Welcome back to 8.033!

James Clerk Maxwell,

1831 - 1879

Image courtesy of Wikipedia.

Less energy ↑

Summary of last two lectures:

Common interaction processes

- Chemical reactions: atoms get rearranged in new ways, perhaps emitting or absorbing photons and electrons. Non-relativistic.
- Nuclear reactions: nucleons get rearranged in new ways, perhaps emitting or absorbing photons, electrons, positrons and neutrinos (electron/positrons and neutrinos must be involved whenever there are conversions betweens protons and neutrons, to conserve charge and lepton number).
- Elementary particle interactions: energy, momentum, charge, lepton number *etc.* gets rearranged in new ways, corresponding to scattering, destruction and creation of particles.

↓ More energy

Harvard Tower Experiment (Pound & Rebka 1960)

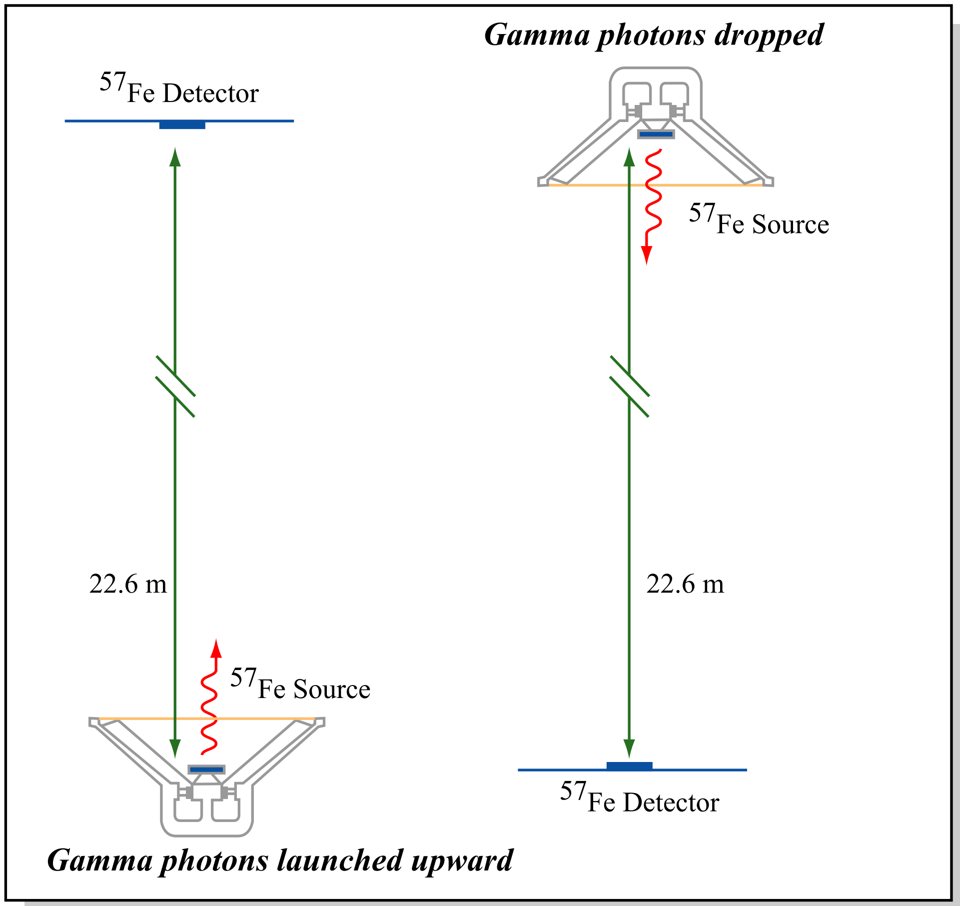
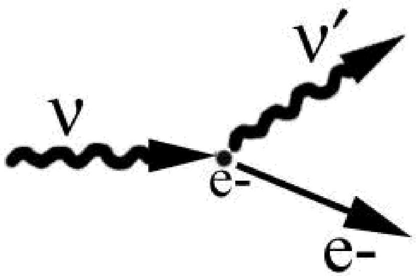


Figure by MIT OCW.

Over 22.6 meters, the gravitational redshift is only 5×10^{-15}, but the Mössbauer effect with the 14.4 keV gγ–ray from iron-57 has a high enough resolution to detect that difference.

