Massachusetts Institute of Technology
Department of Physics

Physics 8.033

Out: Friday 29 September 2006
Due: Friday 6 October 2006

Problem Set 4

Due: Friday 6 October 2006 at 4:00PM. Please deposit the problem set in the appropriate 8.033 bin, **labeled with name and recitation section number and stapled** as needed (3 points).

Readings: Supplementary Topics A & B in Resnick (pages 188–209). No new parallel reading assignments in French.

Problem 1: "Identifying the Lines and Determining the Redshift of a Quasar" (3 pts):

Figure 1 is the visible spectrum of a distant quasar. Five prominent spectral lines can be seen in the spectrum. Also provided is a table of the laboratory wavelengths of the most observed emission lines in the spectra of quasars. Identify the lines and calculate the redshift $z = (\lambda - \lambda_0)/\lambda_0$ of this quasar. (λ is the wavelength measured in the laboratory).

Hint: First measure λ for the five lines in the spectrum. Then look for five lines in the table such that λ/λ_0 has a nearly constant value.

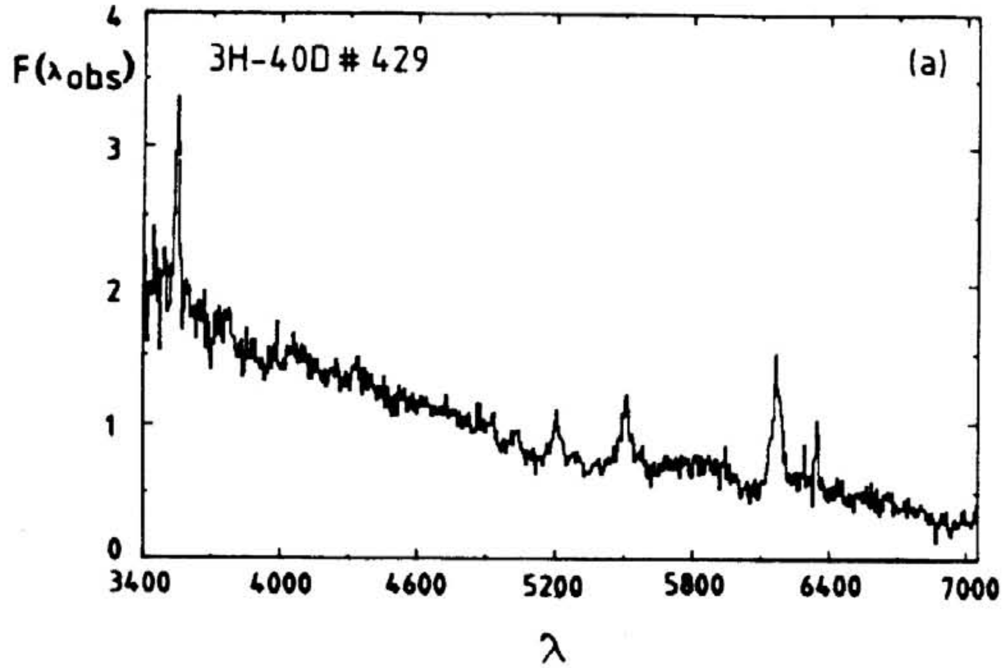


TABLE I
Laboratory wavelengths of the most observed emission lines in the spectra of quasars

λ_{lab}	ion
6563 Å	H alp
5007	[O III]
4956	[O III]
4861	H bet
4363	[O III]
4340	H gam
4102	H del
3969	H eps
	[Ne III]
3869	[Ne III]
3728	[O II]
2799	Mg II
1909	C III]
1549	C IV
1402	Si IV
	O IV]
1241	N V
1216	Ly alp

Problem 2: "Relativistic Jets from the Astrophysical Object SS433" (3+3+3+3):

The astronomical object SS433 is a binary system consisting of a collapsed star orbiting a normal companion star from which it accretes matter. This process results in the expulsion of two relativistic beams ("jets") of matter which leave the system in opposite directions (180° apart). The atoms in these beams emit characteristic spectral lines which can be used to measure the Doppler shifts of these beams as a function of time. The time varying nature of the Doppler curves indicates that the two beams are periodically precessing so that we are viewing them from different angles at different times. The two intersecting Doppler curves correspond to the two oppositely directed beams.

(a). Determine the Doppler shift $(\lambda - \lambda_0)/\lambda_0$ at the times when both beams exhibit the same wavelength.

(b). Show from the relativistic Doppler equation $\lambda/\lambda_0 = \gamma(1 - \beta\cos\theta)$ that this will occur only when $\theta = 90°$ (i.e., when viewing the beams perpendicular to their direction). Note that λ and θ are measured in the Earth's frame, while λ_0 is measured in the frame of the moving beam.

