Chapter 4

Renormalization

Cut off dependence in a few basic quantities this into redefinition:

$$L = -\frac{1}{4}\bar{t}^{a}_{\mu\nu 0}\bar{t}^{a\mu\nu}_{0} + \bar{\psi}_0(ip_0 m_0)\psi_0 \tag{4.1}$$

$$A_{ren.} = \frac{1}{\sqrt{Z_A}}A_0 \tag{4.2}$$

$$\psi_{ren.} = \frac{1}{\sqrt{Z_\psi}}\psi_0 \tag{4.3}$$

$$Z_{T_{\bar{\psi}\psi A}} g_{ren.}\bar{\psi}_{ren.}\gamma^\mu\psi_{ren.}A_{\mu ren.} = g_0\bar{\psi}_0\gamma^\mu\psi_0 A_{\mu 0} \tag{4.4}$$

$$g_{ren.} = \frac{g_0 Z_\psi Z_A^{\frac{1}{2}}}{Z_{T_{\bar{\psi}\psi A}}} \tag{4.5}$$

$$Z_{\bar{\psi}\psi}\bar{\psi}_{ren.}\psi_{ren.}m_{ren.} = \bar{\psi}_0\psi_0 m_0 \tag{4.6}$$

$$m_{ren.} = \frac{m_0 Z_\psi}{Z_{\bar{\psi}\psi}} \tag{4.7}$$

The wave-function renormalizations are absorbed into normalizing the 1–particle states. The renormalized couplings are physical particles matrix of renormalized states. Indeed, these are the parameters define the theory, at least perturbatively. We fix these to experiment. Then, other quantities must be expressed finite terms using them (and we remove the cut off).

Notice that in a gauge theory, we must use a gauge-invariant regulator to get the simplicity. Otherwise, there are A^2 (no derivatives res is like), $A\sigma A, AAA, A^4$, etc. coupling appearing. We may set them to zero and entree conspiracies to record gauge symmetry (throwing their a gauge invariant regulation).

$$\begin{aligned}
g_{ren.} &= \frac{Z_A^{\frac{3}{2}}}{\Gamma_{A^3}} g_0 \quad vector\ 3-pt. && (4.8)\\
&= \frac{Z_A}{\sqrt{\Gamma_{A^4}}} g_0 \quad vector\ 4-pt. \propto g^2 && (4.9)\\
&= \frac{Z_A^{\frac{1}{2}} Z_\psi}{\Gamma_{\bar{\psi}\psi A}} && (4.10)\\
&\vdots \\
&= Z && (4.11)\\
\frac{Z_a}{Z_{\bar{\nu} m A}} &= universal\ (Ward\ identity) && (4.12)
\end{aligned}$$

In Abelian theory are equal 1. In N.A. theory, not generally (+ gauge dependent).

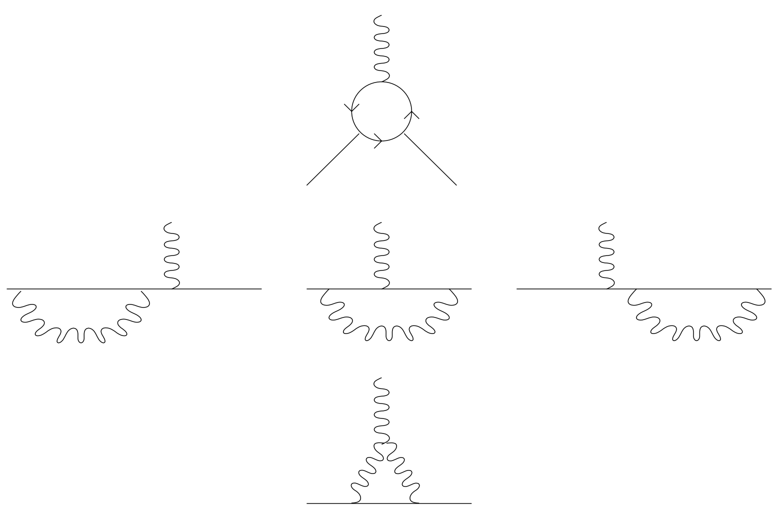

Figure 4.1: Abelian Theory.

To clarify this structure, let's work an example. Correlation function of Bare $\bar{\psi}, \psi$ (inverse propagator).

