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The Franck-Hertz Experiment and the Ramsauer-Townsend Effect: Elastic and Inelastic Scattering of Electrons by Atoms

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The main purpose of these experiments is to measure two phenomena encountered in collisions between electrons and atoms: inelastic scattering resulting in quantized excitation of the target atom, and ionization, resulting in the removal of an electron from the atom. In addition, the experiments provide an opportunity to explore the phenomena of thermionic emission of electrons and space charge limited current in a vacuum tube.

1. PREPARATORY QUESTIONS

1. Considering that the energy of the first excited state of the mercury atom is about 5.0 eV above that of the ground state, what is the maximum amount of energy that an electron with 4.0 electron volts of kinetic energy can impart to a mercury atom with which it collides? Same question for a 6.0 eV electron.
2. Prepare a plot of the mean free path (in mm) of electrons in mercury vapor against the temperature (in degrees centigrade) of the tube in the region of interest. Assume 1) that the cross section of a mercury atom for collision with an electron is geometrical (i.e. $\sigma = \pi R^2$ where R is the radius of the outer electronic shell of the mercury atom), and 2) that the mercury vapor in the tube is in thermodynamic equilibrium with liquid mercury. The vapor pressure of mercury is tabulated in [1].
3. Plot the expected curve of current against accelerating voltage, in the Franck-Hertz experiment with mercury vapor, showing the positions of the peaks and valleys on an absolute voltage scale.

2. INTRODUCTION

Franck and Hertz described the first observation of quantized excitation in 1914, one year after Bohr published his theory of the hydrogen atom with its concept of quantized energy states. They discovered that electrons moving through mercury vapor with an energy equal to or greater than a certain critical value near 4.9 eV can excite the 2,536Å line of the mercury spectrum. Electrons with less than the critical energy merely bounce elastically when they collide with mercury atoms and fail to excite any electromagnetic emission at all. The experiment provided crucial evidence in favor of the Bohr theory.

A version of the Franck-Hertz experiment, employing a mercury-filled vacuum tube with four electrodes made by the Leybold Company, is described in References [2, 3] to which the reader is referred for a discussion of the physical principles and the measurement objectives of the Franck-Hertz experiment. The present Junior Lab

version of the experiment uses a later version of the Leybold equipment which consists of a mercury-filled triode and an oven with glass windows through which one can view the action. A power supply and control circuit built in the Junior Lab shop provides adjustable filament voltage for heating the cathode, adjustable accelerating grid voltage, and an adjustable retarding voltage. The same triode tube and power supply are used in the measurement of the ionization potential.

3. SCATTERING OF ELECTRONS BY MERCURY ATOMS

The experiments on the excitation and ionization of mercury atoms are carried out with a mercury-filled triode which is a sealed glass envelope containing three electrodes and a drop of mercury.¹

The experiments will be performed with the three configurations of the tube and associated circuits illustrated in Figure 1 and in more detail in Figures 2, 3, and 6. The three electrodes are 1) a cathode which emits electrons when raised to a temperature of several hundred degrees by application of a voltage V_F to a heater filament, 2) a grid which is set at a positive potential V_{gc} relative to the cathode so that electrons emitted by the cathode are drawn toward it, and 3) an anode which is connected to an ultra-sensitive Keithley electrometer for the measurement of the anode current I_a .

All of the measurements in these experiments can be made with total tube currents (i.e. cathode current = anode + grid current) of 1000 nA or less. To prolong the useful life of the tube care should be taken not to exceed 1000 nA.

It is wise to start each session by disassembling whatever wired-up circuit the previous users may have left, and then rewiring it yourself. Only in this way can you know for sure how the circuit is connected.

¹ Mercury is an ideal element for a study of excitation phenomena because its vapor is monatomic and its vapor pressure can be readily controlled over the range useful for this experiment by adjusting the temperature in the range from room temperature to 200°C.

