MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF PHYSICS

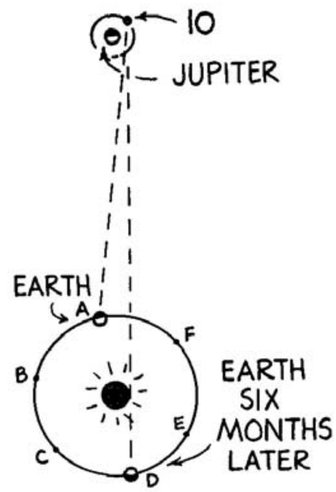
Physics 8.033 September 8, 2006

Problem Set 1

Due: Friday September 15 4:00PM. Please deposit the problem set in the appropriate 8.033 bin, **labeled by recitation number**.
Reading: Resnick Chapter 1 & beginning of Chapter 2. Parallel reading: Chapters 1 & 2 in French.

- **Problem 0(3 points)**: Write your name, section number and staple your problem set!

- **Problem 1(3 points):** "First Measurement of Speed of Light"
In 1675, the Danish astronomer Olaus Roemer showed for the first time that the speed of light is finite. He did this using measurements by French the astronomer Giovanni Cassini of the orbital period of Jupiter's moon Io. The time between two successive appearances of Io from behind Jupiter was observed to be slightly longer when Earth was moving away from Jupiter rather than approaching. Relative to the orbital period when Earth was closest to Jupiter (around A), all periods were thus measured to be slightly longer during the six months when Earth moved from B to C to the point farthest from Jupiter (around D). The sum of these six months of period excesses was was measured to be 22 minutes. How would you interpret this result and estimate the speed of light? (Neglect the motion of Jupiter, which takes 12 years to orbit the Sun.)



Hint: This effect has nothing to do with doppler shifting of light! The difference in observed period values is related to the finite speed of light i.e, the time taken for light to reach Earth varies with its orbital position.

- **Problem 2(6 points):** "This Goodly Frame, the Earth..." (Hamlet)
For some purposes the Earth cannot be taken as a totally "goodly" inertial frame because its motion is accelerated. Calculate the accelerations associated with

  a) the Earth's rotation about its axis (assume an equatorial point),
  b) the Earth's orbital motion about the Sun
  c) the orbital motion of the solar system about the Galactic Center and
  d) motion around Earth-Moon center of mass.

The Earth-Sun distance is about 1.5×10^{11}m and the Earth-Moon distance is about 3.8×10^{8}m. Mass ratio of Earth and the Moon is about 80. Assume that the Sun is about 10 kpc from the Galactic center and orbits around it at about 300 km/s. One parsec $= 10^{-3}$ kpc $\approx 3.09 \times 10^{16}$m.

- **Problem 3(6 points):** "The Galilean Transformation Generalized"

  a) Resnick, Chapter 1, problem number 7, page 46. In other words, generalize equations 1.1a and 1.1b in the book.

  b) Write down the equivalent matrix equation.

  Note: We will return to this problem in Special Relativity where it becomes substantially more complicated.

- **Problem 4(9 points):** "Momentum conservation"
  An observer on the ground watches a collision between two particles whose masses are m_1 and m_2 and finds, by measurement, that momentum is conserved. Use the classical velocity addition theorem to show that an observer on a moving train will also find that momentum is conserved in this collision.

  Repeat this calculation under the assumption that a transfer of mass from one particle to the other takes place during the collision, the initial masses being m_1 and m_2 and the final masses being m_1' and m_2'. Again, assume that the ground observer finds, by measurement, that momentum is conserved. Show that the train observer will also find that momentum is conserved *only* if mass is also conserved, that is, if $m_1' + m_2' = m_1 + m_2$.

  Note: In this course we generally reserve u for velocities of objects within a given frame, and v for velocities of transformation.

- **Problem 5(9 points):** "The Invariance of Elastic"
  A collision between two particles in which energy is conserved is described as *elastic*. Show, using the Galilean velocity transformation equations, that if a collision is found to be elastic in one inertial reference frame, then it will also be found to be elastic in all other such frames.
  Could this result have been predicted from the conservation of energy principle?
  Note: You may restrict your analysis to the special case where all velocities are along the x-axis.

- **Problem 6(6 points):** "The work-energy theorem holds in all inertial frames"
  Observer G is on the ground and observer T is on a train moving with uniform velocity $\mathbf{v}$ with respect to the ground. Each observes that a particle of mass m, initially at rest with respect the train, is acted on by a constant force $\mathbf{F}$ applied to it in the forward direction for a time t.

  a) Show that the two observers will find, for the work done on the particle by the force $\mathbf{F}$,
  $$W_T = \frac{1}{2}ma^2t^2 \quad \text{and} \quad W_G = \frac{1}{2}ma^2t^2 + mvat,$$
  respectively, where a is the common acceleration of the particle.

  b) Show that ΔK_T and ΔK_G, the changes in kinetic energy calculated by each observer, are also given by these same two expressions.

  Thus the work-energy theorem $W = \Delta K$ is valid in all inertial reference frames.

- **Problem 7(6 points):** "Initial Conditions Depend on the Frame"
  A person standing on a train moving with speed v (with respect to the ground) releases a ball from rest at a height h above the floor. The ball falls straight down and hits the floor at a time $t = (2h/g)^{1/2}$, where g is the gravitational acceleration. Sketch the trajectory that an observer at rest on the ground measures. (Assume that one side of the train is made of transparent material.) How do you reconcile, quantitatively, the different trajectories with the fact that Newton's second law, $\mathbf{F} = m\mathbf{a}$, is supposed to work equally well in both of these inertial frames?

- **Problem A: (Optional)** "Vacuum Speed of Light is Independent of Frequency"
  X-ray pulses, visible-light pulses, and radio pulses (the latter corrected for dispersion in the interstellar plasma) emitted by an astronomical object called a "pulsar" are all observed to arrive simultaneously at the Earth — with an uncertainty of only 200 microseconds. The particular pulsar in question is located at a distance from the Earth of 6000 light years. Use this information to make a quantitative estimate of how much the speed of electromagnetic radiation can vary with frequency (or wavelength). Express your answer as a limit on the fractional difference in speed over this wide range of electromagnetic frequencies.

- **Feedback:** Roughly how much time did you spend on this problem set?