Massachusetts Institute of Technology
Physics Department

Physics 8.20 **IAP 2005**
Introduction to Special Relativity
Final Exam Solutions

1.

(a) c — meters per second — when $v/c \sim 1$
(b) A reference frame in which Newton's laws are observed to hold
(c)

$$U = \left(\frac{dct}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau}\right)$$

$$d\tau = dt/\gamma$$

$$\Rightarrow U = \gamma(u)(c, \overrightarrow{u})$$

(d) 4-jerk

$$J = \left(\frac{d^3ct}{d\tau^3}, \frac{d^3x}{d\tau^3}, \frac{d^3y}{d\tau^3}, \frac{d^3z}{d\tau^3}\right)$$

(e) Proper acceleration is the acceleration of an object in a frame in which the object is instantanously at rest.

(f)

$$\overrightarrow{f} = \frac{d\overrightarrow{p}}{dt}$$

$$\overrightarrow{p} = \gamma m \overrightarrow{v}$$

(g)

$$a \cdot b = a_0 b_0 - a_1 b_1 - a_2 b_2 - a_3 b_3$$

(h) See figure (1)
(i) See figure (1)
(j) See figure (1)
(k) You may write the answer in several forms: An observer in a gravitational field is equivalent to an observer in an accelerated frame. Inertial mass is the same as gravitational mass. The laws of physics may be written such that they take the same form in all inertial frames.
(l) Yes. $\overrightarrow{f} = \frac{d\overrightarrow{p}}{dt}$
(m) Michelson-Morley experiment; nearly all particle and nuclear physics data (particle lifetimes, Compton effect); the global positioning system; transverse doppler effect; conversion of mass to energy (e.g.,nuclear power); Fizeau experiment.

(n) Precession of the perihelion of Mercury; the bending of light by the sun; gravitational by compact stellar objects red shift of light; correct operation of the global positioning system; Pound-Reblca experiment showing gravitational redshift on earth; time measured by clocks on a satellite compared with clocks on earth; existence of black holes.

