m_s \tag{7.25}$$

$m_u - m_d$ is not much smaller than $m_u + m_d$. It is still rather poorly determined. Gets mixed up with QED corrections.

Probably

$$\frac{m_u}{m_d} \approx 0.4 \tag{7.26}$$

$$\frac{m_u + m_d}{m_s} \approx \frac{1}{25} \tag{7.27}$$

$\pi - n$ mixing is $\sim \frac{m_u+m_d}{m_s}$.

From all this we get

$$3m^2_\eta + m^2_{\pi^\circ} = 2(m^2_{K^+} + m^2_{K^\circ}) \tag{7.28}$$

which works very well.

However, if we include a singlet

$$M = \begin{pmatrix} \sigma + \frac{n}{\sqrt{6}} & & \\ & \sigma + \frac{n}{\sqrt{6}} & \\ & & \sigma - \frac{2n}{\sqrt{6}} \end{pmatrix} \tag{7.29}$$

however normalized, $\sigma - \frac{2n}{\sqrt{6}}$ gets no contribution from $m_s \Rightarrow$ extra light pseudoscalar mesons Has not been seen (n' won't do). This is the $U_A(1)$ problem.