Welcome back to 8.033!

Image courtesy of Wikipedia.

Albert A. Michelson,
1852-1931

1st American to win
Nobel Prize (1907)

SUMMARY OF LAST LECTURE: SYMMETRY IN PHYSICS, I:

- **Key concepts:** frame, inertial frame, transformation, invariant, invariance, symmetry, relativity
- **Symmetry examples:** translation, rotation, parity, boost
- **Million Dollar question:** what are the symmetries of physics?

TODAY'S TOPIC: SYMMETRY IN PHYSICS, II

• Symmetry of electromagnetism (wave equation, light propagation)

• Does speed of light depend on wavelength, motion of source or motion of observer?

• How reconcile 8.01 with 8.02?

• How transform between inertial frames?

• **Key people:** Michaelson & Morley

WHAT’S THE SYMMETRY OF CLASSICAL MECHANICS?

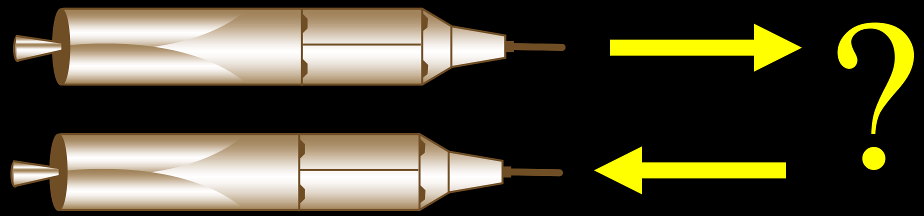

Figure by MIT OCW.

ANSWER:

- Translation:

$$\begin{cases} \mathbf{r}' = \mathbf{r} + \Delta r \\ t' = t + \Delta t \end{cases}$$

- Rotation:

$$\begin{cases} \mathbf{r}' = \mathbf{R}\mathbf{r} \\ t' = t \end{cases}$$

- Galilean:

$$\begin{cases} \mathbf{r}' = \mathbf{r} + \mathbf{v}t \\ t' = t \end{cases}$$

- Combined:

$$\begin{cases} \mathbf{r}' = \mathbf{R}\mathbf{r} + \Delta\mathbf{r} + \mathbf{v}t \\ t' = t + \Delta t \end{cases}$$

WHAT'S THE SYMMETRY OF ELECTRO-MAGNETISM?

The classical wave equation

- Classical wave equation (8.03):

$$\nabla^2 \mathbf{E} - \frac{1}{c_w^2}\ddot{\mathbf{E}} = 0.$$

  For example, E could denote:

  - One of the three component of the electric field
  - One of the three component of the magnetic field
  - Air density
  - Height of water surface (2D)
  - Deflection of guitar string (1D)

- 1D special case:

$$\frac{d^2E}{dx^2} - \frac{1}{c_w^2}\frac{d^2E}{dt^2} = 0.$$

- General solution (show on PS2):

$$y = Af(x - c_w t) + Bf(x + c_w t),$$

  for arbitrary smooth function f and constants A & B.

- More complicated in 3D, but wavefronts still propagate with speed c_w.

Transforming the wave equation

- In the last lecture, we learned that classical mechanics was invariant under Galilean transformations.
- The wave equation can be derived from classical mechanics.

Question: is the classical wave equation invariant under Galilean transformations?

1. Yes
2. No
3. Yes, but only if wave speed $c_w \ll c$

Transforming the wave equation

- Apply Galilean transformation to 1D wave equation:

$$\frac{d^2E}{dx^2} - \frac{1}{c^2}\frac{d^2E}{dt^2} = 0.$$

- Do this on PS2 - hints:
  - $x' = x + vt$
  - $t' = t$
  - Use chain rule for derivatives:

$$\begin{aligned}\frac{\partial}{\partial x} &= \frac{\partial x'}{\partial x}\frac{\partial}{\partial x'} + \frac{\partial t'}{\partial x}\frac{\partial}{\partial t'} = \frac{\partial}{\partial x'} \\ \frac{\partial}{\partial t} &= \frac{\partial x'}{\partial t}\frac{\partial}{\partial x'} + \frac{\partial t'}{\partial t}\frac{\partial}{\partial t'} = v\frac{\partial}{\partial x'} + \frac{\partial}{\partial t'}\end{aligned}$$

  - Work out 2nd derivatives too

- Result:

$$\left(1 - \frac{v^2}{c^2}\right)\frac{d^2E}{dx'^2} - \frac{1}{c^2}\frac{d^2E}{dt'^2} + 2\frac{v}{c}\frac{d^2E}{x'dt'} = 0.$$

- Wave equation *not* invariant under Galilean transformation

- Show on PS2: the new equation has solution

$$y = Af(x - c't) + Bf(x + c't),$$

  where $c' = c + v$ or $c - v$.