(c). Use the results of (a) and (b) to find β for the beam.

(d). Measure the maximum Doppler shift for the precessing beam. Use the relativistic Doppler equation and the value of β from part (c) to find the maximum angle between the beam and the direction to the Earth.

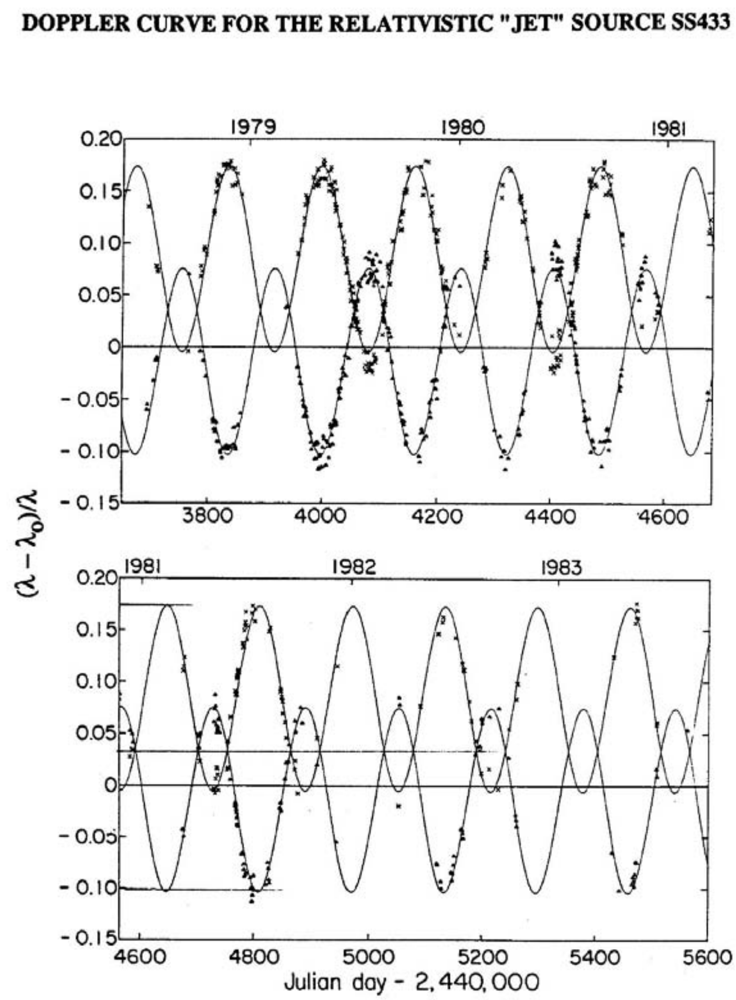


Figure 2: Doppler Curves.

Problem 3: "The Headlight Effect—A High-Speed Limit" (3+3+3 pts)

A source of light, at rest in the S' frame, emits uniformly in all directions. The source is viewed from frame S, the relative speed parameter relating the two frames being β. (a) Show that at high speeds (that is as $\beta \to 1$), the forward-pointing cone into which the source emits half of its radiation has a half-angle $\theta_{0.5}$ giveen closely, in radian measure by $\theta_{0.5} = \sqrt{2(1-\beta)}$. ($b$) What value of $\theta_{0.5}$ is predicted for the gamma radiation emitted by a beam of energetic neutral pions, for which $\beta = 0.993$? (c) At what speed would light source have to move toward an observer to have half of its radiation concentrated into a narrow forward cone of half-angle 5.0°?

Hint: Recall the approximation $(1-\beta) \approx 1/(2\gamma^2)$.

Problem 4: (2004 Exam Question (3 pts))

A lamp moves past you in a straight line. At what angle relative to its direction of motion (draw a picture and label this angle!) do you see it when it is neither blueshifted nor redshifted?

Problem 5: (2004 Final Question (3+4+1+1+3 pts))

Consider three intertial frames, S, S', and S''. S' moves with speed $v' = \frac{3}{5}c$ along the x-axis of frame S and S'' moves with speed $v'' = \frac{4}{5}c$ along the y'-axis of frame S'. The origins and orientations of the frames are such that they coincide at time $t = t' = t'' = 0$ and the origin $x = x' = x'' = y = y' = y'' = z = z' = z'' = 0$.

(a). Write down the Lorentz tranformations for $S \to S'$ and $S' \to S''$, *i.e.*, express (x', y', z', ct') as functions of (x, y, z, ct) and (x'', y'', z'', ct'') as functions of (x', y', z', ct'). You may find it convenient to express your two transformations in matrix form.)