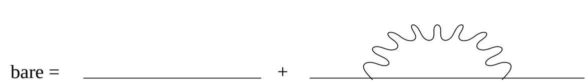

Figure 4.2: Bare.

$$bare = -i(\not{p} - m) + \int^{\Lambda} \frac{d^4k}{(2\pi)^4} (ie)^2 \frac{(-i)(i)\gamma_\mu(\not{p} - \not{k} + m)\gamma^\mu}{k^2[(p-k)^2 - m^2]} \tag{4.13}$$

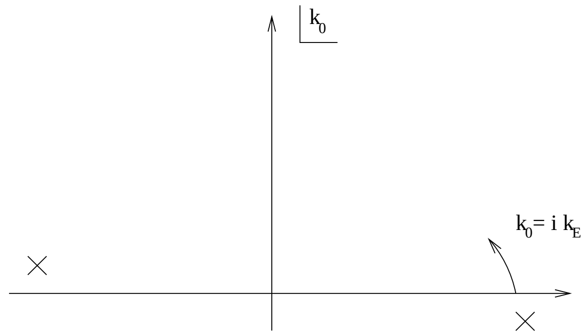

Figure 4.3: Bare Diagram.

Renormalized:

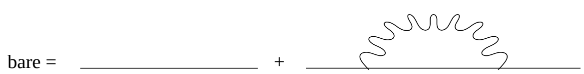

Figure 4.4: Bare.

$$bare = -iZ(\not{p} - m_R) + e_i^2(no\ cut - off) \tag{4.14}$$

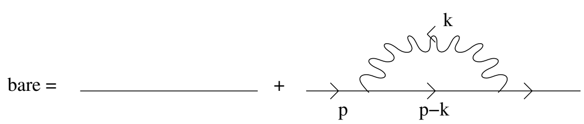

Figure 4.5: Renormalized.

$$bare = -i(\not{p} - m) + ie^2 \int^{\Lambda} \underbrace{\frac{d^4k}{(2\pi)^4} \frac{\gamma_\mu(\not{p} - \not{k} + m)\gamma^\mu}{k^2[(p-k)^2 - m^2]}}_{new\ Eulidec} \tag{4.15}$$

$$S_3 = 2\pi^2 \tag{4.16}$$

$$bare = -i(\not{p} - m) + \frac{ie^2}{8\pi^2} \int^{\Lambda} d\Omega dk k^3 \frac{-2(\not{p} - \not{k}) + 4m}{k^2[(p-k)^2 - m^2]} \tag{4.17}$$

We are going to keep only the divergent part. We shall subtract by normalizing the log divergence (only) at μ.

$$linear + \log\ divergent \tag{4.18}$$

$$\begin{aligned}
\log \quad &: \quad \int^{\Lambda} d\Omega dk k^3 \frac{-2\not p + 4m}{k^2[(p-k)^2 - m^2]} && (4.19)\\
&= \int^{\Lambda} \frac{d\Omega dk k^3}{k^4}[-2\not p + 4m] && (4.20)\\
&= \ln\Lambda[-2\not p + 4m] && (4.21)\\
linear \quad &: \quad \int^{\Lambda} dr dk k^3 \frac{-2\not k}{k^2[(p-k)^2 - m^2]} && (4.22)\\
&= \int^{\Lambda} d\Omega dk k^3 \frac{-2\not k}{k^2 - 2pk + p^2 + m^2} && (4.23)\\
&= \int^{\Lambda} d\Omega dk k^3 \frac{-2\not k}{k^2(1 - \frac{2pk}{(}k^2) + \frac{p^2}{k^2} + \frac{m^2}{k^2}} && (4.24)\\
&= \int^{\Lambda} d\Omega dk k^3 \frac{-2\not k}{k^6} 2pk && (4.25)\\
\int d\Omega \not k k_\mu &= \int d\Omega k_\mu k_\nu \gamma_\nu && (4.26)\\
&= \frac{1}{4}\gamma k^2 && (4.27)\\
\int^{\Lambda} d\Omega dk k^3 \frac{-2\not k}{k^6} 2pk &= \int^{\Lambda} dk k^3 \frac{\not p}{k^4} && (4.28)\\
&= \not p \ln\Lambda && (4.29)
\end{aligned}$$

Putting it all together (as discussed):

$$\begin{aligned}
bare &= -i(\not p - m_0) - i\frac{e^2}{\gamma\pi^2}\ln\frac{\Lambda}{\mu}[-2\not p + 4m_0 + \not p] && (4.30)\\
&= iZ_\psi(\not p - m_r) && (4.31)\\
Z_\psi &= 1 - \frac{e^2}{8\pi^2}\ln\frac{\Lambda}{\mu} && (4.32)
\end{aligned}$$

Finite matrix $d't'$ at $|p| = \mu$

$$m_R(\mu) = m_0[1 - 3\frac{e^2}{8\pi^2}\ln\frac{\Lambda}{\mu}] \qquad (4.33)$$

Check Ward identify:

$$\begin{aligned}
bare &= -ie\gamma_\mu + \int^{\Lambda} \frac{(-ie)^3\gamma_\nu(\not p - \not k)\gamma_\mu(\not p - \not k)\gamma_\nu(i^2)(-i)}{k^2((k-p)^2)^2} && (4.34)\\
&= -ie\gamma_\mu - \frac{ie^3}{8\pi^2}\int^{\Lambda} d\Omega dk k^3 \frac{-2\not k\gamma_\mu\not k}{k^6} && (4.35)
\end{aligned}$$

