MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics Department

Earth, Atmospheric, and Planetary Sciences Department

Astronomy 8.282J-12.402J April 13, 2005

Quiz 2

Name ______________________________ **(please print)**

Last **First**

1. Work any **7** of the **10** problems - indicate clearly which 7 you want graded.
2. Spend 10 or 15 minutes reviewing the problems, and select 7.
3. Immediately after this you have two continuous hours to work the problems.
4. All problems are worth 14 points; everyone gets 2 points just for taking the exam.
5. Closed book exam; you may use two pages of notes, and a calculator.
6. Wherever possible, try to solve the problems using general analytic expressions. Plug in numbers only as a last step.
7. If you have any questions, e-mail the Instructor.
8. Turn exam in during lecture, Friday, April 15.

Time Started: Time Stopped:

Signature ______________________________

Problem	Grade	Grader
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
Total		

APPROXIMATE VALUES OF USEFUL CONSTANTS

Constant	cgs units		mks units	
c (speed of light)	3×10^{10}	cm/sec	3×10^{8}	m/sec
G (gravitation constant)	7×10^{-8}	dyne-cm^2/g^2	7×10^{-11}	N-m^2/kg^2
k (Boltzmann's constant)	1.4×10^{-16}	erg/K	1.4×10^{-23}	J/K
h (Planck's constant)	6.6×10^{-27}	erg-sec	6.6×10^{-34}	J-sec
$m_{\rm proton}$	1.6×10^{-24}	g	1.6×10^{-27}	kg
eV (electron Volt)	1.6×10^{-12}	erg	1.6×10^{-19}	J
$M_\odot$ (solar mass)	2×10^{33}	g	2×10^{30}	kg
$L_\odot$ (solar luminosity)	4×10^{33}	erg/sec	4×10^{26}	J/sec
$R_\odot$ (solar radius)	7×10^{10}	cm	7×10^{8}	m
σ (Stefan-Boltzmann cons)	6×10^{-5}	erg/cm^2-sec-K^4	6×10^{-8}	J/m^2-sec-K^4
Å(Angstrom)	10^{-8}	cm	10^{-10}	m
km (kilometer)	10^{5}	cm	10^{3}	m
pc (parsec)	3×10^{18}	cm	3×10^{16}	m
kpc (kiloparsec)	3×10^{21}	cm	3×10^{19}	m
Mpc (megaparsec)	3×10^{24}	cm	3×10^{22}	m
year	3×10^{7}	sec	3×10^{7}	sec
day	86400	sec	86400	sec
AU	1.5×10^{13}	cm	1.5×10^{11}	m
$1'$ (arc minute)	1/3400	rad	1/3400	rad
$1''$ (arc second)	1/200,000	rad	1/200,000	rad

Problem 1

A star in the Andromeda galaxy yields a *bolometric* flux at the Earth of $F = 1.0 \times 10^{-13}$ ergs cm^{-2} sec^{-1} (1.0×10^{-16} Watts m^{-2}). It has a B-V color index of -0.24. Take the distance to Andromeda to be 1 Mpc. Make use of the table below, where appropriate, to answer the following questions (interpolate between entries using any interpolation scheme that is reasonable). The reference flux for a *bolometric* magnitude of 0.0 is $F_0 = 2.5 \times 10^{-5}$ ergs cm^{-2} sec^{-1} (2.5×10^{-8} Watts m^{-2}).

a. Find the bolometric magnitude, $M_{\rm bol}$ of the star.

b. What is the approximate effective temperature, $T_{\rm effective}$, of the star.

c. Calculate the approximate radius of the star.

Table 1: Abbreviated Table of Main-Sequence Star Properties

Spectral Type	B-V (color)	Mass ($M_\odot$)	$T_{\rm effective}$ (°K)
O5	-0.45	40	35,000
B0	-0.31	17	21,000
B5	-0.17	7	13,500
A0	0.00	3.5	9,700
A5	+0.16	2.1	8,100
F0	+0.30	1.8	7,200
F5	+0.45	1.4	6,500

Problem 2

The binary system called "Cygnus X-1" consists of a black hole (Star 1; for this problem considered to be a point mass) orbiting a normal star (Star 2) with a period $P_{\rm orb} = 5.6$ days. Optical astronomers measure the orbital motion of the normal star via Doppler shifts and determine a "projected" velocity of $v_2 \sin i = 75$ km sec^{-1}. This can be combined with the orbital period to determine the mass function:

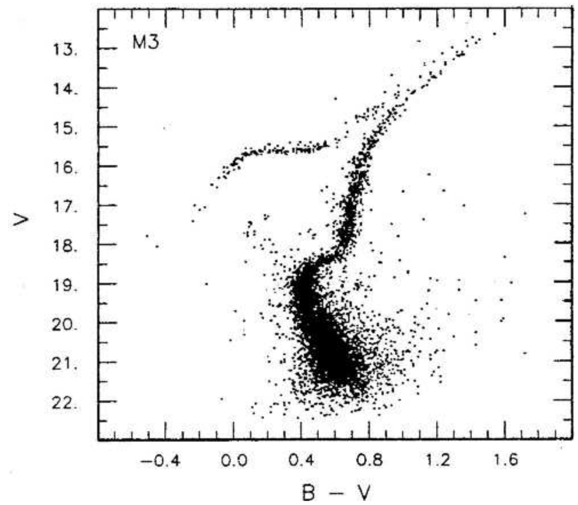
$$f(M) = 0.25\ M_\odot \quad .$$

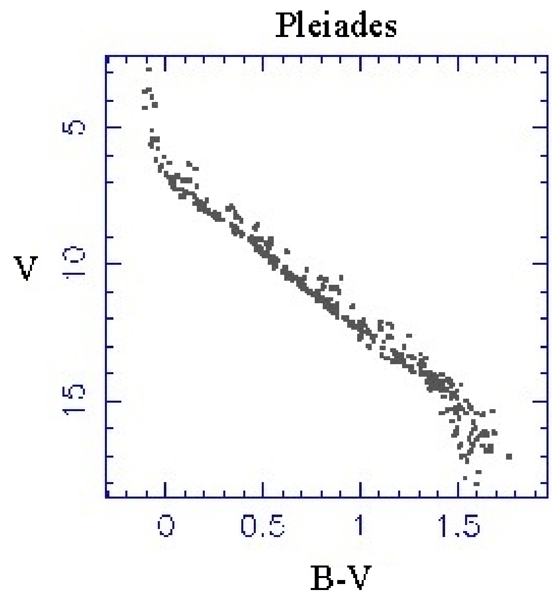
The spectral type of the normal star is found to be B0. Other studies have led to a determination of the orbital inclination angle which turns out to be $i = 30°$.

Use the information above to determine the mass of the black hole. A table of stellar properties is given in Problem 1; use this to help you determine the mass of Star 2. If you have done the problem correctly you will end up with the equivalent of a cubic equation in the mass of the black hole, $M_{\rm BH}$. You can either solve this with Mathematica or some other similar computer program, or simply plug in a handful of trial solutions and "zero in" on an approximate answer (10% is good enough).

Problem 3

The figures below show H–R diagrams for the globular cluster, M3, and the open cluster - the Pleiades.





a. Use these HR diagrams to find the ratio of the distances between M3 and the Pleiades.
b. If we take the absolute visual magnitude of a star with color B-V=0 to be $M_V = 0$, find the actual distance to the Pleiades.
c. Explain why the two HR diagrams look so different.
d. How much intrinsically more luminous are stars at the top of the HR diagram for M3 than those at the bottom?

Problem 4

The image below is of the galaxy NGC 4565. Assume that this is a typical spiral galaxy seen nearly edge on.

Credit: Russell Croman

a. Identify as many generic features of this galaxy as you can. You may write on the white space provided and draw arrows to the appropriate places on the figure.

b. Indicate a likely size scale beside the drawing. How thick would you guess this galaxy is half way out from the center?

c. Indicate approximately where the Sun would be located if this were the Milky Way.

Problem 5

The disk of a galaxy can be modeled as a uniform slab of material of mass density, ρ, that is of (full) thickness, $2H$, in the $\hat{z}$ direction, and is effectively infinite in the $\hat{x}$ and $\hat{y}$ directions. Assume that the mass density for $z > H$ is zero.

a. Compute the effective gravity, $\vec{g}$, at an arbitrary distance, z, inside and above the disk. Sketch $\vec{g}(z)$ for all z (i.e., for + and − values of z). Hint: make use of Gauss' law for gravity $\int \vec{g} \cdot d\vec{A} = -4\pi GM$.

b. Find the speed, v_z that a star must have, starting at the middle of the disk, to get above height H, i.e., just outside of the mass distribution. Express your answer in terms of ρ, G, and H.

Problem 6

A galaxy is found to have a rotation curve, $v(r)$, given by

$$v(r) = \frac{\left(\frac{r}{r_0}\right)}{\left(1 + \frac{r}{r_0}\right)^{3/2}} v_0 \quad ,$$

where r is the radial distance from the center of the galaxy, r_0 is a constant with the dimension of length, and v_0 is another constant with the dimension of speed. The rotation curve is defined as the orbital speed of test stars in circular orbit at radius r.

a. Find an expression for $\omega(r)$, where ω is the angular velocity. What Oort A coefficient would an astronomer living in this Galaxy at $r = r_0$ measure? [Recall: $A = -\frac{1}{2} r_0 \left(\frac{d\omega}{dr}\right)_0$]

b. Find an expression for the mass, $M(< r)$, contained in this galaxy inside of radius r. Assume a spherically symmetric mass distribution.

Problem 7

Short answer questions:

a. The density of stars in a particular globular star cluster is 10^6 pc^{-3}. Take the stars to have the same radius as the Sun, and to have an average speed of 10 km sec^{-1}. Find the mean free path for collisions among stars. Find the corresponding mean time between collisions. (Assume that the stars move in straight-line paths, i.e., are not deflected by gravitational interactions.)

b. A white dwarf star composed entirely of carbon ($_6C^{12}$) reaches a mass of 1.4 $M_\odot$ and all the carbon burns rapidly to magnesium ($_{12}Mg^{24}$). Compute the energy released in this reaction (you should consult the table of Atomic Mass Excess in Problem 10). Compare the nuclear energy released with the gravitational binding energy, U, of the white dwarf. For U you can use $U \simeq GM^2/R$, and choose some reasonable value for R. Is there sufficient nuclear energy to disrupt the white dwarf, i.e., to blow it apart?