Compton scattering

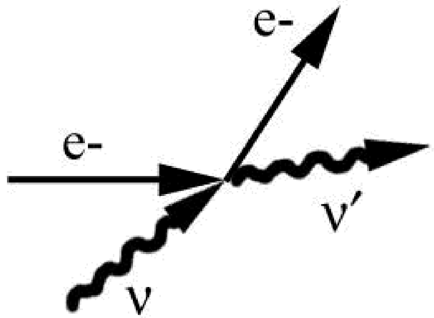


$\nu' < \nu$

Electron is initially at rest

e- gains energy

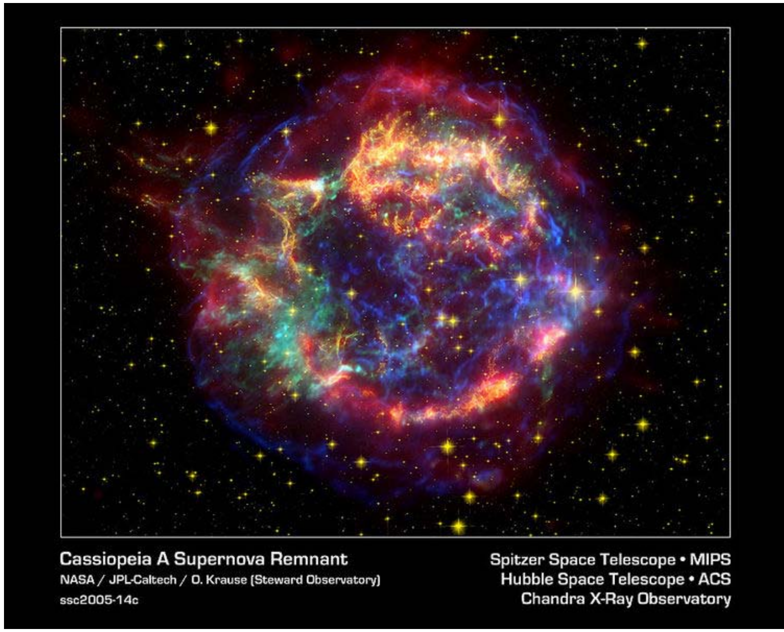
Inverse Compton scattering



$\nu' > \nu$

High energy e- initially

e- loses energy



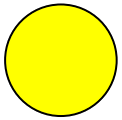
MIT Course 8.033, Fall 2006, Lecture 14

Max Tegmark

Practical stuff:

- Good news: Hubble telescope gets stay of execution
- Resnick vs. French

Today's topics:



- Electromagnatism I

Feedback form

&

EM wave

The electromagnetic force:
Ancient history…

- 500 B.C. – Ancient Greece
  - Amber (ελεχτρον="electron") attracts light objects
  - Iron rich rocks from μαγνεσια (Magnesia) attract iron
- 1730 - C. F. du Fay: Two flavors of charges
  - Positive and negative
- 1766-1786 – Priestley/Cavendish/Coulomb
  - EM interactions follow an inverse square law:
  - Actual precision better than $2/10^9$!

$$F_{em} \propto \frac{q_1 q_2}{r^2}$$

- 1800 – Volta
  - Invention of the electric battery

(From 8.02T S05)

N.B.: Till now Electricity and Magnetism are disconnected!

The electromagnetic force: …History… (cont.)

- 1820 – Oersted and Ampere
  - Established first connection between electricity and magnetism
- 1831 – Faraday
  - Discovery of magnetic induction
- 1873 – Maxwell: Maxwell's equations
  - The birth of modern Electro-Magnetism
- 1887 – Hertz
  - Established connection between EM and radiation
- 1905 – Einstein
  - Special relativity makes connection between Electricity and Magnetism as natural as it can be!

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Let's look at Einstein's 1905 paper again!