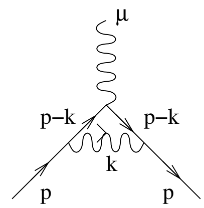

Figure 4.6: Ward Identity.

$$
\begin{aligned}
&= -ie\gamma_\mu - ie^3 \int \frac{dk}{8\pi^2} \frac{-2}{4} \frac{k^5}{k^6} \gamma_\alpha \gamma_\mu \gamma^\alpha && (4.36) \\
&= -ie\gamma_\mu - \frac{ie^3}{8\pi^2} \gamma_\mu \ln \Lambda && (4.37) \\
Z^{-1}_{T_{\bar\psi\psi A}} &= 1 + \frac{e^2}{8\pi^2} \ln \frac{\Lambda}{\mu} && (4.38) \\
Z_{T_{\bar\psi\psi A}} &= 1 - \frac{e^2}{8\pi^2} \ln \frac{\Lambda}{\mu} && (4.39)
\end{aligned}
$$

Check gauge-invariance of $m_R(\mu)$:

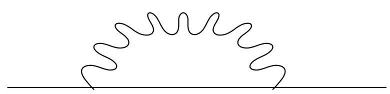

Figure 4.7: Added Stuff.

$$
\begin{aligned}
added\ stuff &\propto \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{\alpha k_\mu k_\nu}{k^2} \frac{\gamma_\mu(\not p - \not k + m)\gamma_\nu}{k^2(p-k)^2} && (4.40) \\
need &\propto \not p - m && (4.41) \\
\log &: 2a \frac{1}{8\pi^2} \int^\Lambda \frac{dk k^3}{k^6} (\frac{1}{4}(-2)\not p + \frac{1}{4} 4m) && (4.42) \\
linear &: \frac{1}{8\pi^2} \int^\Lambda \frac{d\Omega dk k_\mu k_\nu (-)(\gamma_\mu \not k \gamma_\nu (2pk) k^3)}{k^8} && (4.43) \\
&= \frac{1}{8\pi^2} \int^\Lambda \frac{d\Omega dk (-) \not k (2pk) k^3)}{k^6} && (4.44) \\
&= \frac{1}{8\pi^2} \int^\Lambda dk \frac{\frac{1}{4}(-2)\not p k^3}{k^4} - \not p + m && (4.45)
\end{aligned}
$$

Effective mass: $m'_R(\mu_1)$ equivalent to $m^2_R(\mu_2)$ if same m_0, Λ

$$\frac{m'_R(\mu_1)}{1-3\frac{e^2}{8\pi^2}\ln\frac{\Lambda}{\mu_1}} = \frac{m_R^2(\mu_2)}{1-3\frac{e^2}{8\pi^2}\ln\frac{\Lambda}{\mu_2}} \tag{4.46}$$

$$\frac{m_R^2(\mu_2)}{m'_R(\mu_1)} = \frac{1-3\frac{e^2}{8\pi^2}\ln\frac{\Lambda}{\mu_1}}{1-3\frac{e^2}{8\pi^2}\ln\frac{\Lambda}{\mu_2}} \tag{4.47}$$

$$\stackrel{!}{=} 1-3\frac{e^2}{8\pi^2}\ln\frac{\mu_2}{\mu_1} \tag{4.48}$$

$$\frac{m_R(\mu)}{\partial\ln\mu} = 3\frac{e^2}{8\pi^2} \tag{4.49}$$

Renormalization group: N.B. : $e^2\mu$ too.
Physical significance:

1. With normalization at μ, no large loop for processes with parameter $a \sim \mu$.

2. Internally, not rotate for μ to Ω.

Anticipate:

$$\frac{dg(\mu)}{d\ln\mu} = -b_0 g^3 \ (b_0 > 0) \tag{4.50}$$

$$\frac{d\frac{1}{g^2}}{dt} = 2t_0 \tag{4.51}$$

$$\frac{1}{g^2(\mu)} - \frac{1}{g^2(\mu_0)} = 2b_0\ln\frac{\mu}{\mu_0} \tag{4.52}$$

$$e^{\frac{1}{2b_0 g^2(\mu)}} = e^{\frac{1}{2b_0 g^2(\mu_0)}}\frac{\mu}{\mu_0} \tag{4.53}$$

$$\mu = [\mu_0 e^{\frac{1}{2b_0 g^2(\mu_0)}}]e^{\frac{1}{2b_0 g^2(\mu)}} \tag{4.54}$$

Take $\mu \to \infty$, dimensional transmutation perturbative coupling.

Lattice: Given $\frac{1}{g^2(\mu)}$, determine $\mu \propto \frac{1}{a}$. Measure $\frac{M}{\mu}$ fixed, independent of μ as $g \to 0$.