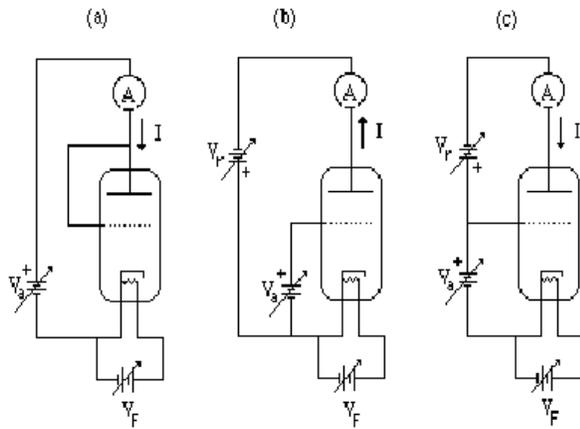


FIG. 1: Schematic illustrations of the mercury-filled triode in the three configuration used in measurements of (a) thermionic emission and space charge limited current, (b) the ionization potential of mercury, and (c) the critical excitation potential of mercury. Note that the arrows indicate the direction of positive current—electrons move in the opposite direction.

An ultra-sensitive Keithley Model 614 electrometer is used to measure the current from anode to laboratory ground. (**Be sure to check the zero set**). It is essential that the input cable be kept stationary during very sensitive measurements. Flexing or moving even high quality cable may generate currents due to the triboelectric effect. Make current measurements as follows:

1. Turn the Model 614 on and allow one hour warm up.
2. Depress ZERO CHECK
3. Select Volts function and the 0.2V range. Adjust the voltage zero potentiometer with a small screwdriver to obtain a displayed reading of 0.00000. Adjustment on this range assures a proper zero on all other functions and ranges.
4. Select Current Function, Units and Range
5. Apply input (be sure to use the BNC-Triaixial adapter!!) and release ZERO CHECK
6. Take reading
7. Overrange is indicated by flashing 0000 on the display

The electrometer has an analog output (2V full scale of display) in the back of its chassis which can be connected by coaxial cable with grounded sheath to the Y-input of an oscilloscope. A Keithley 177 voltmeter is used to measure the accelerating potential relative to laboratory ground. It too has an analog output in the back which can be connected to the X input of the oscilloscope. **Both of these meters have their chassis at laboratory ground potential and therefore can only be**

used to measure currents or voltages from a given point to laboratory ground. In most of the measurements neither the grid nor the cathode are at ground potential in which case the electronic voltmeter cannot register the accelerating potential directly. A separate isolated Fluke multimeter is provided to enable measurement of the potential difference between any two points.

You will have control of four parameters—the filament voltage V_F which controls the temperature of the cathode, the accelerating voltage V_{cg} between cathode and grid, the retarding voltage between the grid and the anode, and the oven temperature T which controls the vapor pressure, and hence, the density of the mercury vapor. Your first job is to explore the effects of filament voltage and oven temperature on the total current.

4. THERMIONIC EMISSION OF ELECTRONS AND SPACE CHARGE LIMITED CURRENT

Consider first the situation in the tube connected as shown in Figure 2. At room temperature the vapor pressure of the mercury is so low that the mean free path of electrons emitted by the cathode is large compared to the dimensions of the tube ($\approx 8\text{mm}$). The cathode, heated by the filament, emits electrons in a process called thermionic emission in which electrons near the top of the Fermi distribution in the metal penetrate the potential barrier at the surface and escape. The emitted electrons have a distribution of kinetic energies which is approximately a Maxwell-Boltzmann distribution with a mean energy E_0 near kT , where T is the cathode temperature. In the steady state, with the grid at a positive potential relative to the cathode the emitted electrons form a cloud of negative charge over the surface of the cathode. This cloud gives rise to a contribution to the electric field that tends to suppress the emission by forcing lower energy electrons back to the cathode, thereby causing a condition known as space charge limited current. The physics of thermionic emission and space charge limited current is discussed in [2] for a tube with ideal cylindrical geometry. Our tube, designed for other purposes, has a different geometry that does not lend itself to a “clean” experiment in thermionic emission. Nevertheless the qualitative characteristics of space-charge limited current can be observed with it.

