

Massachusetts Institute of Technology

Department of Physics

Physics 8.033

Out: Friday 15 September 2006

Due: Friday 22 September 2006

Problem Set 2

Due: Friday 22 September 2006 at 4:00PM. Please Write your name deposit the problem set in the appropriate 8.033 bin, labeled by recitation section number and stapled as needed (3 points).

Readings: Chapter II of Resnick. Parallel reading: Chapters 3 & 4 in French.

Problem 1: (3 points) Consider a general wave equation of the form:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{V^2} \frac{\partial^2 y}{\partial t^2}. \quad (1.1)$$

- (a) Show that any function of the quantity $(x \pm Vt)$ will satisfy this equation.
- (b) Make an argument to show why V represents the propagation speed of waves or other disturbances described by this equation (x is the space and t is time).

Problem 2: “Maxwell versus Galileo” (6 points)

Maxwell’s wave equation for the z -component of an electric field propagating in the x -direction is

$$\frac{\partial^2 E_z}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E_z}{\partial t^2}, \quad (2.1)$$

where $E(x, t)$ is the amplitude of the electric field. Show that this equation is not invariant under a Galilean transformation to a reference frame moving with relative speed v in the x direction.

Hint: Use the chain rule in which, if $f = f(x, t)$, then

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'} \left(\frac{\partial x'}{\partial x} \right) + \frac{\partial f}{\partial t'} \left(\frac{\partial t'}{\partial x} \right). \quad (2.2)$$

Optional: Show that any function $E_z(x + Vt)$ is a solution to the transformed wave equation, where $V = v \pm c$.

Problem 3: “Michelson-Morley Experiment With a Real Wind” (3+3+3 points)

A pilot plans to fly due east from A to B and back again. If u is his airspeed and if l is the distance between A and B , it is clear that his round-trip time t_0 —if there is no wind—will be $2l/u$.

- (a). Suppose, however, that a steady headwind of speed v blows from the east (or from the west). Show that the round-trip time will now be

$$t_1 = \frac{t_0}{1 - (v/u)^2}. \quad (3.1)$$

- (b). If the wind is from the north (or from the south), show that the expected round-trip travel time is

$$t_2 = \frac{t_0}{\sqrt{1 - (v/u)^2}}. \quad (3.2)$$

- (c). Note that these two travel times are *not* equal. In the Michelson-Morley experiment, however, the experiment seems to show that (for arms of equal length), the travel times for light *are* equal; otherwise these experimenters would have found a fringe shift when they rotated the interferometer. What is the essential difference between the two situations?

Problem 4: “Not Much of a Contraction” (3 points)

In the Michelson-Morley experiment, by about how many atomic diameters would the appropriate arm of the interferometer have to shrink with respect to the the other arm according to the Lorentz-Fitzgerald contraction hypothesis? Assume that the Earth is moving along the direction of the first arm. The actual arm length of the interferometer was 2.8m. Take 0.1 nm as a typical atomic diameter.

Problem 5: (3 points) In Special Relativity we will often want to evaluate the relativistic factor $\gamma = (1 - v^2/c^2)^{-1/2}$ for: (1) small values of speed v , (2) for values of v close to the speed of light c . While this can be done with a calculator, it is often more instructive to derive an approximate analytic result explicitly in terms of powers of $\beta = v/c$ for case (1), and powers of $1/\gamma$ for case (2).

- (a). Expand γ^2 and γ in a Taylor series in powers of β^2 , keeping the first 3 terms (constant plus next two terms). If you have not yet learned about Taylor series, simply adopt the expression below:

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots \quad (5.1)$$

Check your answers:

$$\gamma^2 = \frac{1}{1 - \beta^2} = 1 + \beta^2 + \beta^4 + \beta^6 + \dots, \quad (5.2)$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} = 1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \dots. \quad (5.3)$$

Evaluate your expression for with values of $\beta \in \{0.1, 0.001, 10^{-5}\}$. Compare to the values you get by evaluating the full expression for γ with a calculator.

