MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics Department

8.231, Physics of Solids I — Due on Wed., Nov. 8.

Problem set #7

1. **Temperature dependence of conductivity – an evidence of Fermi statistics of electrons**:

   We treat the electrons in a metal as free electrons of mass m_e (ie ignore the periodic potential from the nuclei). The electron density is n_e. We like to calculate the temperature dependence of the conductivity of the metal.

   (a) Electrons in the metal may be scattered by the impurities. To calculate the effect of scattering, we treat the electrons as point particles and treat the impurities as an hard sphere of radius r_0. The impurity density is n_i. Calculate the mean free path l of the electrons. The mean free path l is the average distance that an electron can travel without any collision. (Hint: you might want to treat electrons as hard spheres of radius r_0 and the impurities at point particles. This allows you to picture the volume that an electron sweeps as it travels in space.)

   (b) Here we treat electrons as a classical gas. Estimate the average thermal velocity v of the electrons at a temperature T. Estimate the mean free time τ. (τ is the average time that an electron can travel without any collision.) If we take τ as the relaxation time, use the Drude model to find the conductivity σ of the metal. Show that $\sigma \propto T^{\gamma_c}$ and give the value of γ_c.

   (c) Here we treat electrons as a quantum Fermi gas and assume that the temperature T satisfies $k_B T \ll \epsilon_F$ where ϵ_F is the Fermi energy of the electrons. Estimate the average velocity v of the electrons in the low temperature limit. Estimate the mean free time τ. If we take τ as the relaxation time, use the Drude model to find the conductivity σ of the metal. Show that $\sigma \propto T^{\gamma_q}$ and give the value of γ_q.

2. **Conductivity in magnetic field**

   (a) We treat the electrons in a metal as free electrons of mass m_e (ie ignore the periodic potential from the nuclei). The electron density is n_e. We like to calculate the conductivity of the metal in a magnetic field. We assume that the magnetic field points in the $\boldsymbol{z}$ direction and has a uniform strength B. Use the Drude model to calculate the conductivity tensor σ_{ij} where $i, j = x, y, z$. (An electric field E_i will induce an current density J_i. Conductivity tensor σ_{ij} is defined as $J_i = \sum_j \sigma_{ij} E_j$.) Calculate the corresponding resistivity tensor ρ_{ij} which is defined as $E_i = \sum_j \rho_{ij} J_j$. Note that ρ_{xy}/B is the Hall coefficient R_H.

   (b) Now consider a system with two types of charge carriers. The two carriers have the same density n and opposite charge (e and $-e$), and their masses and relaxation times are m_1, m_2 and τ_1, τ_2, respectively. (You may want to use the mobility, $\mu = \tau/m$, instead of τ and m.)
   (i) Calculate the resistance, ρ_{xx}, as a function of magnetic field B in the $\boldsymbol{z}$ direction.

(ii) Calculate the Hall coefficient $R_H = \rho_{xy}/B$.
(iii) An undoped semiconductor corresponds to the above system. $n = n_0 e^{-\Delta/k_B T}$, describes the temperature dependence of the carrier concentration. What will the temperature dependence of the Hall coefficient be?

3. **1D electron band for weak potential**:
Electrons of mass m are confined to one dimension. A weak periodic potential of period a, $V(x) = V_0 \cos(2\pi x/a)$, is applied.

   (a) Under what conditions will the nearly free-electron approximation work? Assuming that the condition is satisfied, sketch the three lowest energy bands in the first Brillouin zone. Number the energy bands (starting from one at the lowest band).

   (b) Calculate (to first-order in V_0) the energy gap at $k = \pi/a$ (between the first and second band) and $k = 0$ (between the second and third band).

4. **Oscillations induced by a static force**:

   An 1D electron band has the following dispersion $\epsilon_k = \epsilon_0 \cos(ka)$ where a is the lattice constant. Under the influence of static force F, the momentum of the electron $p = \hbar k$ increases according to the Newton law $\hbar \frac{dk}{dt} = F$. Assume the electron is at $x = 0$ and has a zero velocity $v = 0$ at time $t = 0$, find the position of electron at time t. (Hint: the velocity of the electron is the group velocity $v = \hbar^{-1} \frac{d\epsilon_k}{dk}$.)