The first part of the experiment is an exploration of the dependence of the total tube current on two quantities: (1) the accelerating potential between the electron emitting cathode and the grid and (2) the filament voltage which controls the temperature of the cathode and thus the emitted current density. The measurements are made at room temperature.

4.1. Procedure for Exploring Thermionic Emission

1. Connect the tube as shown in Figure 2 with the filament voltage set to its lowest value (~ 1.2 V). In order to measure the TOTAL tube current, the grid and anode currents should be summed at the electrometer input with coaxial cables and a ‘BNC-T’ connector. Use a two-prong banana plug-to-BNC adaptor at the grid terminal on the oven taking care as to which side of the banana plug adaptor is connected to the grid’s banana jack.
2. Set the accelerating voltage control so that V_{cg} , the voltage of the cathode relative to the grid (which is grounded through the electrometer), is about -60 V. Set the retarding voltage control so that $V_r=0$. Increase V_F in small steps until the total current is between 100 and 1000 nA. Tabulate and plot the total current (grid + anode) I_{ag} versus V_F as you proceed.
3. Set V_F at a value for which I_{ag} is in the range 100 - 1000 nA when $V_{cg} = -60$ V. Now, tabulate and plot I_{ag} as a function of V_{cg} , shifting scales on the electrometer as needed. You may find that the current drifts substantially at any given setting of V_F and V_{cg} . Don’t panic. **The purpose of this section is to obtain a rough idea of how the cathode current, i.e. the total tube current, is affected by the cathode temperature and the accelerating voltage. Don’t spend a lot of time trying to perfect these particular data. Since the tube characteristics change significantly with usage, you should start each session with a similar rough check of I_{ag} versus V_F with V_{cg} set to ~ -60 V and the oven at room temperature.**

5. IONIZATION POTENTIAL OF MERCURY

In this next experiment the tube is connected as shown in Figure 3 so that the anode is maintained at a negative potential of a couple of volts with respect to the cathode. In this condition it is energetically impossible for any electrons to reach the anode. When the grid potential is made positive with respect to the cathode, electrons are accelerated to the grid. Some pass through the grid into the space between the grid and anode, but all are eventually pulled back and collected by the grid. However, if the electrons that enter the space between the grid and anode have sufficient energy to ionize the mercury atoms, then the resulting positive mercury ions are drawn to the anode and a positive current is registered by the electrometer. The experiment consists of measuring the anode current as a function of the grid to cathode potential difference. Analysis of the current-voltage relation yields a measure of the ionization potential.

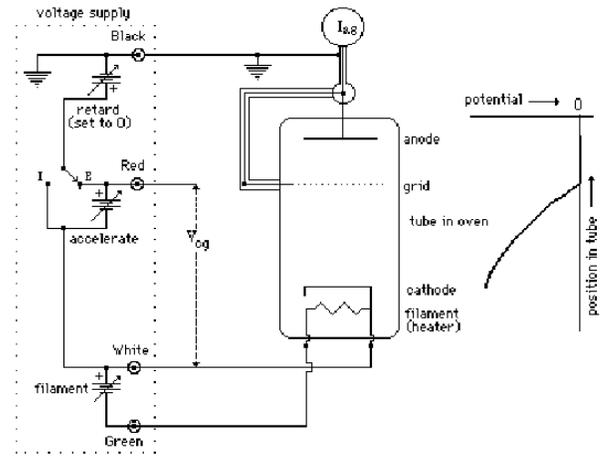


FIG. 2: Schematic diagram of the tube and circuit for measurements of space charge limited current from thermionic emission. The variation of potential with position in the tube is indicated in the plot at the right. For this measurement the toggle switch should be set to the ‘E’ (excitation) position and the ‘retarding’ voltage control should be set to zero. The colors of the external banana plug female connectors are indicated in the diagram. The voltmeter labeled V_{cg} is the Keithley electronic voltmeter connected so as to measure the accelerating voltage between the cathode and grid. **Note that the point marked ‘RED’ in this diagram is at ground potential and therefore must be connected to the bottom input connector on the Keithley. The meter will read a negative voltage for the accelerating potential.**