- (b). Show that for large γ , $1 - \beta \approx 1/(2\gamma^2)$. Use this expression to find the values of β when $\gamma \in \{2, 10, 10^3\}$. Compare the approximate results with those obtained from the full expression using a calculator.

Problem 6: “Aberration and Relativity” (3+1.5+3 points)

Consider the aberration arrangement of Fig. 1-9(Resnick page 29).

- (a). At what speed does light pass along the telescope axis according to the ether hypothesis?
(b). According to the special theory of relativity?
(c). Show that, according to relativity theory, the classical aberration equation (see Eq. 1-11)

$$\tan \alpha = \frac{v}{c} \quad (\text{classical theory}) \quad (6.1)$$

must be replaced by

$$\sin \alpha = \frac{v}{c} \quad (\text{relativity theory}). \quad (6.2)$$

Thus, the ether theory and the theory of relativity make different predictions for the aberration of starlight.

Optional: If α_c and α_r are the predictions of these two theories, find their fractional difference $(\alpha_r - \alpha_c)/\alpha_r$. Assume that $v = 30$ km/s (the earth’s orbital speed).

Hint: The predictions of the two theories are so close that their difference may be beyond the scope of your hand calculator. Use the series expansions of \sin^{-1} and \tan^{-1} .

Problem 7: “The Speed of Light Really is the Same in All Frames” (3+3 points)

(a). Start with the equation:

$$x^2 + y^2 + z^2 - c^2t^2 = 0, \tag{7.1}$$

and apply the Lorentz-Einstein transformations to yield equation:

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = 0. \tag{7.2}$$

(b). The corresponding matrix equation is

$$X^t \eta X = 0 \tag{7.3}$$

Also we saw in class that $\Lambda^t \eta \Lambda = \eta$ for any Lorentz matrix Λ . Start here and show that

$$X'^t \eta X' = 0 \tag{7.4}$$

where $X' = \Lambda X$. You can see that the matrix formalism is easier.

Problem 8: “A Moving Clock” (3 points)

A clock moves along the x -axis at a speed of $0.6c$ and reads zero as it passes the origin. What time does it read (to an observer stationary w.r.t the clock) as it passes the 180 m mark on the axis? What is the time difference with a stationary clock synchronized at the origin?

Problem 9: “A Moving Rod” (1.5 points)

A rod lies parallel to the x -axis of reference frame S , moving along this at a speed of $0.6c$. Its rest length is 1.0 m. What will its measured length be in frame S' ?

Problem 10: “Inverse Lorentz transformation” (3 points)

As you know the Lorentz transformation along x -axis may be written in matrix form as

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}. \tag{10.1}$$

Find the inverse transformation, that is, find the coordinates (ct, x, y, z) in terms of (ct', x', y', z') , by the physical expectation of the inverse being given by changing the sign of the relative velocity v . Show by explicit matrix multiplication, that the transformation found in this way is indeed inverse to the matrix in (10.1).

Optional Problem 11: “Fascinating Thought Experiment”

Ehrenfest (1880 - 1933) proposed the following thought experiment to illustrate the different behavior expected for light under the ether-wind hypothesis and under Einstein’s second postulate.

Imagine yourself seated at the center of a spherical shell of radius 3×10^8 m, the inner surface being diffusely reflecting. A source at the center of the sphere emits a sharp pulse of light, which travels outward through the darkness with uniform intensity in all directions. What would you see during the 3-s interval following the pulse under the assumptions that (a) there is a steady ether wind blowing through the sphere at 100 km/s and (b) that there is no ether and Einstein’s second postulate holds. (c) Discuss the relationship of this thought experiment to the Michelson-Morley experiment.

Source: Problems 2, 3, 4, 6, 7, 8, 9, 10, 11 are taken from *Basic Concepts in Relativity* by Robert Resnick, David Halliday.

Feedback: Roughly how much time did you spend on this problem set?