Good news: Check approach to continuous limit. Bad news: Decreasing $g(\mu)$ required exponentially bigger lattice.

With opposite sign for b_0, coupling blows up as $\mu \to \infty (a \to 0)$, no limiting theory guaranteed.

Running coupling and dimensional transmutation:

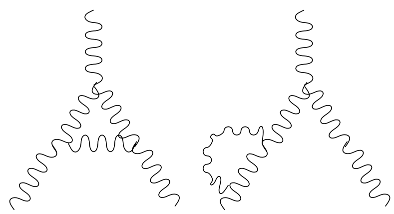

Figure 4.8: Running Coupling

$$\frac{dg(\mu)}{d\ln\mu} = -b_0 g^3 \quad (b_0 > 0) \tag{4.55}$$

$$\frac{d\frac{1}{g^2}}{d\ln\mu} = 2b_0 \tag{4.56}$$

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(\mu_0)} + 2b_0 \ln\frac{\mu}{\mu_0} \tag{4.57}$$

Bare charge $\rightarrow 0$

$$\frac{\mu}{\mu_0} = e^{\frac{1}{2b_0 g^2(\mu)}} e^{\frac{1}{2\overline{b_0} g^2(\mu_0)}} \tag{4.58}$$

$$\mu_0 = e^{\frac{1}{2b_0 g^2(\mu)}} \mu e^{\frac{1}{2\overline{b_0} g^2(\mu_0)}} \tag{4.59}$$

Identify from $\mu \rightarrow \infty$ limit [finite].
More accurate:

$$\frac{dg}{d\ln\mu} = -b_0 g^3 + b_1 g^5 \tag{4.60}$$

$$\frac{d\frac{1}{g^2}}{d\ln\mu} = 2b_0 - 2b_1 \frac{1}{\frac{1}{g^2}} \tag{4.61}$$

$$\frac{du}{dt} = 2b_0 - \frac{2b_1}{u} \tag{4.62}$$

$$u_0 = 2b_0 t \tag{4.63}$$

$$\frac{du}{dt} = 2b_0 - \frac{b_1}{b_0 t} \tag{4.64}$$

$$u \simeq 2b_0 t - \frac{b_1}{b_0} \ln t \tag{4.65}$$

$$\begin{aligned}
\frac{1}{g^2(\mu)} &\simeq 2b_0 \ln\frac{\mu}{\mu_0} - \frac{b_1}{b_0}\ln\ln\frac{\mu}{\mu_0} && (4.66)\\
&\simeq 2b_0 \ln\frac{\mu}{\mu_0} - \frac{b_1}{b_0}\ln\frac{1}{2b_0 g^2} && (4.67)\\
\ln\mu_0 &= \ln\mu - \frac{1}{2b_0 g^2} + \frac{b_1}{2b_0^2}\ln\frac{1}{2b_0 g^2} && (4.68)\\
\mu_0 &\simeq \mu e^{-\frac{1}{2b_0 g^2(\mu)}}[\frac{1}{g^2(\mu)}]^{\frac{b_1}{2b_0^2}}\ finite\ as\ \mu \to \infty && (4.69)
\end{aligned}$$

Relation to lattice (fundamental): Hold μ_0 fixed (reference mass)

$$a(g) \sim \frac{1}{\mu} \sim e^{-\frac{1}{2b_0 g^2}}[\frac{1}{g^2}]^{\frac{b_1}{2b_0^2}}\frac{1}{\mu_0} \qquad (4.70)$$

Small a corresponds to small g. N.B.: Exponential dependence. Test: Physical masses stay constant as $g \to 0$. Explicitly: Measures correlation fall-off $e^{-\lambda n}$, where n is the lattice sparcing.

This is e^{-ML} with $L = an$ for $M = \frac{\lambda}{a}$.

$$\begin{aligned}
e^{-\lambda n} &= e^{-\frac{\lambda}{a}an} && (4.71)\\
&\to \mu_0 e^{\frac{1}{2b_0 g^2}[\frac{1}{g^2}]^{\frac{b_1}{2b_0^2}}}\lambda && (4.72)
\end{aligned}$$

Fixing μ_0, we see $\lambda \to 0$ as $g \to 0$. Large correlation lengths, variating of lattice artifacts. Alternatively, $aM \to 0$ without asymptotic freedom we would be lost here, in tertudation theory. Long correlation lengths can occur at critical points. To nail things down let's look at this more closely:

$$\begin{aligned}
\frac{dg}{d\ln\mu} &\cong -b_0(g - g_C) && (4.73)\\
\ln(g - g_C) &= -b_0 \ln\mu && (4.74)\\
\frac{g(\mu) - g_C}{g(\mu_0) - g_C} &= (\frac{\mu_0}{\mu})^{b_0} && (4.75)\\
\mu &= \mu_0(\frac{g(\mu) - g_C}{g(\mu_0) - g_C})^{\frac{1}{b_0}} && (4.76)
\end{aligned}$$

so we have, $g(\mu) \to g_C$

$$\begin{aligned}
e^{-n\lambda} &= e^{-\frac{\lambda}{a}na} && (4.77)\\
M &= \frac{\lambda}{a} && (4.78)
\end{aligned}$$