The first task is to determine the optimum oven temperature for measurement of the ionization effect. If the vapor pressure of the mercury is too low, then electrons entering the grid-to-anode region with sufficient energy to cause ionization will have a small probability of collision (long mean free path) and the ion current collected by the anode will be too small for accurate measurement. If the vapor pressure is too high, then electrons will suffer inelastic collisions in the space between the cathode and grid as soon as their energies slightly exceed the energy required to raise mercury atoms to their first excited (not ionized) state and will therefore never attain sufficient energies to cause ionization. In the Junior Lab apparatus, the distance from the cathode to the perforated anode (grid) is 8mm while the distance between the perforated anode and the counter electrode plate is small (of order 1mm).

5.1. Ionization Potential: Procedure

1. Connect the tube as shown in Figure 3.
2. Set the filament voltage at the value which gives a total tube current in the range 100-1000 nA when $V_{cg} = \sim -60$ V, as determined in section 4 above.

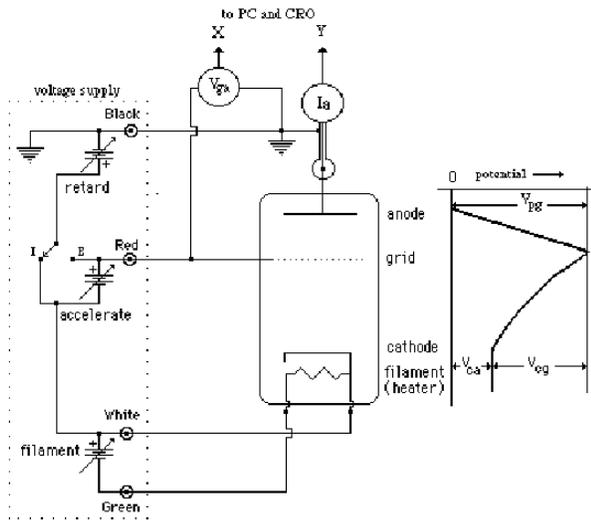


FIG. 3: Schematic diagram of the mercury-filled triode in the configuration for measurement of the ionization potential of mercury. After you complete the connections check all the potential differences in your setup with the floating Fluke voltmeter and verify that they are consistent with the voltage plot shown in the figure. Note that the mode-selecting toggle switch should be in the 'Ionization' position.

3. Set the accelerating voltage V_{cg} to 15 V (measure with the floating voltmeter connected between cathode and grid), which is a few volts above the ionization potential.
4. Set the retarding voltage so the cathode potential is 2 V above the anode potential.
5. Measure and plot the anode current as a function of oven temperature between 30 and 130 degrees Celsius in steps of 10 degrees and then repeat as it cools down back to room temperature. (**DO NOT EXCEED 200°C**). Explain what you observe in terms of the effect of temperature on the mean free path of the electrons. Discuss the time constants involved in the determination and setting of the desired operating temperature and how you might optimize this in future sessions. In light of your results select an optimum temperature for the following measurement of the ionization potential and adjust the variac so as to maintain that temperature. When you repeat the experiment in later sessions, you might consider varying the oven temperature to measure its effect.
6. Set the variac at the optimal temperature setting and tabulate and plot by hand the anode current I_a as a function of the accelerating voltage V_{cg} , using the picoampere and nanoampere scales. **To avoid damaging the tube by a runaway discharge do not exceed an accelerating voltage of 15 V in this part of the experiment.**

5.2. Ionization Potential: Analysis

Estimate the value and uncertainty of V_{cg} when ionization just begins in the region between the grid and the anode. Using results from the subsequent experiment on the excitation potential, you will be able to correct your result for the effect of the contact potential.

