

Massachusetts Institute of Technology

Department of Physics

Physics 8.033

Out: Friday 27 October 2006

Due: Friday 3 November 2006

Problem Set 7

Due: Friday 3 November 2006 at 4:00PM. Please deposit the problem set in the appropriate 8.033 bin, labeled with name and recitation section number and stapled as needed (3 points).

Readings: Chapter IV in Resnick and Chapter 8 in French. (As stressed in lecture, please don't get bogged down with their excessive E&M algebra.)

Problem 1: Key concepts (3+3+3 points):

- Professor D. Ubious tells you that he has observed a 1 GeV photon decay into an electron-positron pair (there was no other particle nearby and no external electromagnetic field). Is this consistent with relativity and standard particle physics? If not, explain which conservation law it violates and give an explicit calculation if needed. (Hint: if you find yourself making a nontrivial calculation, try working in a frame where things are simpler.)
- Consider a neutron undergoing β -decay. For each of the three decay products, list at least one conservation law that would be violated if it weren't produced.
- Give an inventory of a water molecule in terms of elementary particles (specify how many quarks and leptons it contains of various types).

Problem 2: "Transforming the electromagnetic field" (6+3+3+3 points):

- Resnick spends pages and pages working out how \mathbf{E} and \mathbf{B} are Lorentz transformed. French spends 37 pages on the subject without even obtaining this key result. Now let's see if you can derive it more elegantly. In class, we showed that Lorentz transforming the Force law gave (in c.g.s. units)

$$\begin{pmatrix} 0 & B'_z & -B'_y & E'_x \\ -B'_z & 0 & B'_x & E'_y \\ B'_y & -B'_x & 0 & E'_z \\ E'_x & E'_y & E'_z & 0 \end{pmatrix} = \Lambda \begin{pmatrix} 0 & B_z & -B_y & E_x \\ -B_z & 0 & B_x & E_y \\ B_y & -B_x & 0 & E_z \\ E_x & E_y & E_z & 0 \end{pmatrix} \Lambda^{-1},$$

where

$$\Lambda = \begin{pmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{pmatrix}$$

is the usual matrix corresponding to a Lorentz transformation in the x -direction, and its inverse Λ^{-1} is the same matrix with $-\beta$ replaced by β . Perform the above matrix multiplications explicitly and solve for the new field quantities $E'_x, E'_y, E'_z, B'_x, B'_y$ & B'_z in terms of the old ones E_x, E_y, E_z, B_x, B_y & B_z . Does your result agree with Resnick's? Which derivation was shorter?

- To get some intuition for what your result means, describe what happens in the following special cases:
 - Both \mathbf{E} and \mathbf{B} point in the x -direction in S (parallel to direction in which S' is moving).
 - S has a pure \mathbf{E} -field in the y -direction (perpendicular to the direction in which S' is moving), *i.e.*, $E_x = E_z = B_x = B_y = B_z = 0$.
- In what sense does your last result imply that one cannot consider electric and magnetic fields as fundamentally separate entities?

- (d). If \mathbf{B} vanishes in S , then in what other frames does it vanish? (In other words, in which directions can the new frame S' move and still have no magnetic field?)

Problem 3: “Center of Mass Collisions” (6 points):

(a) In modern experimental physics, energetic particles are made to circulate in opposite directions in so-called storage rings and permitted to collide head-on. In this arrangement each particle has the same kinetic energy K in the laboratory. The collisions may be viewed as totally inelastic, in that the rest energy K in the laboratory. The collisions may be viewed as totally inelastic, in that the rest energy of the two colliding protons, plus all available kinetic energy, can be used to generate new particles and to endow them with kinetic energy. Show that the available energy in this arrangement can be written in the form.

$$\epsilon = 2m_0c^2\left(1 + \frac{k}{m_0c^2}\right)$$

(b) How much energy is made available when 100 GeV protons are used in fashion? (c) What proton energy would be required to make 100 GeV available?

Problem 4: “Atomic Decay With Recoil” (9 points): French, problem #6-10, Chapter 6, page 201.

Problem 5: “Compton Scattering” (9 points): French, problem #6-17, Chapter 6, page 202.

Note: The block of matter serves only as a source of “free” electrons; it does not absorb any momentum as a collective unit.

Problem 6: “Creation of Pions” (6 points): A 10^{15} eV cosmic-ray (proton) collides with a proton (at rest) in the Earth’s atmosphere. Compute the maximum number of pions (rest mass = 140 MeV) that can be created in such a collision. [Hint: Either work in the center of mass reference frame or make use of the fact that $E_{\text{TOT}}^2 - p_{\text{TOT}}^2c^2$ is Lorentz invariant.]

Problem 7: “Pair Creation” (6 points): Two protons of mass ~ 1 GeV collide to produce a particle of rest mass 300 GeV. The two protons remain after the collision. Find the threshold energy for this particle production to occur if: (i) one of the protons is initially at rest, and, (ii) both protons have the same energy and collide head on. Comment on the relative efficiency of colliding beam accelerators versus those using fixed targets.

Optional Problem 8: “Proton-Antiproton Collision”: French, problem #6-12, Chapter 6, page 202.

Optional Problem 9: “Scattering Angles in Elastic Collisions”: French, problem #7-5, Chapter 7, page 226.

Optional Problem 10: “Cyclotron”: French, problem #7-9, Chapter 7, page 226.

Optional Problem 11: “Relativistic Elastic Collision”: A relativistic particle of rest mass M_0 moves along the x -axis of S with motion characterized by β_0 and γ_0 . It collides with an identical particle that is at rest. After the collision it is observed that both particles are moving at angles $\pm\theta$ with respect to the x -axis.

(a). Show that the velocity of the CM frame (S' frame) with respect to S is given by

$$\beta_{\text{CM}} = \frac{\gamma_0\beta_0}{\gamma_0 + 1}, \quad \gamma_{\text{CM}} = \sqrt{\frac{\gamma_0 + 1}{2}}. \quad (11.1)$$

Hint: one way to proceed is to use the relation: $\beta_{\text{CM}} = p_{\text{TOT}}c/E_{\text{TOT}}$.

- (b). Argue that before the collision, both particles have the same energy and magnitude of momentum in the CM frame, and that these quantities are characterized by β_{CM} and γ_{CM} .
- (c). Argue that after the collision both particles are moving at right angles to the x' -axis in the CM frame.
- (d). For each particle, after the collision, find p_x and p_y back in the S frame; express your answers in terms of β_{CM} and γ_{CM} .
- (e). Compute $\tan(\theta) = p_y/p_x$ and show that $\tan(\theta) = 1/\gamma_{\text{CM}} = \sqrt{2/(\gamma_0 + 1)}$.

Source: Problem 3 is taken from *Basic Concepts in Relativity* by Robert Resnick, David Halliday.

Feedback: Roughly how much time did you spend on this problem set?