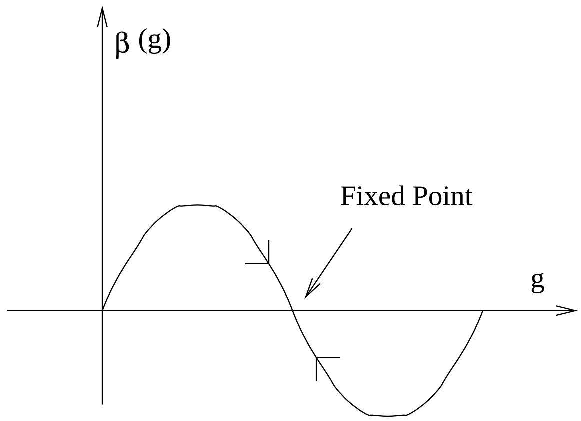

Figure 4.9: Fixed Point

$$a(g(\mu)) \sim \frac{1}{\mu} \sim \frac{1}{\mu_0}(\frac{g(\mu) - g_C}{g(\mu_0) - g_C})^{\frac{1}{b_0}} \tag{4.79}$$

$M = \frac{\lambda}{a}$ fixed $\Rightarrow \lambda \to 0, \frac{1}{M} \gg a$. Alternatively, IR fixed points, but these do not support non-zero masses.

Antiscreening as paramagnetism:

Vacuum polarization:

$$E.D \propto \epsilon F_{oi}F^{oi} \tag{4.80}$$
$$B.H \propto \mu^{-1}F_{ij}F^{ij} \tag{4.81}$$

In reality

$$\epsilon\mu = 1 \tag{4.82}$$

either is varying g.

Antiscreening ($\epsilon < 1$) = paramagnetism ($\mu > 1$) magnetism is easier to think about, since particles do not get produced.

$$E_{class.} = \frac{1}{2g^2}B^2 \tag{4.83}$$

But zero point energies, $\frac{1}{2}h\omega$ (Bose), $-\frac{1}{2}h\omega$ (Fermi) would like to throw it out, but it is field-dependent and cut off dependent.

Interpretation using running coupling:

Wilson approach: Contribution of modes between $\mu_0 + \mu$ (say $\mu > \mu_0$) is

$$= -n \ln \frac{\mu}{\mu_0} B^2 \tag{4.84}$$

$$E = \underbrace{\frac{1}{2g^2(\mu_0)}}_{cut\ off\ at\ \mu_0} B^2 \tag{4.85}$$

$$= \underbrace{(\frac{1}{2g^2(\mu)} - n \ln \frac{\mu}{\mu_0})}_{cut\ off\ at\ \mu} B^2 \tag{4.86}$$

$$n = b_0 \tag{4.87}$$

Result and physical interpretation:

$$n = \frac{1}{96\pi^2}[-\{T(R_0) - 2T(R_{\frac{1}{2}} + 2T(R_1))\}] + \frac{1}{96\pi^2}[3\{-2T(R_{\frac{1}{2}} + 8T(R_1))\}] \tag{4.88}$$

To aid in physical interpretation, pick a particular direction (e.g., τ_3) in internal space for B, and pretend it is electromagnetism.

First term from orbital diamagnetism (Lambda). Note 2 for polarizations, $-$ for fermion.

N.B. This applies to 0–point energy of virtual particles. Real particles do not have this.

Second term from spin paramagnetism (Pauli). Note g–factor 2 (also for vectors), $-$ for fermion, $\frac{3}{1}$ ratio as in solid state.

$$\frac{S^2}{S^2_{\frac{1}{2}}} = 4 \tag{4.89}$$

AF comes from gluon paramagnetism. Reference: F. W., RMP 71 S85 (Ceturean Issue).

Learning from QED: With these insights, we could extract the non-Abelian β function from the Abelian one.