6. THE EXCITATION POTENTIAL OF MERCURY

Consider, now, the following idealized description of the excitation potential experiment. We make the following assumptions (all inaccurate in varying degrees as the previous experiments have shown):

- All electrons emerge from the cathode with zero velocity.
- The number of electrons emerging from the cathode is unaffected by the accelerating field between the cathode and grid.
- The energy of an electron is unaffected by collisions with the mercury atoms as long as their energy is below that of the first excited state. Therefore all electrons at a given position between the cathode and grid have the same energy.
- All electrons that pass through the grid with energies less than the retarding potential between the grid and anode are pulled back and collected by the grid; all electrons with energy more than the retarding potential are collected by the anode.
- The inelastic cross section of mercury for electrons with energies above the critical value is so large that such electrons travel a negligible distance before suffering inelastic collision.

Were all these assumptions valid, a plot of anode current I_a against accelerating voltage V_{cg} would appear as shown in Figure 4. The separation between successive current increases (or decreases) is equal to the excitation potential of the first excited state of the Hg atom. The width of the minima is determined by the retarding potential between the grid and the anode. The difference between the accelerating voltage at the first drop and the excitation potential is the difference between the work functions of the cathode and grid metals, i.e. the contact potential between the two metals. In reality the I_a versus V_{cg} curve will depart considerably from the ideal for a variety of reasons.

Figure 5 illustrates the electric field with the voltages set for the measurement of the excitation potential in the idealized case of plane parallel electrodes. Also shown are the trajectories of two electrons, each interrupted by elastic and inelastic collisions. At gas pressures so low

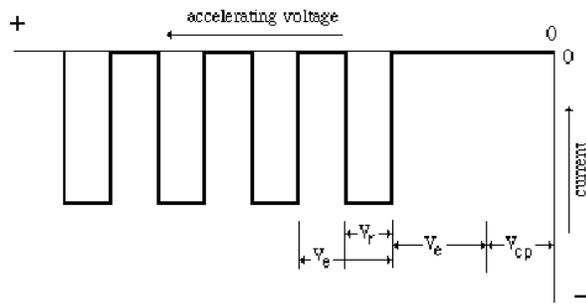


FIG. 4: Plot of anode current against accelerating voltage V_{cg} implied by set of idealized assumptions.

that the mean free path of the electrons is large compared with all relevant distances the motion is ballistic and the fraction of the electron current intercepted by the grid is approximately the fraction of area covered by the fine grid wires. At gas pressure so high that the mean free path is short compared with all dimensions, including the wire diameter the electron orbits jitter along the field lines. Regardless of their energy they are mostly intercepted by the grid since most of the field lines end there.

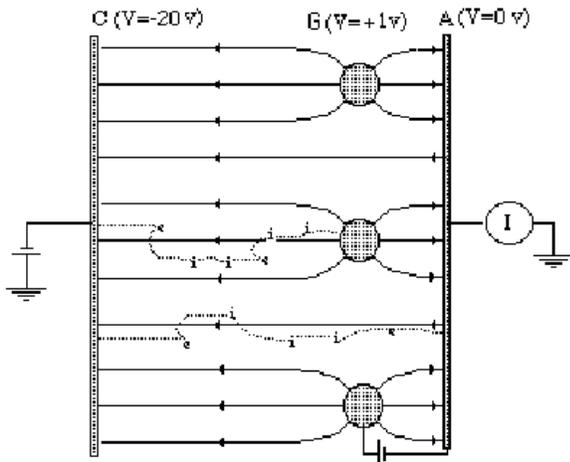


FIG. 5: Schematic illustration of the electric field lines in the measurement of the excitation potential. Two electron trajectories are illustrated. Elastic and inelastic scatterings are indicated by “e” and “i”, respectively. One electron is caught by the grid G. The other makes it to the anode A.

