MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department
Earth, Atmospheric, and Planetary Sciences Department

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Quiz 1 - Solutions

Name ______________________________ **(please print)**
Last **First**

1. Be sure to attempt all problems.
2. The point values of the problems are indicated at the top of each page.
3. Closed book exam; you may use one page of notes.
4. Wherever possible, try to solve the problems using general analytic expressions. Plug in numbers only as a last step.

Problem	Grade	Grader
1		
2		
3		
4		
Total		

APPROXIMATE VALUES OF USEFUL CONSTANTS

Constant	**cgs**	**units**	**mks**	**units**
c (speed of light)	3×10^{10}	cm/sec	3×10^{8}	m/sec
G (gravitation constant)	7×10^{-8}	dyne-cm^2/g^2	7×10^{-11}	N-m^2/kg^2
k (Boltzmann's constant)	1.4×10^{-16}	erg/K	1.4×10^{-23}	J/K
h (Planck's constant)	6.6×10^{-27}	erg-sec	6.6×10^{-34}	J-sec
$m_{\rm proton}$	1.6×10^{-24}	g	1.6×10^{-27}	kg
eV (electron Volt)	1.6×10^{-12}	erg	1.6×10^{-19}	J
$M_\odot$ (solar mass)	2×10^{33}	g	2×10^{30}	kg
$L_\odot$ (solar luminosity)	4×10^{33}	erg/sec	4×10^{26}	J/sec
$R_\odot$ (solar radius)	7×10^{10}	cm	7×10^{8}	m
σ (Stefan-Boltzmann cons)	6×10^{-5}	erg/cm^2-sec-K^4	6×10^{-8}	J/m^2-sec-K^4
Å(Angstrom)	10^{-8}	cm	10^{-10}	m
km (kilometer)	10^{5}	cm	10^{3}	m
pc (parsec)	3×10^{18}	cm	3×10^{16}	m
kpc (kiloparsec)	3×10^{21}	cm	3×10^{19}	m
Mpc (megaparsec)	3×10^{24}	cm	3×10^{22}	m
year	3×10^{7}	sec	3×10^{7}	sec
day	86400	sec	86400	sec
AU	1.5×10^{13}	cm	1.5×10^{11}	m
$1'$ (arc minute)	$1/3400$	rad	$1/3400$	rad
$1''$ (arc second)	$1/200,000$	rad	$1/200,000$	rad

Problem 1 (25 points)

Orbital Dynamics

A binary system consists of two stars in circular orbit about a common center of mass, with an orbital period, $P_{\mathrm{orb}} = 10$ days. Star 1 is observed in the visible band, and Doppler measurements show that its orbital speed is $v_1 = 20$ km s^{-1}. Star 2 is an X-ray pulsar and its orbital *radius* about the center of mass is $r_2 = 3 \times 10^{12}$ cm $= 3 \times 10^{10}$ m.

(a) Find the orbital radius, r_1, of the optical star (Star 1) about the center of mass.

$$v_1 = \frac{2\pi r_1}{P_{\mathrm{orb}}}$$

$$r_1 = \frac{P_{\mathrm{orb}} v_1}{2\pi} = 2.75 \times 10^{11} \quad \mathrm{cm}$$

(b) What is the total orbital separation between the two stars, $r = r_1 + r_2$?

$$r = r_1 + r_2 = 2.75 \times 10^{11} + 3 \times 10^{12} = 3.3 \times 10^{12} \quad \mathrm{cm}$$

(c) Compute the total mass of the system, $M_1 + M_2 = M_{\mathrm{tot}}$.

$$\frac{GM_T}{r^3} = \left(\frac{2\pi}{P_{\mathrm{orb}}}\right)^2$$

$$M_T = \left(\frac{2\pi}{P_{\mathrm{orb}}}\right)^2 \frac{r^3}{G} = 2.8 \times 10^{34} \quad \mathrm{g} = 14\ M_\odot$$