Problem 8

The equation of state for cold (non-relativistic) matter may be approximated as:

$$P = a\rho^{5/3} - b\rho^{4/3}$$

where P is the pressure, ρ the density, and a and b are fixed constants. Use a dimensional analysis of the equation of hydrostatic equilibrium to estimate the "radius-mass" relation for planets and low-mass white dwarfs whose material follows this equation of state. Specifically, find $R(M)$ in terms of G and the constants a and b. You should set all constants of order unity (e.g., 4, π, 3, etc.) to 1.0. [Hint: solve for $R(M)$ rather than $M(R)$]. You can check your answer by showing that for higher masses, $R \propto M^{-1/3}$, while for the lower-masses $R \propto M^{+1/3}$.

Problem 9

Once a star like the Sun starts to ascend the giant branch its luminosity, to a good approximation, is given by:

$$L = \frac{10^5 L_\odot}{M_\odot^6} M_{\rm core}^6 \quad ,$$

where the symbol $\odot$ stands for the solar value, and $M_{\rm core}$ is the mass of the He core of the star. Further, assume that as more hydrogen is burned to helium – and becomes added to the core – the conversion efficiency between rest mass and energy is:

$$\Delta E = 0.007\ \Delta M_{\rm core} c^2 \quad .$$

a. Use these two expressions to write down a differential equation, in time, for $M_{\rm core}$.

b. Solve the differential equation for the core mass, $M_{\rm core}(t)$, as a function of time. To make the problem easier, do not evaluate either $L_\odot$ or $M_\odot$ until the next step.

c. Find the time for the star to ascend the giant branch when its core mass increases from $M_{\rm core} = 0.2\ M_\odot$ to $M_{\rm core} = 0.5\ M_\odot$.

Problem 10

This problem relates to the principal nuclear burning chain that powers the Sun, the $p-p$ chain.

a. Write down the 3 nuclear reactions in the $p-p$ chain.

b. Use the table on the *following page* to compute the energy released from either reaction involving ${}_2He^3$ in part (a) – 3 significant figures are sufficient. (The table gives the atomic mass excesses, expressed in MeV.)

c. Compute how much energy is released, in total, from the conversion of 4 hydrogen nuclei into 1 helium nucleus (you may ignore the electrons). Hint: you may bypass intermediate reactions.

Table 4-1 from "Principles of Stellar Evolution and Nucleosynthesis" by Donald Clayton, published by McGraw-Hill.

Table 4-1 Atomic mass excesses†

Z	*Element*	*A*	*M − A, Mev*	*Z*	*Element*	*A*	*M − A, Mev*
0	*n*	1	8.07144			19	3.33270
1	H	1	7.28899			20	3.79900
	D	2	13.13591	9	F	16	10.90400
	T	3	14.94995			17	1.95190
	H	4	28.22000			18	0.87240
		5	31.09000			19	−1.48600
2	He	3	14.93134			20	−0.01190
		4	2.42475			21	−0.04600
		5	11.45400	10	Ne	18	5.31930
		6	17.59820			19	1.75200
		7	26.03000			20	−7.04150
		8	32.00000			21	−5.72990
3	Li	5	11.67900			22	−8.02490
		6	14.08840			23	−5.14830
		7	14.90730			24	−5.94900
		8	20.94620	11	Na	20	8.28000
		9	24.96500			21	−2.18500
4	Be	6	18.37560			22	−5.18220
		7	15.76890			23	−9.52830
		8	4.94420			24	−8.41840
		9	11.35050			25	−9.35600
		10	12.60700			26	−7.69000
		11	20.18100	12	Mg	22	−0.14000
5	B	7	27.99000			23	−5.47240
		8	22.92310			24	−13.93330
		9	12.41860			25	−13.19070
		10	12.05220			26	−16.21420
		11	8.66768			27	−14.58260
		12	13.37020			28	−15.02000
		13	16.56160	13	Al	24	0.1000
6	C	9	28.99000			25	−8.9310
		10	15.65800			26	−12.2108
		11	10.64840			27	−17.1961
		12	0			28	−16.8554
		13	3.12460			29	−18.2180
		14	3.01982			30	−17.1500
		15	9.87320	14	Si	26	−7.1320
7	N	12	17.36400			27	−12.3860
		13	5.34520			28	−21.4899
		14	2.86373			29	−21.8936
		15	0.10040			30	−24.4394
		16	5.68510			31	−22.9620
		17	7.87100			32	−24.2000
8	O	14	8.00800	15	P	28	−7.6600
		15	2.85990			29	−16.9450
		16	−4.73655			30	−20.1970
		17	−0.80770			31	−24.4376
		18	−0.78243			32	−24.3027