Note that the lowest excited state of mercury is the 6^3P_0 state, which is metastable, i.e. dipole transition to the ground state is forbidden. Its lifetime is $\sim 10^5$ times that of ordinary allowed transitions. As the electrons accelerate toward the grid they reach a position where they have sufficient energies to excite the 6^3P_0 state, and will do so. Because the excited atoms decay slowly, their number builds up to the point where most of the electron-mercury atom interactions become elastic collisions off of excited atoms. The 6^3P_1 state is not metastable and de-

cays immediately by emission of a 2536 \AA photon. Franck and Hertz detected these photons through a window in their tube. Having measured the threshold voltage V at which the photons appeared, they could derive a value for $h = Ve\lambda/c$. The agreement with the values obtained previously by Planck, Einstein, and Bohr from their theories of the blackbody spectrum, the photoelectric effect, and the hydrogen spectrum, respectively, was a striking and historic confirmation of the new quantum theory.

6.1. Excitation Potential of Mercury: Procedure

1. For the excitation potential measurement connect the tube as shown in Figure 6. Check all the potentials with the floating voltmeter.
2. Set the oven temperature at so that the mean free path of the electrons is short compared to the cathode-grid distance.
3. Set the filament voltage to a value that would give a total tube saturation current (anode + grid) in the range 100-1000 nA at room temperature, as measured above in section 4.
4. Set the retarding voltage at a value between 0.1 and 2 volts. Electrons that pass through the grid with less than enough energy to overcome the retarding potential are turned back to be collected by the grid. Higher energy electrons reach the anode and are registered as a negative current by the Keithley electrometer.
5. By hand or by using the ramp function, sweep the accelerating voltage, V_{cg} , from 0 to 60 volts and note the occurrence of maxima and minima in the anode current I_a (probably best done on the picoampere scale). Using the Fluke multimeter, measure and tabulate the values of V_{cg} at the maximum and minimum values of the current, and the corresponding maximum and minimum anode currents. The electrometer must be connected between the source of current and laboratory ground; since the resistance of the electrometer is very small, the anode potential remains very close to ground potential. In the measurement of both the excitation and ionization potentials the accelerating voltage is between the cathode and grid. Neither of these electrodes is at ground potential. In the excitation potential determination (i.e. the Franck-Hertz experiment) the Keithley electronic voltmeter is arranged to record the anode-to-cathode voltage. The accelerating potential can be measured directly with a floating (Fluke) voltmeter connected between cathode and grid.

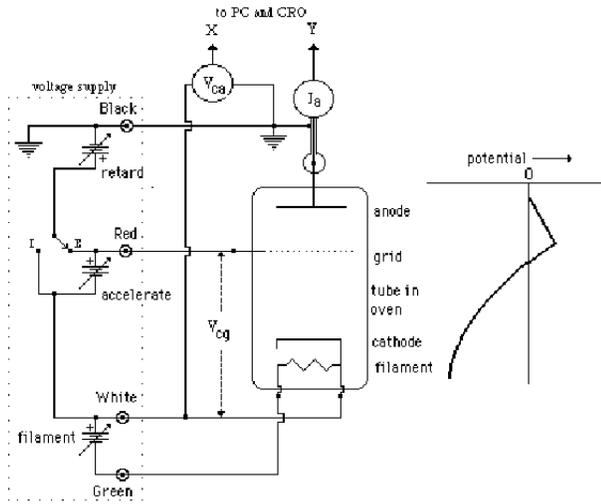


FIG. 6: Circuit arrangement for measurement of the critical excitation potential.

6.2. Excitation Potential: Analysis

Derive from your data the value and error of the excitation potential. Referring to the plot of I_a vs. V_{cg} , at what point between each successive pair of minima do you think the next set of excitation events begins? Derive an estimate of the value and error of the contact potential (see Melissinos for a discussion of the physics of contact potential) between the cathode and grid. You will need to correct the measurement of the ionization potential in section 5 by this quantity.

7. THE RAMSAUER-TOWNSEND EFFECT

“The Ramsauer-Townsend effect may be thought of as a diffraction of the electron around the rare-gas atom, in which the wave function inside the atom is distorted in just such a way that it fits on smoothly to an undistorted wave function outside” [4].