Numerical value:

$$QCD: \ T(R_{\frac{1}{2}}) = f\frac{1}{2} \tag{4.90}$$

$$T(R_1) = 3 \tag{4.91}$$

$$b_0 = \frac{1}{96\pi^2}[+2\frac{f}{2} - 2 \times 3 + 3(-2\frac{f}{2} + 8 \times 3)]$$

$$= \frac{1}{96\pi^2}[66 - 4f] = \frac{1}{16\pi^2}[11 - \frac{2}{3}f] \tag{4.92}$$

$b_0 > 0$ for $f \leq 16$. Gluons heavily dominate for $f = 3$.
Application for unified theories:

$$\begin{aligned} SU(3) &\quad -8F + 66 && (4.93) \\ SU(2) &\quad F(\underbrace{-4}_{fermion\ factor} \times \underbrace{4}_{number\ of\ doublets} \times \underbrace{\frac{1}{2}}_{T_{doublet}}) + 44 \\ &= -8F + 44\ \frac{1}{2}\quad Higgs && (4.94) \\ U(1) &\quad normalized\ charge: \begin{pmatrix} 2 &&&& \\ & 2 &&& \\ && 2 && \\ &&& -3 & \\ &&&& -3 \end{pmatrix} \frac{1}{\sqrt{60}} && (4.95) \\ \sum q^2 &= F(\frac{1}{2} + \frac{1}{60}(3 \times 4^2 + 6(-1)^2 + 1(-6)^2)) && (4.96) \\ &= 2F \stackrel{W.S.B.}{\longrightarrow} -8F\ - \frac{3}{10}\quad Higgs && (4.97) \end{aligned}$$

3 equations: constraint, t_u, g_u

$$\frac{1}{g_u^2} = b^i t_u + \frac{1}{g_i^2} \tag{4.98}$$

$$\frac{1}{g_2^2} - \frac{1}{g_3^2} = (b^3 - b^2) t_u \tag{4.99}$$

Note: unchanged by complete multiplets (no F)

$$\frac{\frac{1}{g_2^2} - \frac{1}{g_3^2}}{\frac{1}{g_1^2} - \frac{1}{g_2^2}} = \frac{b^3 - b^2}{b^2 - b^1} \tag{4.100}$$

Fixing scale: note exponential dependence

$$t_u = \frac{\frac{1}{g_2^2} - \frac{1}{g_3^2}}{b^3 - b^2} = \frac{1}{b^3 g_u^2} - \frac{1}{b^3 g_3^2} = \frac{1}{b^2 g_u^2} - \frac{1}{b^2 g_2^2} \tag{4.101}$$

$$\frac{1}{g_u^2} = \frac{\frac{1}{b^2 g_2^2} - \frac{1}{b^3 g_3^2}}{\frac{1}{b^2} - \frac{1}{b^3}} = \frac{b^3 \frac{1}{g_2^2} - b^2 \frac{1}{g_3^2}}{b^3 - b^2} = \frac{1}{2}(\frac{1}{g_2^2} + \frac{1}{g_3^2}) + \frac{1}{2}\frac{b^3 + b^2}{b^3 - b^2}(\frac{1}{g_2^2} - \frac{1}{g_3^2}) \tag{4.102}$$

4.1 SUSY Modifications

1. Gauge Modification

$$2R_{\frac{1}{2}} - 2R_1 \tag{4.103}$$
$$-6R_{\frac{1}{2}} - 24R_1 \tag{4.104}$$
$$R_{\frac{1}{2}} = \underbrace{11}_{group\ theory} \times \underbrace{2}_{chialitic} \times \underbrace{\frac{1}{2}}_{majorna} \Rightarrow factor\frac{9}{11} \tag{4.105}$$
$$66 \rightarrow 54 \tag{4.106}$$
$$44 \rightarrow 36 \tag{4.107}$$

2. Higgs Modification

$$2\ doublets \tag{4.108}$$
$$fermion\ partner \tag{4.109}$$
$$-1 \rightarrow 2 \times (-1 + \underbrace{2-6}_{\times\frac{1}{2}})\ like\ 6\ Higgs\ doublet \tag{4.110}$$

3. Matter Modification

$$\times\frac{3}{2}\ by\ same\ reasoning \tag{4.111}$$

Example: $N = 4$ gauge field. 4 adjoint fermions. 6 Higgs.

$$\begin{array}{rcc} diamagnetism: & 2 & -8 \\ paramagnetism: & 8 & -8 \end{array} \tag{4.112}$$

Numerical aspects:

$$\frac{b^3 - b^2}{b^2 - b_1} = \frac{22 + \frac{1}{2}}{44 - \frac{1}{2} + \frac{3}{10}} = \frac{75}{146} \simeq 0.514 \tag{4.113}$$

SUSY:

$$\frac{b^3 - b^2}{b^2 - b_1} = \frac{18 + \frac{6}{2}}{36 + 6(-\frac{1}{2} + \frac{3}{10})} = \frac{35}{58} \simeq 0.604 \tag{4.114}$$

$\frac{1}{b^3 - b^2}$:

$$\frac{1}{22+\frac{1}{2}} \quad \rightarrow \quad \frac{1}{18+\frac{6}{2}} \tag{4.115}$$

$$0.044 \quad \rightarrow \quad 0.048 \tag{4.116}$$

Raising scale looks small but appears in exponential (p–decay).

