

# Classical Mechanics III (8.09) Fall 2014

## Assignment 8

Massachusetts Institute of Technology  
Physics Department  
Mon. November 10, 2014

*Due Mon. November 17, 2014  
6:00pm*

### **Announcements**

This week we will continue our study of fluids, now including viscosity.

### **Reading Assignment**

- For ideal fluids and sound waves, read sections 8.6–8.10, 8.13, and 8.14 from the Mechanics book by **Symon** (scanned and available on the 8.09 website).
- For fluids with viscosity the readings are from Landau and Lifshitz, Fluid Mechanics, Chapter II, sections 15-17 and 19-20 (which will be scanned and available on the 8.09 website).
- Further reading on fluids, including lots of worked examples and plenty of problems, can be found in the book “A physical introduction to Fluid Mechanics” by Alexander Smits, which is on reserve in the physics reading room.

## Problem Set 8

In these 6 problems we explore both ideal and viscous fluids. The last two problems use dimensional analysis.

### 1. Flow Geometries [12 points]

The velocity potential for certain potential flows of an ideal fluid is given in cylindrical coordinates  $(r, \theta, z)$  by a function  $\phi = \phi(r, \theta)$ . Find the geometry of a fixed surface that is consistent with each of these potential flows [4 points for each]:

- (a)  $\phi = Cr^2 \cos(2\theta)$
- (b)  $\phi = Cr^4 \cos(4\theta)$
- (c)  $\phi = Cr^{2/3} \cos\left(\frac{2\theta}{3}\right)$

Identify any stagnation points at finite  $r$ , and roughly sketch a streamline near the surface for each case.

### 2. Ideal Fluid Flow around a Cylinder [16 points]

Consider an infinitely long solid cylinder of radius  $R$  with its axis along  $\hat{z}$ , which is immersed in an incompressible irrotational ideal fluid of density  $\rho$ . At  $x = -\infty$  the fluid flows with a constant velocity  $\vec{u} = u\hat{x}$ .

- (a) [6 points] Using Laplace's equation find the steady potential flow around the cylinder. Solve for the velocity field  $\vec{v}$ . Be sure that you satisfy the boundary conditions.
- (b) [4 points] Derive a result for the pressure on the cylinder.
- (c) [6 points] Derive a differential equation for the streamlines for this flow. Show that  $x^2 + y^2 = R^2/(1 - K/y)$  is a solution of your equation, where  $K$  is an arbitrary constant.

### 3. A Spherical Sound Wave [10 points]

A sound wave is generated in air by a point source.

- (a) [3 points] Show by direct computation that the spherical pressure wave

$$p = \frac{1}{r} f(r - c_s t)$$

satisfies the sound wave equation for any differentiable function  $f$ .

- (b) [4 points] Find the corresponding velocity field  $\vec{v}$ . To facilitate grouping the time dependence of your solution into the combination  $r - c_s t$ , let  $g(x) = \int^x dx' f(x')$ .
- (c) [3 points] Cross check your answer in (b) by verifying that your  $\vec{v}$  also satisfies the wave equation. Note that in spherical coordinates the Laplacian on a vector field  $\nabla^2(\phi(r)\hat{r}) = [\nabla^2\phi(r)]\hat{r} - \frac{2}{r^2}\phi(r)\hat{r}$ .

**4. Viscous Fluid Velocity between Coaxial Cylinders** [10 points]

Two very long cylinders are coaxial with  $\hat{z}$ . The inner cylinder is held at rest and has a radius  $R_1$ . The outer cylinder has radius  $R_2$  and moves parallel to its axis with a constant velocity  $u\hat{z}$ . A viscous fluid between the two cylinders exhibits a steady flow, and there is no pressure gradient in the  $z$  direction. What is the velocity profile of the fluid? What is the friction force per unit length for each cylinder?

**5. Vortex Shedding with Dimensional Analysis** [6 points]

Wind blows over a cylindrical chimney and vortices are shed into the wake. The frequency of vortex shedding  $f$  depends on the chimney diameter  $D$ , its length  $L$ , the wind velocity  $V$ , and the kinematic viscosity of air  $\nu$ .

- (a) [3 points] Use dimensional analysis to write down a general functional relation for  $f$  in terms of the other parameters.
- (b) [1 point] If on another day we observe a second chimney whose diameter and length are  $1/2$  those of the original one, then what must the velocity of the wind be for there to be a chance to use a similarity analysis for the two chimneys?
- (c) [2 points] Taking the velocity of wind for the second chimney as in (b), what is the frequency of vortex shedding for the second chimney relative to the first?

**6. Golf Ball Drag with Dimensional Analysis** [6 points]

The drag force  $F_D$  on a golf ball depends on its velocity  $V$ , its diameter  $D$ , its spin rate  $\omega$  measured in radians/sec, the air density  $\rho$  and air viscosity  $\eta$ , and the speed of sound  $c_s$ .

- (a) [3 points] Use dimensional analysis to find a general functional relation for  $F_D$  in terms of the other parameters.
- (b) [3 points] Suppose that the speed of sound is not important. How does your result in (a) change? With this assumption, if an experiment is carried out with air in a wind tunnel at velocity  $2V$ , what diameter and spin rate are needed for the results to be dynamically similar to those for the velocity  $V$ ?

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