

# Massachusetts Institute of Technology

Department of Physics

Physics 8.033

Out: Friday 20 October 2006

Due: Friday 27 October 2006

## Problem Set 6

**Due:** Friday 27 October 2006 at 4:00PM. Please deposit the problem set in the appropriate 8.033 bin, labeled with name and recitation section number and stapled as needed (3 points).

**Readings:** Chapter 3 in Resnick and Chapter 6 and 7 in French.

**Problem 1:** (2004 Exam Question) (5+3+1+1+1 points)

- A particle of rest mass  $m_0$  moving with Lorentz factor  $\gamma$  collides with an identical particle that is initially at rest. Compute the *total* (rest+kinetic) energy  $E_{\text{tot}}$  of the two-particle system both in the lab frame  $S$  and in the CM frame  $S'$ . Compute the ratio  $E_{\text{tot}}/E'_{\text{tot}}$ .
- CERN's Large Hadron Collider will use colliding particle beams rather than beams hitting fixed targets to make new forms of matter — in one sentence, why is this a good idea based on your result in (a)?
- In the Hindenburg blimp disaster, the fire was dominated by the chemical reaction  $2H_2 + O_2 \rightarrow 2H_2O$ . Based on this fact, the combined rest masses of two hydrogen molecules and one oxygen molecule is (**circle one**) SMALLER/LARGER than that of a two water molecules.
- Order the following particles by increasing rest mass:  
proton, neutron, photon,  $Z$ -boson, electron.
- You consist of quarks called \_\_\_\_\_ and \_\_\_\_\_ and leptons called \_\_\_\_\_.

**Problem 2:** (2003 Final Question) (4 points)

Consider the following reactions:

- \_\_\_\_\_ An isolated 1 GeV photon decays into an electron-positron pair.
- \_\_\_\_\_ A neutron decays into a proton and an electron
- \_\_\_\_\_ A neutron decays into an electron-positron pair
- \_\_\_\_\_ A neutron decays into a proton and a photon

For each one, write **one** of the letters from the option list below.

- L violates lepton number conservation
- B violates baryon number conservation
- P violates parity conservation
- E violates energy-momentum conservation
- Q violates charge conservation
- N violates none of the above conservation laws

**Problem 3:** Concept questions (6 points): For each of these pairs, circle the one with the largest rest mass.

- (A) a proton  
(B) a neutron
- (A) a proton  
(B) 1000 electrons at rest far from each other

- (c). (A) a 10 GeV muon  
(B) a 100 GeV electron
- (d). two neutrons and two protons in a fixed arrangement where all pairs are separated by of order  
(A) 1 m  
(B)  $10^{-10}$ m
- (e). two neutrons and two protons arranged so that all pairs are separated by of order  
(A) 1 m  
(B)  $10^{-15}$ m (bound as an  $\alpha$ -particle, *i.e.*, a Helium nucleus)
- (f). (A) two hydrogen molecules ( $H_2$ ) at rest and one oxygen molecule ( $O_2$ ) at rest  
(B) two water molecules at rest

For the systems involving more than one particle, interpret the rest mass of the system as its total relativistic mass  $E/c^2$  in the frame where the total momentum is zero. When answering these questions, you can ignore any kinetic energy due to internal motions (from the particles in the system moving relative to each other).

**Problem 4:** “Transforming a photon” (3+3+3+3+3 points):

- (a). Write down the momentum 4-vector  $\mathbf{P}$  for a photon of angular frequency  $\omega$  traveling in an arbitrary direction specified by angles  $\theta$  and  $\phi$  in polar coordinates.
- (b). Write down an equation relating  $\mathbf{P}$  to the wave 4-vector  $\mathbf{K}$  for light of the same frequency traveling in this same direction.
- (c). Compute the momentum 4-vector  $\mathbf{P}'$  in a frame  $S'$  that is moving with velocity  $v = \beta c$  in the  $z$ -direction.
- (d). Derive the energy ratio  $E'/E$  and the new direction of motion ( $\phi'$  and  $\theta'$ ) in terms of  $\phi$ ,  $\theta$  and  $\beta$ .
- (e). Comment on any similarities between your results and the Doppler effect and aberration formulas that we derived in class.

**Problem 5:** This problem appeared on the second 8.033 quiz in 2001. (6+3 points)

A particle with rest mass  $M_0$  is kept moving in a circular orbit of fixed radius  $R$ , by a magnetic field,  $B$ , whose field lines are perpendicular to the orbit. A force of constant amplitude  $F$  is applied to the particle in a direction that is always along its velocity vector (*i.e.*, in the  $\theta$  direction). As the particle is accelerated to higher  $(\gamma, \beta)$ , the magnetic field strength is adjusted so that the particle continues to move in its orbit with constant radius  $R$ .