(From 8.02T S05)

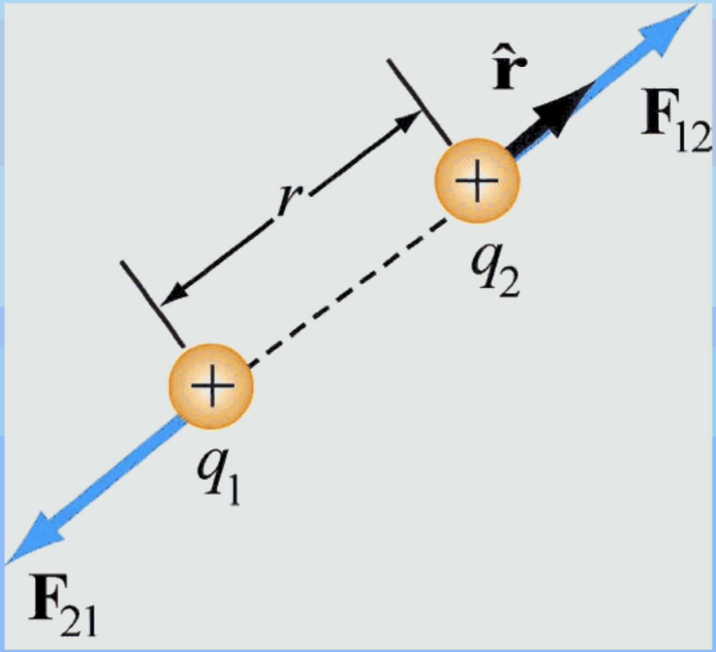
Coulomb's Law

Coulomb's Law:

Force by q_1 on q_2

$$\vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

Why of this form?



$$k_e = \frac{1}{4\pi\varepsilon_0} = 8.9875\times 10^9 \text{ N m}^2/\text{C}^2$$

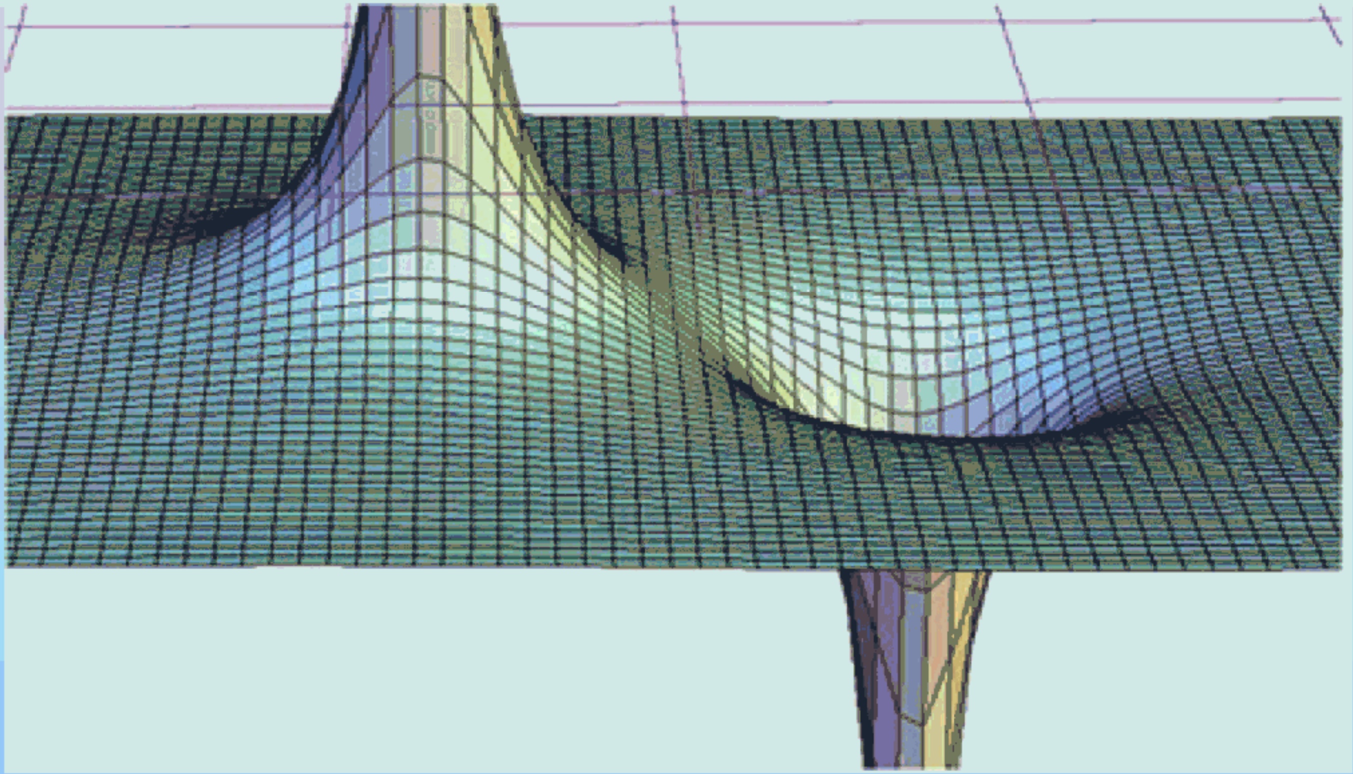
$\hat{\mathbf{r}}$: unit vector from q_1 to q_2

$$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{r} \Rightarrow \vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r^3}\vec{\mathbf{r}}$$