- Just what you'd expect for waves in a substance, "aether" (velocities add).

- How can this be consistent with the wave equation being derived from classical mechanics, which is Galilean invariant?

SO WHICH DO YOU TRUST MORE:

Classical mechanics, or E&M?

Observed properties of speed of light

- Does speed depend on wavelength?
- Does speed depend on motion of source?
- Does speed depend on motion of observer (frame)?

Does c depend on wavelength?

- Does light speed through glass depend on wavelength?
- But what about light speed through vacuum?
- Gamma-ray bursts provide great test
- Gamma-ray bursts last a few seconds to minutes
- Old speculations: nefarious nukes, civilization annihilation, nearby neutron stars
- Recently shown to originate at cosmological distances (few billion light years $\sim 10^{17}$ light-seconds).
- Flash seen also at x-rays and optical wavelengths, all within of order a minute $\sim 10^2$ seconds, so
$$\frac{\Delta t}{t} \lesssim \frac{10^2 \mathrm{s}}{10^{17} \mathrm{s}} = 10^{-15}.$$
- $c = d/t$, so relative speed variation with wavelength is
$$\frac{\Delta c}{c} \approx \frac{\Delta t}{t} \lesssim 10^{-15}.$$

Answer:

No, at least not more than about $10^{-15} c \approx 300$ nm/s.

Does c depend on source motion?

- Does speed of a bullet depend on speed of rifle?
- Does sound speed of a gun shot depend on speed of rifle?
- Binary stars provide great test
- If velocities add, then

$$\begin{aligned} t_1 &= \frac{d}{c-v} \\ t_2 &= \frac{d}{c+v} \\ \Delta t &\equiv t_1 - t_2 = \frac{2dv}{c^2 - v^2} \approx 2\frac{d}{c}\frac{v}{c} = 2t\frac{v}{c} \approx 200 \text{ years, say} \end{aligned}$$

  (for a pulsar in the Large Magellanic Cloud with $v = 300$ km/s, $d = 100000$ lightyears)

- But half an orbit takes only 2 days, say
- You'd see new "Doppler effect" $\propto a$ rather than v
- You'd see things moving backward in time whenever $a > \frac{c^2}{d}$ towards you

Answer:

No dependence on source motion observed (and should be dramatic)

Does c depend on observer motion (frame)?

Albert A. Michelson, 1852-1931

Edward Williams Morley, 1838-1923

1st American to win Nobel Prize (1907)

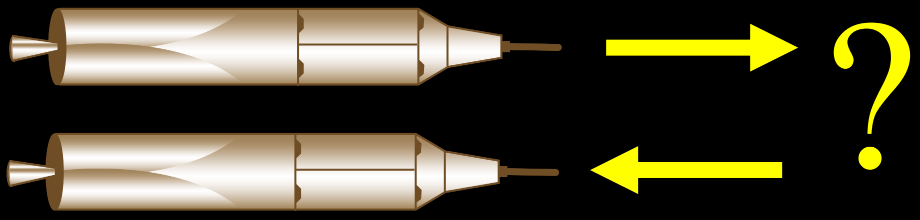

Figure by MIT OCW.

Does c depend on observer motion (frame)?

- No 1st order effect had been seen
- Michelson-Morley experiment hammered it - let's see how
- Consider interferometer moving with velocity $\mathbf{v}$ w.r.t. aether and compute round trip flight times parallel ($t_\parallel$) and perpendicular ($t_\perp$) to $\mathbf{v}$.
- For light traveling in direction $\pm\mathbf{v}$,

$$
\begin{aligned}
ct_\pm &= L_\parallel \pm vt_\pm \\
t_\pm &= \frac{L_\parallel}{c \mp v} \\
t_\parallel &= t_+ + t_- = \frac{L_\parallel}{c-v} + \frac{L_\parallel}{c+v} = \frac{2L_\parallel}{c}\gamma^2,
\end{aligned}
$$

where we have defined the quantity

$$
\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}.
$$

- For light traveling perpendicularly to $\mathbf{v}$,

$$
\begin{aligned}
(ct_\perp/2)^2 &= \sqrt{L_\perp^2 + (vt_\perp/2)^2} \\
t_\perp &= \frac{2L_\perp}{c}\gamma
\end{aligned}
$$