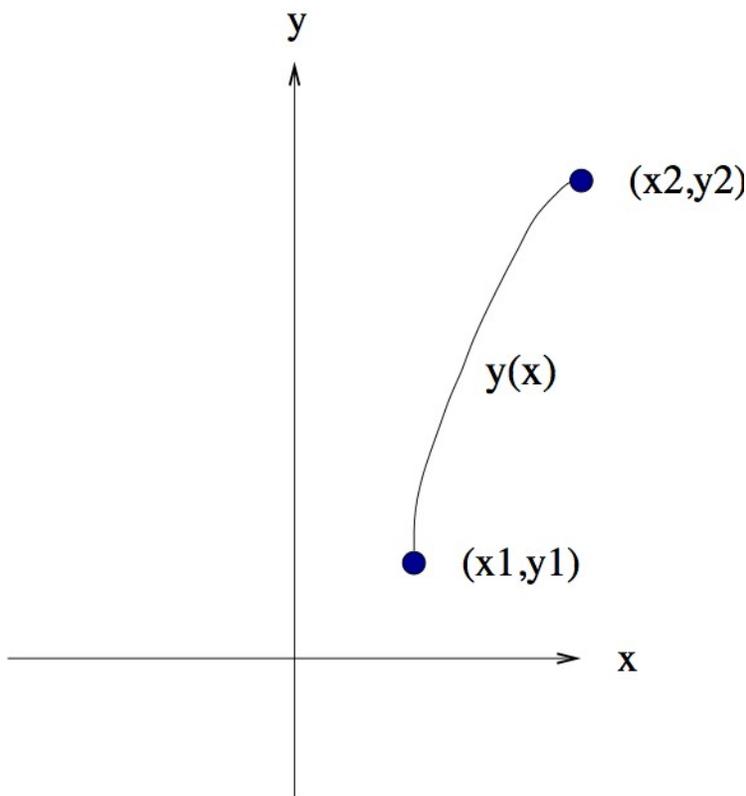
- (a). Starting with  $\vec{F} = d\vec{p}/dt$ , derive an equation for the increase in particle speed,  $u$ , with time. Express your answer in terms of  $du/dt$  equals a function of  $\gamma$ ,  $M_0$ , and  $F$ . You need not solve the equation.
- (b). Find how the magnetic field must change as a function of  $\gamma$ . Note, this part of the problem can be done independently of part (a).

**Problem 6:** Photoelectric effect: “Calculating the Planck Constant and More”: (3+3+3 points)

In a photoelectric experiment in which a sodium surface is used, one finds a stopping potential of 1.85 V for a wavelength of 300 nm and a stopping potential of 0.82 V for a wavelength of 400 nm. From these data find (a) a value for the Planck constant, (b) the work function for sodium and (c) the cutoff wavelength for sodium.

**Problem 7:** “Photon Rocket” (9 points): Review and discuss French’s Photon Rocket problem on pages 183-184. Derive a quadratic equation for  $f$ , the payload’s mass as a fraction of the rest mass of the rocket. How feasible is the photon rocket? Note that he seems to have written this in a hurry and gets into lots of totally unnecessary algebra towards the end.

**Optional Problem 8:** “More on the Calculus of Variations”: Consider two points in the  $x - y$  plane as shown in the sketch:  $(x_1, y_1)$  and  $(x_2, y_2)$ . The two points are connected by a curve specified by  $y(x)$ . This curve is then rotated about the  $y$  axis to form a surface. Find the shape of the curve  $y(x)$  that minimizes the area of the resultant figure of rotation.



- (a). Show that the infinitesimal arc length along the path is  $ds = dx\sqrt{1 + (dy/dx)^2}$ .  
 (b). Show that the integral for the surface area of the figure of rotation is:

$$A = 2\pi \int_{x_2}^{x_1} x\sqrt{1 + y'^2}dx, \quad (8.1)$$

where  $y' = dy/dx$ .

- (c). Apply the appropriate Euler equation:

$$\frac{\partial f}{\partial y} - \frac{d}{dx} \frac{\partial f}{\partial y'} = 0 \quad (8.2)$$

to find the differential equation for the shape of the curve  $y(x)$ . Show that

$$\frac{xy'}{\sqrt{1 + y'^2}} = a = \text{constant}. \quad (8.3)$$

- (d). Solve for  $dy/dx$  and show that the result is:

$$y = \int \frac{a}{\sqrt{x^2 - a^2}} dx. \quad (8.4)$$

- (e). Look up the integral and show that it can be written as:

$$y = a \cosh^{-1} \left( \frac{x}{a} \right) + b, \quad (8.5)$$

or

$$x = a \cosh\left(\frac{y-b}{a}\right), \quad (8.6)$$

where  $b$  is a second constant of integration. This function is called a “catenary”. It has the same shape as a flexible chain hanging freely between two points of support.

**Feedback:** Roughly how much time did you spend on this problem set?