$$\frac{b^3+b^2}{b^3-b^2} = \frac{100+16F-\frac{1}{2}}{22+\frac{1}{2}} \stackrel{F\equiv 3}{=} \frac{62-\frac{1}{2}}{22+\frac{1}{2}} = \frac{41}{15} \simeq 2.73 \tag{4.117}$$

SUSY:

$$\frac{b^3+b^2}{b^3-b^2} = \frac{90+24F-3}{22+3} \stackrel{F\equiv 3}{=} \frac{3}{5} \simeq 0.60 \tag{4.118}$$

b^2: (very little remaining)

$$44-8F-\frac{1}{2} = 19\frac{1}{2} \rightarrow 36-12F-3 = -3 \tag{4.119}$$

Other applications:

$$\frac{m_b}{m_\tau} \rightarrow 3 \ from\ minimal\ SU(5) \tag{4.120}$$

Vacuum instability and Colman–Weinberg:

$$\varphi^4 \lambda(\frac{\varphi}{\mu}) \tag{4.121}$$

interpret as energy from extra nodes.
Too many fermions. moment μ, field strength φ (as in our magnetic calculation).

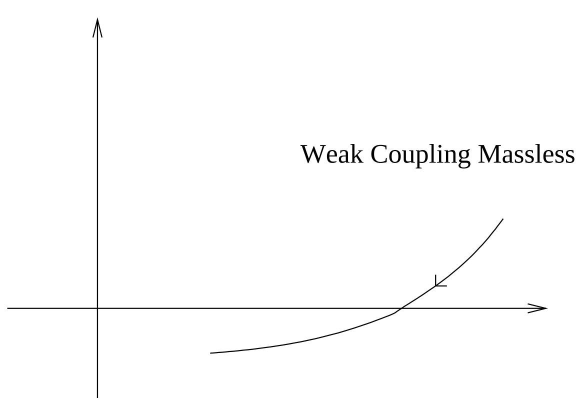

Figure 4.10: Weak Coupling Massless

Table 4.1: Running of coupling

	SM	$MSSM$	
$SU(3)_{b_3^3}$	$66-8F$	$54-12F$	
$SU(2)_{b_3^2}$	$44-8F-\frac{1}{2}$	$36-12F-3$	nearly flat
$U(1)_{b_0^1}$	$-8F-\frac{3}{10}$	$-12F-\frac{9}{5}$	
$\frac{\frac{1}{g_2^2}-\frac{1}{g_3^2}}{\frac{1}{g_1^2}-\frac{1}{g_2^2}}=\frac{b^3-b^2}{b^2-b^1}$	0.514	0.604	agrees with expectation
$t_u=\frac{\frac{1}{g_2^2}-\frac{1}{g_3^2}}{b^3-b^2}$	0.044	0.048	larger scale
			important for p-decay
			close to Planck scale
$\frac{1}{g_u^2}=\frac{\frac{1}{b^2g_2^2}-\frac{1}{b^3g_3^2}}{\frac{1}{b^2}-\frac{1}{b^3}}$	2.73	0.60	families and extra stuff
			appear here
			bigger coupling desirable?

Running of coupling – summary sheet

Other applications: $N=4$ SUSY, glue, 4 Majorna fermions, 6 scalars. Dia: count DOF. Para: 4 fermion, $\beta=0$

Normal SU(5):

$$\frac{m_b}{m_\tau}=1\ at\ unification \tag{4.122}$$

Normalizes to

$$\frac{m_b}{m_\tau}\simeq 3 \tag{4.123}$$

Vacuum instability

$$\lambda(\frac{\mu}{\mu_0})\varphi^4\ negates\ (AF)\ contribution \to \lambda(\frac{\varphi}{\mu_0})\varphi^4\ from\ fermions \tag{4.124}$$

Variational interpretation?

$$<\varphi>\leftarrow a_k^+, a_k \tag{4.125}$$

mix mode in $|k|$–dependent manner. C.F. our magnetic field calculation.

Colman-Weinberg:

Electroweak breaking through leavry sTop. t instability ulicrod when $\hat{t}$ comes in.

PQCD:

Simplest case: $e^+e^-\to$ hadrons

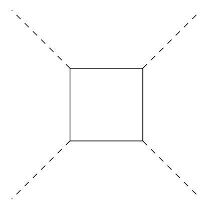

Figure 4.11: Vacuum Instability

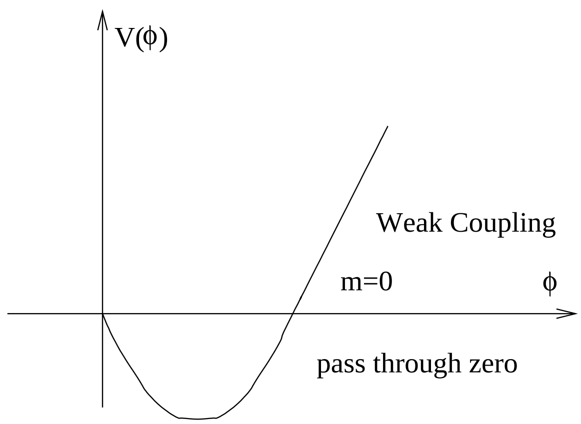

Figure 4.12: Spontanous Symmetry Breaking

$$\frac{\sigma(p, g^{(\mu)}, \mu)}{\sigma(\lambda p, g^{(\mu)}, \mu)} = \sigma(\lambda p, g(\lambda\mu), \lambda\mu) \tag{4.126}$$

$$= \frac{1}{\lambda^2} \underbrace{\sigma(p, g(\lambda\mu), \mu)}_{expand\ perturbatively} \tag{4.127}$$