- The difference is

$$\Delta t \equiv t_\perp - t_\parallel = \frac{2L_\perp}{c}\gamma - \frac{2L_\parallel}{c}\gamma^2$$

- Rotating the interferometer by 90° changes this to

$$\Delta t' = \frac{2L_\perp}{c}\gamma^2 - \frac{2L_\parallel}{c}\gamma,$$

- *i.e.*, changes it by an amount

$$\Delta t' - \Delta t = \frac{2L_\perp}{c}\gamma^2 - \frac{2L_\parallel}{c}\gamma - \frac{2L_\perp}{c}\gamma + \frac{2L_\parallel}{c}\gamma^2 = 2\gamma(\gamma-1)\frac{L_\parallel + L_\perp}{c}.$$

- To lowest order in v/c, we have

$$\begin{aligned}
\gamma &\approx 1 + \left(\frac{v}{c}\right)^2 \\
\Delta t' - \Delta t &\approx \frac{L_\parallel + L_\perp}{c}\left(\frac{v}{c}\right)^2, \\
\frac{\Delta t' - \Delta t}{t} &\approx \left(\frac{v}{c}\right)^2.
\end{aligned}$$

- $v \approx 30$ km/s, so $(v/c)^2 \sim 10^{-8}$ — tough to measure!
- But their $L_\parallel + L_\perp = 11$m was about 2×10^7 wavelengths $\lambda \sim 500$nm, and they could see fringe shifts as small as 0.01λ.
- But they saw no fringe shift at all! So c appears *not* to depend on frame.

Observed properties of speed of light

- Does speed depend on wavelength?
- Does speed depend on motion of source?
- Does speed depend on motion of observer (frame)?

No, no and no!

Aether rescue attempts (see Resnick Table 1-2)

$$\Delta t \equiv t_{\perp} - t_{\parallel} = \frac{2L_{\perp}}{c}\gamma - \frac{2L_{\parallel}}{c}\gamma^2$$

Aether rescue attempts (see Resnick Table 1-2)

- Lorentz-Fitzgerald contraction: $L_{\parallel}$ contracts to $L_{\parallel}/\gamma$.
- Ruled out by Kennedy & Thorndike (1932) using interferometer with $L_{\parallel} \neq L_{\perp}$
- Aether drag hypothesis
- Ruled out by stellar aberration
- Also by light propagation in moving water (Fizeau 1851)
- Emission theories (v depends on source speed)
- Ruled out by binary stars (above)
- Also ruled out by Michelson-Morley with extraterrestrial light
- Also ruled out by measuring speed of γ-rays from CERN particle decays

Quantity	Invariance		
	Translational?	Rotational?	Gallilian?
t	N	Y	Y
$\mathbf{r}$	N	N	N
Δt	Y	Y	Y
$\Delta \mathbf{r}$	Y	N	Y
$\|\Delta \mathbf{r}\|$	Y	Y	Y
d/dt	Y	Y	Y
∇	Y	N	Y
∇^2	Y	Y	Y
$\mathbf{v}$	Y	N	N
$\mathbf{p}$	Y	N	N
$\mathbf{a}$	Y	N	Y
$\mathbf{F}$	Y	N	Y
m	Y	Y	Y
E_{kin}	Y	Y	N
W	Y	Y	N
$\mathbf{F} = m\mathbf{a}$	Y	Y	N
Newt. mechanics	Y	Y	Y
Electromagnetism	Y	Y	N

We've seen that classical mechanics is invariant under Galilean transformations but electromagnetism isn't.

Question: What is wrong?

1. The idea that all inertial frames are equivalent
2. Our theory of mechanics (8.01)
3. Our theory electromagnetism (8.02)
4. Nothing, because of Bohr's complementarity principle

What are we to make of this?

- Parity symmetry applied to some things, not others.
- Is it the same with galilean symmetry?
- **An experimental question:** Is physics the same in all inertial frames?
- A: Experiments suggest **YES**, both for mechanics and electromagnetism

- **A theoretical question:** How describe this invariance mathematically, *i.e.*,
  what is the tranformation law that leaves physics invariant?
- Galilean transformation? Works for mechanics but fails for $E\&M$
- Lorentz transformation? Works for $E\&M$ (PS3) but fails for mechanics
- No transformation works for both $E\&M$ and mechanics
- So at least one of the two must be wrong!
- Changing $E\&M$ to be have Galilean invariance is experimentally ruled out
- So let's try changing mechanics to be Lorentz invariant!
- BINGO! Not only OK with old experiments, but triumphed with new ones.