(d) Compute the individual masses of Star 1 and of Star 2.

$$\frac{M_1}{M_2} = \frac{r_2}{r_1} = \frac{3 \times 10^{12}}{2.8 \times 10^{11}} = 10.9$$

$$M_1 + M_2 = M_T = 14\ M_\odot \Rightarrow M_2 = 1.2\ M_\odot; \quad M_1 = 12.8\ M_\odot$$

Problem 2 (25 points)

Geometric and Physical Optics

(a) Compare the angular resolution of the The Very Large Array (VLA), the Hubble Space Telescope, and the Spitzer Space Telescope. The effective diameters of the three instruments are 36 km, 2.5 m, and 85 cm, respectively, while typical wavelengths used for observation might be 6 cm, 0.6 μm, and 5 μm, respectively.

$$\Theta_{\rm min} \simeq \frac{\lambda}{D}$$

$$\text{VLA}: \quad \frac{6 \text{ cm}}{36 \times 10^5 \text{ cm}} = 0.33''$$

$$\text{HST}: \quad \frac{0.6 \times 10^{-6} \text{ m}}{2.5 \text{ m}} = 0.05''$$

$$\text{Spitzer}: \quad \frac{5 \times 10^{-6} \text{ m}}{0.85 \text{ m}} = 1.2''$$

(b) Each of the two Magellan telescopes has a diameter of 6.5 m. In one configuration the effective focal length is 72 m. Find the diameter of the image of a planet (in cm) at this focus if the angular diameter of the planet at the time of the observation is 45″.

Start with:

$$s = \alpha f \quad ,$$

where s is the diameter of the image, f the focal length, and α the angular diameter of the planet. For the values given in the problem:

$$s = \frac{45}{3600}\frac{\pi}{180} 7200 = 1.6 \text{ cm}$$

(c) A prism is constructed from glass and has sides that form a right triangle with the other two angles equal to 45°. The sides are L, L, and H, where L is a leg and H is the hypotenuse. A parallel light beam enters side L normal to the surface, passes into the glass, and then strikes H internally. The index of refraction of the glass is $n = 1.5$. Compute the critical angle for the light to be internally reflected at H. Does it do so in the geometry described here?

From Snell's law we have:

$$n_g \sin(\theta_g) = n_{\rm air} \sin(\theta_{\rm air})$$

$$\sin(\theta_{\rm crit}) = \frac{1}{1.5}\sin(90^\circ) \Rightarrow \theta_{\rm crit} = 41.8^\circ$$

But, from the geometry, $\theta_{\rm incident}$ for the light striking side H is 45°. Thus, since $\theta_{\rm incident} > \theta_{\rm crit}$, the light *does* undergo total internal reflection.

Problem 3 (25 points)

Doppler Effect and Bohr Atom

(a) A binary system with circular orbit is viewed edge on (i.e., in the plane of the orbit), and the spectrum of one of the stars is recorded on a regular basis (the other star is undetectable). The orbital speed of the visible star is 300 km s^{-1}, and the orbital period is $P_{\rm orb}$. The center of mass of the binary system is moving *toward* the Solar System at 300 km s^{-1}. Compute an expression for the Doppler shift of the Hα line (at 6563 Å) as a function of time, and sketch $\Delta\lambda$ as a function of orbital cycle. You may ignore the orbital speed of the Earth around the Sun.

The time-dependent Doppler shift of the star is given by:

$$\frac{\Delta\lambda}{\lambda} \simeq -\frac{v}{c}\sin(\theta) \quad ,$$

where $\theta = 0$ corresponds to the source moving toward us. Thus,

$$\frac{\Delta\lambda}{\lambda} \simeq -\frac{300}{3\times10^5} - \frac{300}{3\times10^5}\sin\left(\frac{2\pi t}{P_{\rm orb}} + \phi_{\rm orb}\right) \quad ,$$

where the first term represents the constant center of mass motion for the binary, while the second term is the time-varying Doppler shift of the star being observed. The parameter $\phi_{\rm orb}$ is the orbital phase. Finally, in terms of a wavelength shift, we have:

$$\Delta\lambda \simeq -6563\mathring{A}\ \left[10^{-3} + 10^{-3}\sin\left(\frac{2\pi t}{P_{\rm orb}} + \phi_{\rm orb}\right)\right] \quad .$$

$$\Delta\lambda \simeq -6.5\mathring{A}\ \left[1 + \sin\left(\frac{2\pi t}{P_{\rm orb}} + \phi_{\rm orb}\right)\right] \quad .$$