The result is a near extinction of the scattered S partial wave and a consequent suppression of the scattering cross section. The effect can be observed only when it occurs at an energy below that of the first excited state of the scattering atom. Otherwise it is masked by inelastic scattering. Noble gases provide specially suitable targets because they are monatomic, their outer electrons are in a closed shell, excitation of their lowest excited state requires a comparatively large amount of energy, and they do not attach electrons to form negative ions.

The Ramsauer-Townsend effect was discovered in 1921, before the invention of wave mechanics in 1926 and the discovery of the electron diffraction by Davisson and Germer in 1927. Its explanation in terms of the scattering of matter waves, first suggested by Bohr and worked out

in detail by Mott and Massey[5] and others on the basis of Schrödinger’s equation, was one of the early successes of the new wave mechanics.

The theoretical treatment of the elastic scattering of an electron by a noble gas atom requires the solution of the three dimensional Schrödinger equation for a free electron incident on a spherically symmetric attractive potential. This can be done by the method of partial waves as explained in various intermediate texts on quantum mechanics (e.g. [4, 6–8])

The well-known model of a 1-dimensional, scattering problem, solved in many elementary quantum mechanics texts, is often put forward as exhibiting the essential features of wave mechanical behavior that one observes in the Ramsauer-Townsend effect. The 1-D model bears a relation to the real 3-D scattering problem that is similar to the relation which the problem of the energy levels of a particle in a 1-D box bears to the 3-D problem of the hydrogen atom, i.e. suggestive, but fairly distant as far as numerical agreement is concerned. In the 1-D scattering problem one finds the transmission coefficient has maxima at a series of discrete energies for both positive and negative potential wells, whereas in the 3-D case there is only one maximum, and that only for an attractive potential.

The present experiment is described in Reference [9]. A xenon-filled tube is commercially available in the form of a thyratron switch tube (2D21). The depression of the scattering cross section at the critical energy is indicated by the reduction in the intensity of an electron beam directed from the cathode to the anode.

The apparatus and circuitry are illustrated schematically in Figure 7. Electrons emitted from the heated cathode are accelerated through a small potential difference toward the “grid” which is actually a metal box inside of which there is no electric field. The electrons that enter the box “drift” through it, and those that emerge from the far end are collected by the anode and measured as a current by the Keithley electrometer. At high vacuum, when the xenon gas is condensed on the glass walls of the thyratron by cooling in liquid nitrogen, the fraction of atoms emitted by the cathode that reaches the anode depends only on the geometry of the electrodes. When the xenon is vaporized, the fraction reaching the anode is reduced by elastic scatterings which deflect electrons from the trajectories that would otherwise bring them to the anode. The amount of the reduction is a measure of the elastic scattering cross section of the xenon atoms. One obtains the required data by measuring the anode current as a function of the accelerating voltage both with and without the xenon vapor.

7.1. Ramsauer-Townsend effect in xenon: Procedure

1. With all voltages set to their minimum values connect the circuit as shown in Figure 7, using the

Keithley electrometer to measure the anode current and the Fluke multimeter to measure the accelerating voltage.

2. Dip the tip of the thyatron in liquid nitrogen to freeze out the xenon. **(To reduce the likelihood of cracking the tube by thermal shock the filament voltage should always be zero when the tube is dipped into the liquid nitrogen.)**
3. With an accelerating voltage of 5 volts increase the filament heater voltage until the anode current is $10 \mu\text{A}$.
4. Leaving the filament voltage at the value just found, tabulate and plot the anode current (shifting to the nanoampere or picoampere scales where necessary) as a function of the accelerating voltage from 5V down to zero both with and without the xenon vapor frozen out, taking special care in the region between 0 and 2V where the interesting phenomenon occurs. It is wise to make your two sets of measurements at the same sequence of voltage values so that you can easily calculate the ratio of currents with and without the xenon frozen out. Compute and plot in your lab book the ratio against the accelerating voltage as you go along to monitor the trend of the data.