(From 8.02T S05)

Potential Landscape

Positive Charge



Negative Charge

(From 8.02T S05)

Why relativity and electricity implies magnetism

- We know that the force $\mathbf{F}$ on charged particle of charge q in an electric field $\mathbf{E}$ is

$$\mathbf{F} = q\mathbf{E},$$

  independent of the velocity $\mathbf{u}$ of the particle.

- We can rewrite this equation in a mathematically equivalent way using 4-vectors:

$$\mathbb{F} = \frac{q}{c}\mathbf{M}\mathbf{U}, \qquad (1)$$

  where $\mathbb{F}$ is the force 4-vector, $\mathbf{U}$ is the velocity 4-vector and $\mathbf{M}$ is the 4×4 matrix

$$\mathbf{M} = \begin{pmatrix} 0 & 0 & 0 & E_x \\ 0 & 0 & 0 & E_y \\ 0 & 0 & 0 & E_z \\ E_x & E_y & E_z & 0 \end{pmatrix}.$$

  The 4th component of this equation reads $P = q\mathbf{E} \cdot \mathbf{u}$, so $P = \mathbf{F} \cdot \mathbf{u}$ as should be.

- Let's Lorentz transform to a frame S' moving with with velocity $v = \beta c$ in the x-direction:

$$\mathbb{F}' \equiv \mathbf{\Lambda}\mathbb{F} = \frac{q}{c}\mathbf{\Lambda}\mathbf{M}\mathbf{U} = \frac{q}{c}\mathbf{\Lambda}\mathbf{M}\mathbf{\Lambda}^{-1}\mathbf{\Lambda}\mathbf{U} = \frac{q}{c}\mathbf{M}'\mathbf{U}',$$

where the transformed matrix is

$$\mathbf{M}' \equiv \mathbf{\Lambda}\mathbf{M}\mathbf{\Lambda}^{-1}. \qquad (2)$$

- Plugging in our **M**-matrix above, this gives

$$\begin{pmatrix} 0 & -\beta\gamma E_y & -\beta\gamma E_z & E_x \\ \beta\gamma E_y & 0 & 0 & \gamma E_y \\ \beta\gamma E_z & 0 & 0 & \gamma E_z \\ E_x & \gamma E_y & \gamma E_z & 0 \end{pmatrix}.$$

FIELD LINE INTERPRETATION

- This shows two things. First we see that, hardly surprisingly by now, the **E**-field is picks up some γ-factors — specifically, $E'_x = E_x$ whereas $E'_y = \gamma E_y$ and $E'_z = \gamma E_z$. Second, we see that new terms appear in the matrix that don't correspond to an E-field! The component M_{23} would also become non-zero if we transformed to a frame moving in a different direction. So to be able to describe the general case, we need to introduce more field components in **M**. It's easy to show that the upper left 3×3 block of the matrix is antisymmetric regardless of how we Lorentz transform, *i.e.*, $M_{11} = M_{22} = M_{33} = 0$ and $M_{32} = -M_{23}$, $M_{13} = -M_{31}$, $M_{21} = -M_{12}$, so we simply need to keep track of the three quantities M_{23}, M_{31} and M_{12}. We could denote these three numbers by whatever symbols we want — let's call them B_x, B_y and B_z. This means that, by definition, the **M**-matrix takes the form

$$\mathbf{M} \equiv \begin{pmatrix} 0 & B_z & -B_y & E_x \\ -B_z & 0 & B_x & E_y \\ B_y & -B_x & 0 & E_z \\ E_x & E_y & E_z & 0 \end{pmatrix}.$$

- Plugging this back into equation (1) now gives

$$\mathbf{F} = q(\mathbf{E} + \mathbf{u} \times B), \tag{3}$$

for the first three components, *i.e.*, the famous Lorentz force law from 8.02! The 4th component gives $P = q\mathbf{E}\cdot\mathbf{u} = F\dot{\mathbf{u}}$ as should be.