(b) If the Bohr energy levels scale as Z^2, where Z is the atomic number of the atom (i.e., the charge on the nucleus), estimate the energy or wavelength of a photon that results from a transition from $n = 3$ to $n = 2$ in Fe, which has $Z = 26$. Assume that the Fe atom is completely stripped of all its electrons except for one.

$$h\nu = 13.6\ Z^2 \left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right] \ \text{eV}$$

$$h\nu = 13.6 \times 26^2 \left[\frac{1}{2^2} - \frac{1}{3^2}\right] \ \text{eV}$$

$$h\nu = 1280 \ \ \text{eV} = 1.28\ \text{keV} \Rightarrow 9.6\mathring{A}$$

Problem 4 (25 points)

Luminosity and Magnitudes

(a) A white dwarf star has an effective temperature, $T_e = 50,000$ degrees Kelvin, but its radius, $R_{\rm WD}$, is comparable to that of the Earth. Take $R_{\rm WD} = 10^4$ km (10^7 m or 10^9 cm). Compute the luminosity (power output) of the white dwarf. Treat the white dwarf as a blackbody radiator.

$$L = 4\pi R^2 \sigma T_e^4 = 4\pi (10^9)^2 (5.7 \times 10^{-5})(50,000)^4 \ \text{ ergs s}^{-1}$$
$$L \simeq 4.5 \times 10^{33} \text{ ergs s}^{-1} \simeq 1\ L_\odot$$

(b) An extrasolar planet has been observed which passes in front of (i.e., transits) its parent star. If the planet is dark (i.e., contributes essentially no light of its own) and has a surface area that is 2% of that of its parent star, find the decrease in magnitude of the system during transits.

The flux goes from a maximum of F_0, when the planet is not blocking any light, to $0.98F_0$ when the planet is in front of the stellar disk. So, the uneclipsed magnitude is:

$$m_0 = -2.5 \log(F_0/F_{\rm ref}) \quad .$$

When the planet blocks 2% of the stellar disk, the magnitude *increases* to:

$$m = -2.5 \log(F/F_{\rm ref}) = -2.5 \log(0.98 F_0/F_{\rm ref}) \quad .$$

Thus, the change in magnitude is:

$$\Delta m = m - m_0 = -2.5 \log(0.98) \simeq 0.022 \quad \text{magnitudes}$$

(c) If a star cluster is made up of 10^4 stars, each of whose absolute magnitude is -5, compute the combined *apparent* magnitude of the cluster if it is located at a distance of 1 Mpc.

The absolute magnitude of one of the stars is given by:

$$M = -2.5 \log(L/L_{\rm ref}) = -5 \quad ,$$

where L is the stellar luminosity, and $L_{\rm ref}$ is the luminosity of a zero magnitude star. This equation implies that $L = 100\ L_{\rm ref}$. Armed with this fact, we can now compute the combined magnitude of the collection of 10^4 stars:

$$M_{\rm TOT} = -2.5 \log[(10^4 \times 100\ L_{\rm ref})/L_{\rm ref}] = -2.5 \log(10^6) = -15$$

Finally, the distance modulus corresponding to 1 Mpc is $5 \log(10^6/10) = 25$. Therefore, the apparent magnitude of the star cluster at this distance is:

$$m = M + \text{distance modulus} \Rightarrow m = -15 + 25 = +10 \quad .$$