8.13-14 Experimental Physics I & II "Junior Lab"
Fall 2007 - Spring 2008

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Department of Physics

8.14 Junior Lab Data analysis Assignment #2 2/6/2008

Due: at the start of session #4

Objective: Distinguish Physical phenomena from resolution effects

Analysis Exercise: Gaussian or Lorentzian ?

In the experiments of Dopplerfree, Moessbauer, QIP, and Zeeman you will have to fit line-shapes (or dips). From the Uncertainty Principle we know: ΔE Δt ≥ h(bar)/2, for a wave-packet, which translates into: Γ τ ≥ h for a resonant line with width Γ (FWHM) from a de-excitation exponential lifetime τ. There is an interesting relation between Γ and τ, by the fact, that the energy Fourier transform of an exponential decay in time results into a Lorentzian non-relativisticaly (as mostly the case in 8.14). An easy derivation follows:

Emission of a Spectral line is described as a damped oscillator with ω_0 represented by a time dependent amplitude:

$$f(t) = C \cdot e^{i\omega_0 t} e^{-\gamma t} \quad \text{with} \quad \int_{-\infty}^{\infty} |f(t)|^2 dt = C^2 \left| \int_{-\infty}^{\infty} e^{-2\gamma t} dt \right| = \frac{C^2}{2\gamma} = 1 \quad \text{hence } C = \sqrt{2\gamma}$$

Complete Amplitude: $$f(t) = \sqrt{2\gamma} e^{i\omega_0 t} e^{-\gamma \cdot t}$$

Fourier transform: $$F(\omega) = \sqrt{\frac{\gamma}{\pi}} \cdot \int_{-\infty}^{\infty} e^{-i\omega t} e^{i\omega_0 t} e^{-\gamma t} dt = \sqrt{\frac{\gamma}{\pi}} \frac{i}{\omega_0 - \omega + i\gamma}$$

Spectral intensity I(ω) $$|F(\omega)|^2 = \frac{1}{\pi} \frac{\gamma}{(\omega_0 - \omega)^2 + \gamma^2} \quad \text{with} \quad \int |F(\omega)|^2 dt = 1$$

Inserting E= hν =h(bar)ω and Γ= 2γ h(bar) we obtain the probability/energy, the

Lorentzian

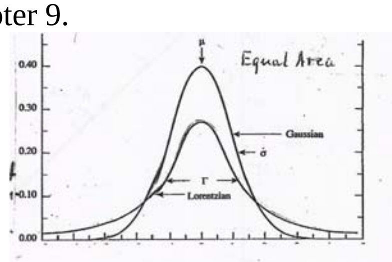
$$I(E) = I_0 \frac{\Gamma/2\pi}{(E_0 - E)^2 + \Gamma^2/4}$$

Compare this line shape to a Gaussian of equivalent: FWHM = $2.354 \cdot \sigma$

you will see a substantial difference.

$$I(E, E_0, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{E - E_0}{\sigma} \right)^2 \right]$$

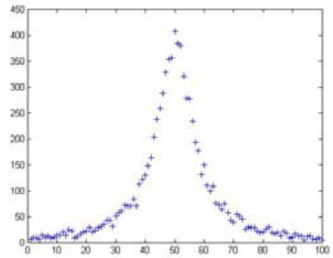
Bevington, page 33. Also read chapter 9.



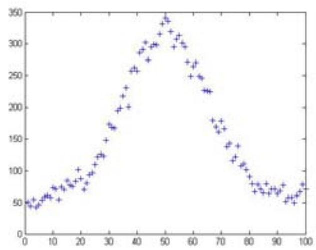
What is the point? Well, the physically interesting quantity is the "**Natural Line Width Γ**", whereas resolution effects like Doppler broadening and/or stochastic errors will make the line-shape appear more Gaussian; – recall the Central Limit Theorem. So, if you want to claim that you measured a Natural Line, better prove that the shape is right. Then you also may state the lifetime of the excited state.

Problem:

1) Fit the dataset 'lineshape1.txt' available from http://web.mit.edu/8.13/www/handouts.shtml under Ulrich Becker to both a Lorentzian PDF and a Gaussian PDF. Compare the fit results and chi-squared values to determine the correct fit hypothesis.



2)Fit the dataset 'lineshapedata.txt' also available from http://web.mit.edu/8.13/www/handouts.shtml to both a lorentzian and Gaussian PDF's, this time with the possible addition of DC and linear background terms. What are the chi-squared values? Which fit hypothesis is justified ?



For each case:

1. Produce a plot of the data with error bars, assuming Poisson statistics.
2. Make an educated guess for initial values for a fit to Gaussians plus background.
3. Perform the fit and retain all values. Give a plot with the fit.
4. Make a subtraction plot Data – Fit, to show the residuals. Comments?
5. As a check repeat with different starting values. Compare the results.
6. Optional: Can you give confidence limits for each hypothesis?