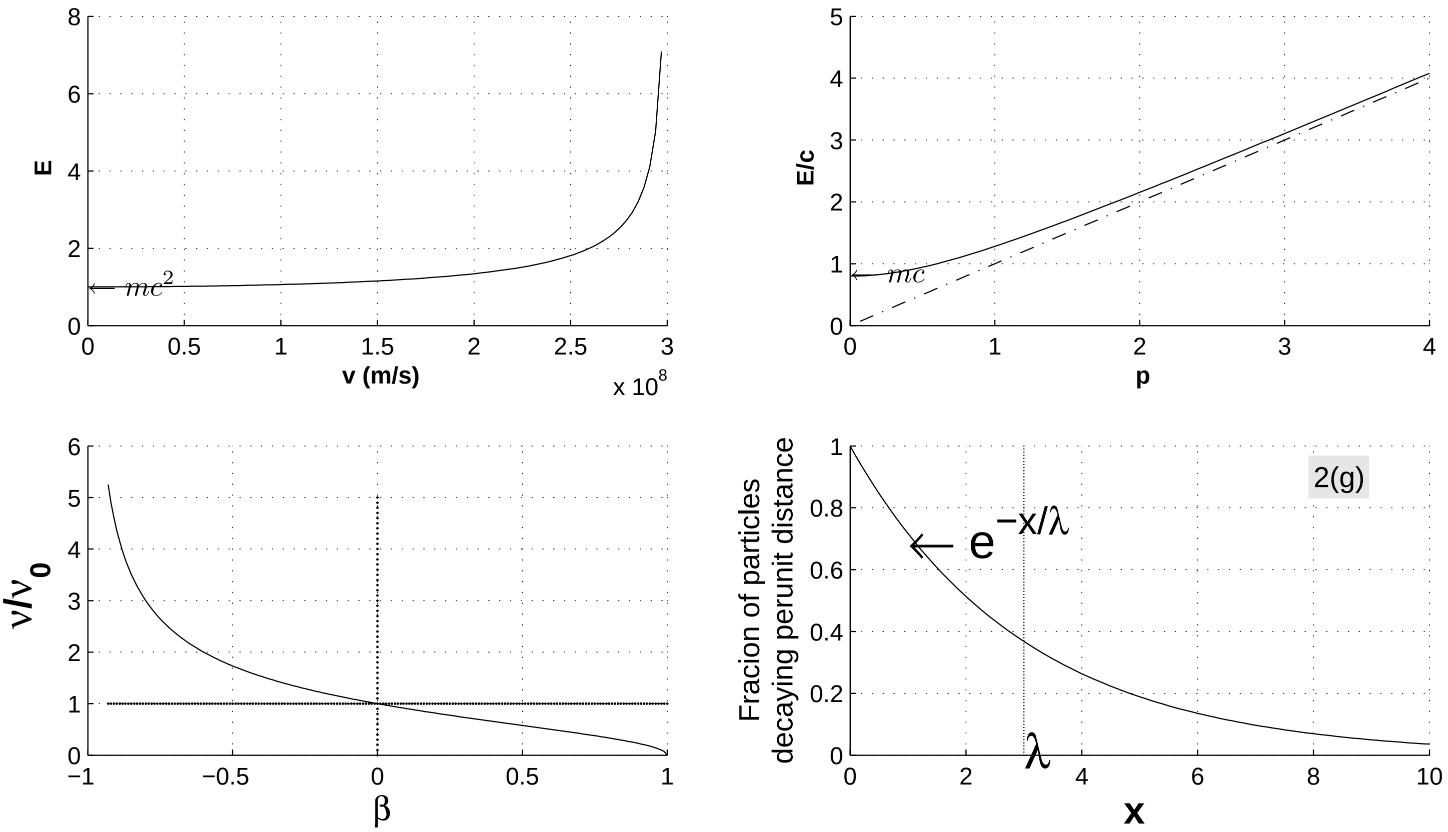

Figure 1: Problem #1 & #2

2.

(a)

$$\tau_\mu = \gamma \tau'_\mu \Rightarrow \gamma = \frac{\tau_\mu}{\tau'_\mu}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \beta = \sqrt{1-\gamma^{-2}}$$

$$\boxed{\beta = 0.95}$$

(b) 100 MeV

`(c)`in Σ

$$E_\mu = \gamma m_\mu c^2 = \frac{6.6}{2.2} 100 = 300 \text{ MeV}$$

`(d)`in Σ

$$\texttt{KE} = (\gamma - 1) m_\mu c^2 = 200 \text{ MeV}$$

`(e)`

$$p = \gamma \beta m_\mu c^2 / c = 3 \times 100 \times 0.95 = 285 \text{ MeV/c}$$

`(f)`

$$\Delta x = \beta c \tau_\mu = 1881 \text{ m}$$

3.

$$\boxed{\gamma = \frac{1}{\sqrt{1-\beta^2}} = 5/4}$$

(a) John's separation from George is 10 km$/\gamma = 8$ km

(b) The speed of the neutrons coming from John's gun is

$$\frac{\beta+\beta}{1+\beta^2}c = \frac{15}{17}c$$

(c) The same report.

(d)

$$\frac{u'_y}{u'_x} = \tan\theta'$$

$$u_y = v/2$$

$$u_x = \frac{\sqrt{3}}{2}v$$

$$u'_y = \frac{v/2}{\gamma(1+u_x v/c^2)}$$

$$u'_x = \frac{\frac{\sqrt{3}}{2}v + v}{1+u_x v/c^2}$$

$$\therefore \theta' = \tan^{-1}\left[\frac{1/2}{5/4(1+\sqrt{3}/2)}\right] \sim 13^\circ$$

(e) John is firing neutrons at the rate of

$$\nu = \nu_0\sqrt{\frac{1+\beta}{1-\beta}} = 20 \text{ kHz}$$

4.