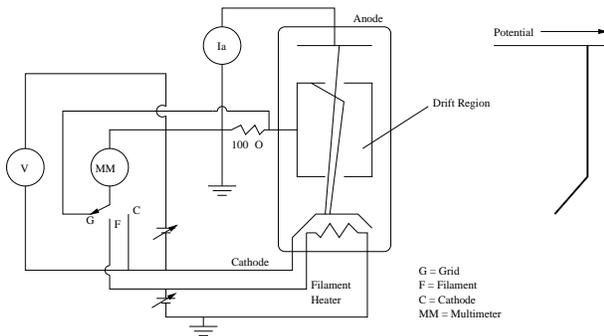


FIG. 7: Schematic diagram of the 2D21 thyatron and circuit used in the measurement of the xenon elastic scattering cross section using low energy electrons. The trajectories of two electrons are shown, one scattered by a Xenon atom and the other reaching the anode undeflected. Adjust the filament voltage so that the anode current measures $\approx 10 \mu\text{A}$ with an accelerating voltage of 5.0 V. The vapor pressure in the tube is approximately 0.05 Torr at room temperature (≈ 0.001 Torr at 77K) and the length of the scattering volume is 0.7cm.

The liquid nitrogen needed to freeze out the xenon is available. Handle it carefully and wear goggles! Do not fill the Dewar flask to the brim! The tube need only be partially immersed to provide a cold place at which the gas can condense.

7.2. Ramsauer-Townsend: Analysis

The discussion about interpretation of the experiment in the article of Kukolich seems a little obscure. The following remarks may help. Call $I_0(V)$ the measured current with the xenon frozen out when the accelerating potential is V , and $I(V)$ the measured current at the same accelerating voltage with the xenon gas free in the tube. If every electron scattered by a xenon atom were effectively removed from the beam, then these two measurements would be related by the equation

$$I(V) = I_0(V)e^{[-n\sigma(V)l]} \quad (1)$$

where n is the number of xenon atoms cm^{-3} , $\sigma(V)$ is the cross section of the xenon atoms for inelastic scattering the electrons of energy eV, and l is the path length from cathode to anode. The desired measure of the cross section would then be

$$\sigma(V) = \frac{1}{nl} \ln \left[\frac{I_0(V)}{I(V)} \right] \quad (2)$$

Actually, not every scattered electron is removed from the beam due to the poor geometry of the thyatron tube in this particular application. Therefore the value of the cross section derived from the simple analysis of the present experiment, assuming the quantity nl were accurately known, is somewhat less than the true value of the total inelastic scattering cross section.

Plot $\ln \left[\frac{I_0(V)}{I(V)} \right]$ against V . Discuss the sources and magnitudes of the errors and indicate your error estimates on the plot. Comment on the position of the minimum and its relation to the size of the xenon atom.

7.3. Possible Theoretical Topics for Oral Exam

- Richardson's law.
- Child's law.
- Distribution in energy of electrons emitted by a hot metal.
- Propagation of electrons through the Hg vapor.
- Transmission of electrons through a rectangular potential well.
- Partial-wave scattering theory of the Ramsauer-Townsend effect.

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- [1] M. Vukalovich and L. Fokin, *CRC Handbook of Chemistry and Physics: Vapor Pressure of Mercury* (CRC Press, 2003-2004), chap. 6, p. 147, 84th ed., available on-line at <http://www.hbcernetbase.com/>.
 - [2] A. Melissinos, *Experiments in Modern Physics* (Academic Press Inc., 1966).
 - [3] A. Melissinos, *Experiments in Modern Physics* (Academic Press Inc., 2003), 2nd ed.
 - [4] L. Schiff, *Quantum Mechanics* (McGraw-Hill, 1950).
 - [5] Mott and Massey, *The Theory of Atomic Collisions* (Oxford University Press, 1971), chap. 18, 3rd ed.
 - [6] D. Bohm, *Quantum Theory* (Prentice Hall, 1951).
 - [7] J. McGervey, *Introduction to Modern Physics* (Academic Press, 1971).
 - [8] R. Dicke and J. Wittke, *Introduction to Quantum Mechanics* (Addison-Wesley, 1960).
 - [9] S. Kukolich, *Am. J. Phys.* **36**, 701 (1968).