- In conclusion, starting with a pure electric field in S, we found that in S', the force on our particle will also depend on its velocity according to equation (3), *i.e.*, there is a magnetic field!

- Having figured out that these three new components correspond to a **B**-field, let us now use equation (2) to derive the transformation properties of an arbitrary electromagnetic field:

$$\begin{pmatrix} 0 & B'_z & -B'_y & E'_x \\ -B'_z & 0 & B'_x & E'_y \\ B'_y & -B'_x & 0 & E'_z \\ E'_x & E'_y & E'_z & 0 \end{pmatrix} =$$

$$= \boldsymbol{\Lambda} \begin{pmatrix} 0 & B_z & -B_y & E_x \\ -B_z & 0 & B_x & E_y \\ B_y & -B_x & 0 & E_z \\ E_x & E_y & E_z & 0 \end{pmatrix} \boldsymbol{\Lambda}^{-1} =$$

$$= \begin{pmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} 0 & B_z & -B_y & E_x \\ -B_z & 0 & B_x & E_y \\ B_y & -B_x & 0 & E_z \\ E_x & E_y & E_z & 0 \end{pmatrix} \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \gamma(B_z - \beta E_y) & -\gamma(B_y + \beta E_z) & E_x \\ -\gamma(B_z - \beta E_y) & 0 & B_x & \gamma(E_y - \beta B_z) \\ \gamma(B_y + \beta E_z) & -B_x & 0 & \gamma(E_z + \beta B_y) \\ E_x & \gamma(E_y - \beta B_z) & \gamma(E_z + \beta B_y) & 0 \end{pmatrix}, \tag{4}$$

so

$$\begin{aligned} E'_x &= E_x \\ E'_y &= \gamma(E_y - \beta B_z) \\ E'_z &= \gamma(E_z + \beta B_y) \\ B'_x &= B_x \\ B'_y &= \gamma(B_y + \beta E_z) \\ B'_z &= \gamma(B_z - \beta E_y) \end{aligned} \tag{5}$$

Key formula summary

- Lorentz force law:

$$\mathbf{F} = q(\mathbf{E} + \frac{1}{c}\mathbf{u} \times \mathbf{B})$$

- Lorentz transforming the electromagnetic field:

$$\begin{aligned}
E'_x &= E_x \\
E'_y &= \gamma(E_y - \beta B_z) \\
E'_z &= \gamma(E_z + \beta B_y) \\
B'_x &= B_x \\
B'_y &= \gamma(B_y + \beta E_z) \\
B'_z &= \gamma(B_z - \beta E_y).
\end{aligned}$$

Transforming charge and current densities

- The theory of electromagnetism consists of two parts: how matter affects fields and how fields affect matter. Above we studied the latter — let us now study the former.

- Analogy: the theory of gravity consists of two parts: how matter affects fields (the gravitational field) and how fields affect matter. In general relativity, the role of the gravitational field is played by the metric, and we will find that both parts of the theory get a geometric interpretation: the former that matter moves along geodesics through spacetime and the latter that matter curves spacetime.

What properties are Lorentz invariant?

Property	Independent of velocity?	
	Classically?	Relativistically?
Charge q	Y	Y
Spin	Y	Y
Lepton number	Y	Y
Duration Δt	Y	N
Length L	Y	N
Mass m	Y	N
Proper duration $\Delta\tau$	Y	Y
Proper length L_0	Y	Y
Rest mass m_0	Y	Y
Momentum p	N	N
Energy E	N	N

Integers!

- The source of electromagnetic fields is matter carrying electric charge, characterized at each spacetime eventy by a *charge density* $\rho(\mathbf{r}, t)$ and a *current density* $\mathbf{J}(\mathbf{r}, t)$.

- These can be combined into the current 4-vector (or "4-current")

$$\mathbb{J} \equiv \begin{pmatrix} J_x \\ J_y \\ J_z \\ \rho c \end{pmatrix} .$$

For a blob of charge of uniform density ρ_0 in its rest frame that moves with velocity 4-vector $\mathbf{U}$, the 4-current is simply

$$\mathbb{J} \equiv \rho_0 \mathbf{U},$$

and the total 4-current from many sources (say electrons and ions moving in opposite directions) is simply the sum of all the individual 4-currents.

- ρ_0 is called the *proper charge density*.