(a)

$x_A(0)$=1 AU$\simeq$ 1.5 $\times$ 10^{11} m

$x_B(0)\simeq$1.5 AU$\simeq$ 2.3 $\times$ 10^{11} m.

(b) Just rewrite the formula in the information sheet with the right initial condition, i.e

$$x_{A(B)}(t=0) = x_{A(B)}(0)$$

$$x_A(t) = \frac{c^2}{g}\sqrt{1+\left(\frac{gt}{c}\right)^2} - \frac{c^2}{g} + x_A(0)$$

$$x_B(t) = \frac{c^2}{g}\sqrt{1+\left(\frac{gt}{c}\right)^2} - \frac{c^2}{g} + x_B(0)$$

(c) Lorentz contraction(See figure(2)) :

$$AD = \gamma AC = \gamma\left(x_B(0) - x_A(0)\right)$$

Therefore

$$\Delta x'_{AB} = AB \geq \gamma l$$

(d)

$$x_B(t) - x_A(t) = x_B(0) - x_A(0)$$

(e) Tangent to A's wordline will be ct' axis of Σ' frame and x' axis will be the axis so that a light like wordline bisects the angle between the x' and ct' axis.

You can choose $x = x' = 0$ at $t = t' = 0$. You can choose $x = x' = 0$ at $t = t' = 0$ and A at $x' = 0$ if and only if the tangent line passes through the origin.

(f) See figure(2). (g) The spaceships start the same distance apart, and undergo the same acceleration (at any particular time t as measured in Σ—this is crucial), so clearly they stay the same distance apart as measured in Σ. (h) In Σ'; B starts accelerating earlier than A; and it's accelerating faster at anytime t'. (i) No (j) No. If spaceship A and spaceship B occupy the same spacetime point in some frame Σ then they will occupy the same spacetime point also in Σ'