(b). Complete the following table of coordinates for the event $A = (1, 2, 3, 4)$:

Frame	x	y	z	ct
S	1	2	3	4
S'				
S''				

(c). A rocket of proper length L_0 moves along the x-axis of frame S with speed $u = \frac{3}{5}c$. What is its velocity in frame S'?

(d). How long is it in frame S'?

(e). List three pieces of observational evidence supporting special relativity.

Problem 6: "Bob Is Older Than Dave This Time" (3+3+3+3 pts)

Here is a different version of the twin paradox, where the twins are Bob and Dave:

Bob, once started on his outward journey from Dave, keeps on going at his original uniform speed of 0.8c. Dave, knowing that Bob was planning to do this, decides, after waiting for three years, to catch up with Bob and to do so in three additional years. (a) To what speed must Dave accelerate to do so? (b) What will be the elapsed time by Bob's clock when they meet? (c) How far will they each have traveled when they meet, measured in Dave's original reference frame? (d) Who is older and by how much when they meet?

Problem 7: "Bob And Dave Are Twins Again" (3 pts)

Bob travels away from his brother Dave with speed 0.8c and Dave stays at rest. After three years, each counting the years by his own onboard clock, Bob come to rest and Dave will accelerate to 0.8c and eventually catch up with Bob. What will be the total elapsed times on each of their clocks when they meet?

Problem 8: (3+3+3 pts) "More General Version of the Twin Paradox": Two clocks start at rest at the same point in space and are set to the same time in frame S. Clock A remains at rest in the S reference frame. Clock B takes an accelerated trip along the x-axis of total duration T as recorded by Clock A, and ends up at rest in S, at the location of Clock A. The trip can be broken up into 6 segments which are described below. *All times, velocities, and accelerations are measured in S.* At the end of the trip, Clock B shows an elapsed time T'.

Outbound portion of trip (segments 1 through 3):

1. Clock B undergoes a constant acceleration a_0 for time $fT/4$, where f is the fraction of the entire trip which is spent accelerating.
2. Clock B then coasts at constant speed β for time $(1-f)T/2$.
3. Clock B decelerates at $-a_0$ for time $fT/4$, and comes to rest.

Return portion of trip (segments 4 through 6): Clock B undergoes all the same motions as in the outbound trip, *but in reverse order*, and ends up at rest in S, at the location of Clock A. You can make use of the symmetry in the problem by noting that steps 4-6 take a time equal to steps 1-3.

(a). Compute T'/T as a function of a_0, β, abd f.

(b). Show that in the limit of $f \to 1$ (i.e., no coasting period), and β approaches unity, the ratio $T'/T \approx \pi/4$.

(c). Show that in the limit of $f \to 0$ (i.e., no significant time spent during the acceleration), and β still large (i.e., large accelerations at the turning points), the "standard" result for the twin paradox is recovered.
Note: the 8.01 relations $v = v_0 + at$ and $x = x_0 + v_0 t + at^2/2$ still hold in S.

Problem 9: "Geodesic on a Sphere" (3+3+3+3 pts): Use the calculus of variations to find the curve representing the shortest path between any two points on a unit sphere (of radius 1).

(a). Show that the infinitesimal distance between two neighboring points is given by:

$$\mathrm{d}s = (\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2)^{1/2}. \tag{9.1}$$

(b). Take the independent variable to be θ. Use Euler's equation (of the first form) to find the following differential equation:

$$\frac{\mathrm{d}\phi}{\mathrm{d}\theta} = \frac{a \csc^2\theta}{(1 - a^2\csc^2\theta)^{1/2}}, \tag{9.2}$$

where a is the first constant of integration.

(c). Integrate this equation to find:

$$\cot\theta = -\frac{\sqrt{1-a^2}}{a}\sin(\phi - \alpha), \tag{9.3}$$

where α is the second constant of integration.

(d). Show that the result of part (c) can be written in the form:

$$Ay - Bx = z, \tag{9.4}$$

in Cartesian coordinates. Note that this is a plane passing through the center of the sphere and the two chosen points on the surface. Use this to deduce the nature of a geodesic curve on a sphere.
Useful relations:

$$x = \sin\theta\cos\phi, \qquad y = \sin\theta\sin\phi, \qquad z = \cos\theta. \tag{9.5}$$

