Physics 8.322, Spring 2003
Homework #7

Due **Monday, April 7** by 4:00 PM in the 8.322 homework box in 4-339B.

1. Sakurai: Problem 1, Chapter 6 (page 377)

2. Sakurai: Problem 2, Chapter 6 (page 377)

3. Sakurai: Problem 4, Chapter 6 (page 377)

4. Sakurai: Problem 5, Chapter 6 (page 378)

5. Consider the "one-dimensional hydrogen atom" with Schrödinger equation

$$-\frac{1}{2}\frac{\partial^2\psi}{\partial x^2} - Z\delta(x)\psi = E\psi.$$

(a) Find the ground state energy and wavefunction

(b) Now consider the "one-dimensional helium atom", with Hamiltonian

$$H = -\frac{1}{2}\frac{\partial^2}{\partial x_1^2} - \frac{1}{2}\frac{\partial^2}{\partial x_2^2} - Z\delta(x_1) - Z\delta(x_2) + \delta(x_1 - x_2)$$

which acts on a wavefunction $\psi(x_1, x_2)$ of the positions of two "electrons". Treat the interaction term $\delta(x_1 - x_2)$ as a perturbation and find the ground state energy to first order. Compare to 3D helium. (You may assume that the "electrons" are spinless and satisfy Bose statistics.)

(c) Use the variational method with a trial wavefunction analogous to that used in the text for the 3D helium atom (Sakurai equation 6.4.13). Find the best approximation to the ground state energy, and compare to 3D helium.

6. For the 1D helium atom of the previous problem, use the Hartree Ansatz

$$\psi(x_1, x_2) = \phi(x_1)\phi(x_2)$$

for the ground state. Find the equation for ϕ and solve it analytically. Show that

$$\langle H\rangle_{\text{Hartree}} = -(Z - \frac{1}{4})^2 - \frac{1}{48}.$$

Compare to the results from the previous problem.