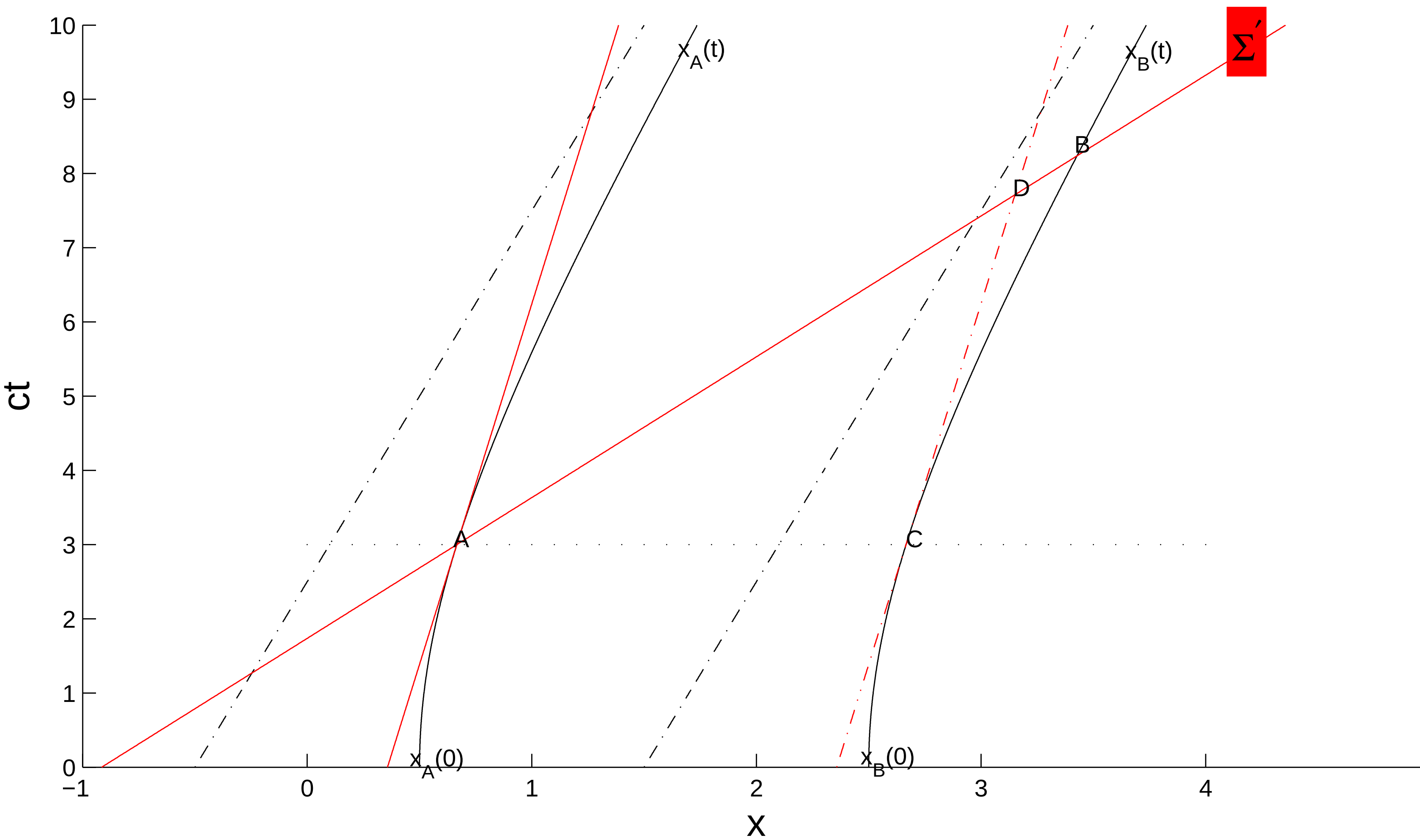

Figure 2: Problem #5

5.

(a)

$$\gamma = \frac{E_p}{m_p c^2} = 1000$$

$$\beta = \sqrt{1 - \gamma^{-2}}$$

$$\gamma^{-2} = 10^{-6} \ll 1$$

$$\beta \simeq 1 - 10^{-6}/2$$

$$c - v = c(1 - \beta) \sim 150 \text{ m/s} \sim 300 \text{ mi/hour}$$

(b)

$$|\vec{F}| = |\frac{d\vec{p}}{dt}| = \gamma m_p |\frac{d\vec{v}}{dt}| = \gamma m_p v^2/R \simeq \gamma m_p \frac{c^2}{R}$$

Note: $d\gamma/dt = 0$ since v is constant for a uniform circular motion.

(c)

$$a' = \gamma^{\mathbf{2}} a_\perp \simeq \gamma^{\mathbf{2}} c^2/R$$

From the information sheet, for proper acceleration perpendicular to the direction of motion.

(d)

$$P_1 = (E_p/c, p)$$

$$P_2 = (m_p c, 0)$$

$$P_X = (E_X/c, p_X)$$

$$P_1 + P_2 = P_X$$

$$(P_1 + P_2)^2 = P_1^2 + P_2^2 + 2P_1 \cdot P_2 = P_X^2 = m_X^2 c^2$$

$$2P_1 \cdot P_2 = 2m_P E_P = m_X^2 c^2 - 2m_P^2 c^2$$

$$m_X = m_P \left[2(1+\frac{E_P}{m_P c^2})\right]^{1/2} = (1\mathrm{GeV/c^2})(2\times 1001)^{1/2} \simeq 45\mathrm{GeV/c^2}$$

(e)

$$(E_p/c, p) + (E_p/c, -p) = (m_X c, 0)$$

Compare zeroth element of LHS and RHS:

$$m_X = 2E_P/c^2 = 2\ \mathrm{TeV/c^2}$$

(f)(ii) since 2 TeV/c^2 is greater than supposed m_X.

(g) In this case, $2E_p/c^2(20\%) = 400$ GeV. We therefore probably need to build a collider (iii). It sure would be nice to know a "few hundred GeV" more precisely.