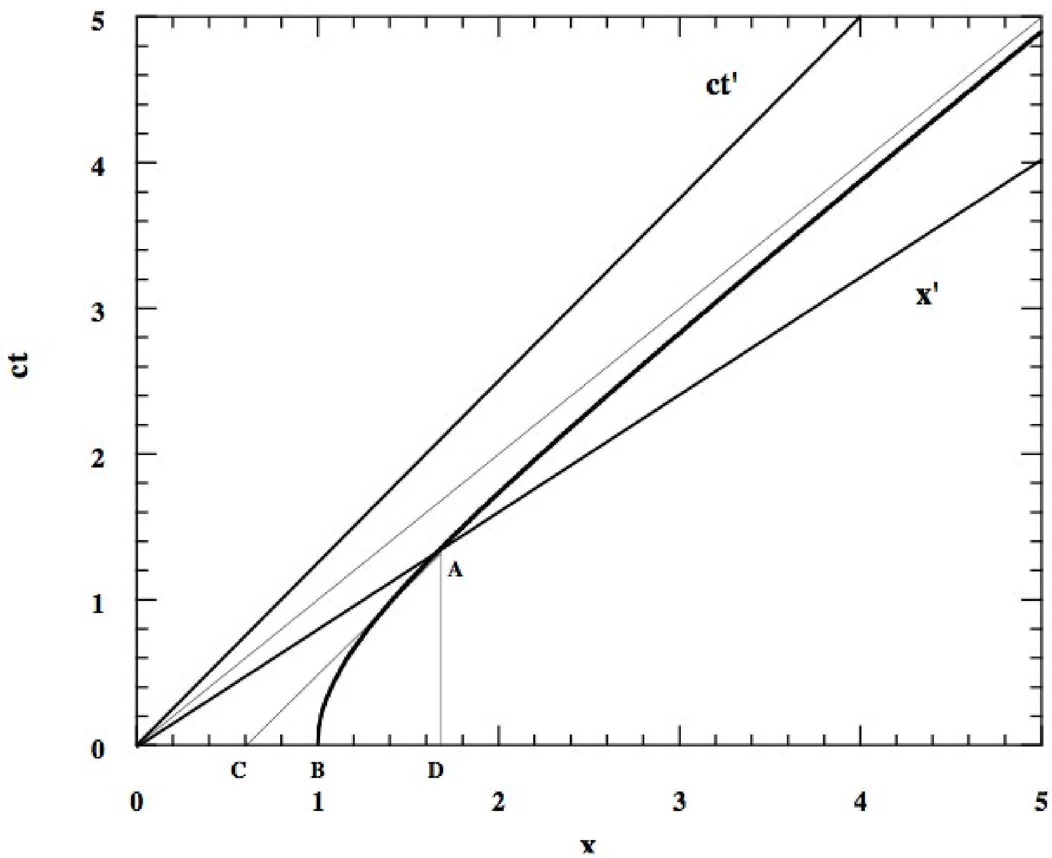
Optional Problem 10: "Fresnel Drag Coeffcient": Use the relativistic addition of velocities to compute the "Fresnel drag coeffcient". Start with the fact that the speed of light in a medium with index of refraction n is c/n. Compute how fast the light appears to travel with respect to a frame in which the medium is moving with speed v (in the same direction as the light is propagating). Show that to the lowest order in v/c this speed is given by $u = c/n + v(1 - 1/n^2)$. The term in brackets on the right hand side is called the "Fresnel drag coeffcient", although there was never any need to assume a "drag" mechanism in the first place. Typical indices of refraction for water and glass are $n \sim 1.5$.

The Fresnel prediction (1817) of light "drag" by a moving medium, and the experimental "verification" of this effect by Fizeau (1851) are discussed on pages 30–33 of Resnick.

Optional Problem 11: "Calibration by Hyperbolas"

(2003 Final Question)

Minkowski Diagrams: The quadrant of the Minkowski diagram shown on the drawing below contains the ct and x axes of the S frame, as well as the ct' and x' axes of an S' frame moving with speed $\beta = 4/5$, ($\gamma = 5/3$). Point A on the x' axis has a coordinate $(x', ct') = (1, 0)$. The figure is drawn accurately to scale.



(i) Line segment AD is parallel to the ct axis. Use the Lorentz transformations to find the coordinate x of point D in S. Confirm your answer by reading the value from the graph.

(ii) Line segment AC is parallel to the ct' axis. Use the Lorentz transformations to find the coordinate x of point C in S. Confirm your answer by reading the value from the graph.

(iii) The heavy curve that connects points B and A and its extrapolation is designed so as to intersect the x axis at a value of 1.0. Furthermore, it would intersect any other x'' axis drawn on the figure (corresponding to β'') at a value of $x'' = 1$. Find the equation, $x(t)$, of this curve.

Source: Problems 3, 6, 7, 11 are taken from *Basic Concepts in Relativity* by Robert Resnick, David Halliday.

Feedback: Roughly how much time did you spend on this problem set?