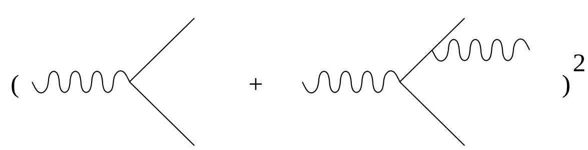

Figure 4.13: Expand Perturbatively.

Potential problem from soft and collisions divergence. They do not occur in inclusive qualities.

Jet phenomena: Observation, heuristic explanation, hard scattering, rate. Sterman-Weinberg comes, energy bite. Stiull no scale – same argument. Extremum pattern. Hadmic gets over 6 nodes of reguitode.

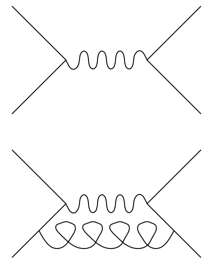

Figure 4.14: W–Exchange.

OPE, DIS: W–exchange.
Nodes up to $k^2 \sim m_\omega^2$.
Simplify to

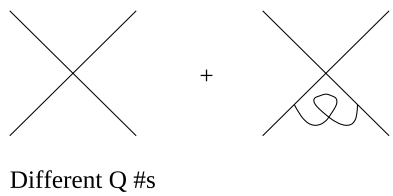

Figure 4.15: Simplify.

Operator versions:

$$J(x)J(\frac{X}{2}) \stackrel{small\ x}{\longrightarrow} \sum C_i(x)O_i(o) \tag{4.128}$$

Singularities of C ordered by dimensions of O_i keep low dimensions. $C_i(x, g, \mu)$ includes renormalization of O.

$$\begin{aligned} C_i(x, g, \mu) &= [\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - \gamma_\theta(\lambda)]C_i = 0 & (4.129) \\ C_i(\lambda^{-1}x, g(\mu_0), \mu_0) &= C_i(x, g(\lambda\mu_0))e^{-\int_{g_0}^{g_1}\gamma_\theta\frac{dg}{\beta}} \\ &= C_i(x, g(\lambda\mu_0))(\frac{\bar{g}}{g})^{-\frac{c_0}{b_0}} & (4.130) \end{aligned}$$

One can also have operators mixing. For DIS, light–cone singularities (tracing quarks and glouns) are important.

$$J^\mu(\frac{x}{2})J^\nu(-\frac{x}{2}) \stackrel{x^2\to 0}{\approx} \sum C_{(x)}^{(i)}O^{\mu\nu\alpha_1\cdots\alpha_n}(0)x_{\alpha_1\cdots x_{\alpha_n}} \tag{4.131}$$

but not individual component.

Twist ≡ dimension – spin

$$< O > \;\rightarrow\; p^{\mu} p^{\nu} \cdots \; (symmetric) \tag{4.132}$$

$$x \;\rightarrow\; \frac{\partial}{\partial g^2} \; or \; \frac{g}{g^2} \tag{4.133}$$

$(\frac{pq}{g^2})^2$ is what appears.
B_j limit:

$$g^2 \rightarrow \infty \tag{4.134}$$

$\frac{2pq}{g^2}$ finite ($\equiv \frac{1}{x}$).
QCD has twist–tor families of operators (respectively chirality).

$$\overline{g}_{(L)} \gamma_{\alpha_1} \overleftrightarrow{\nabla}_{\alpha_2} \cdots \overleftrightarrow{\nabla}_{\alpha_n} q_{(L)} \qquad D = 3 + n - 1, J = n \tag{4.135}$$

$$G^a_{\mu\alpha_1} \overleftrightarrow{\nabla}_{\alpha_2} \cdots \overleftrightarrow{\nabla}_{\alpha_{n-1}} G^a_{\alpha_n \mu} \qquad D = 4 + n - 2, J = n \tag{4.136}$$

First family includes nonsinglets, currents. $T_{\mu\nu}$ also appears (but not by itself). Twist J governs J^{th} moment of appropriate structure function.

Many testable predictions: Parton model sum rules and g^2 correlations. CG relation and g^2 correlations. E: Max with o^{th} order correlations. B_j scalar deviations.