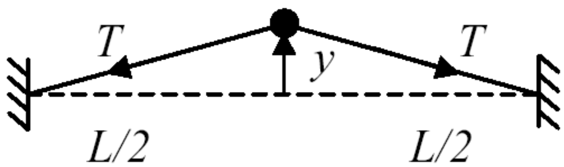
October 5, 2004 – mini-Quiz #4 – 8.03

Your name . . . .

Circle your recitation – R01 – R02 – R03 – R04 – R05

A string of length L is fixed at both ends. The tension is T. A very small mass, m, is attached to the string in the middle (see the figure). The mass of the string is negligibly small compared to m.

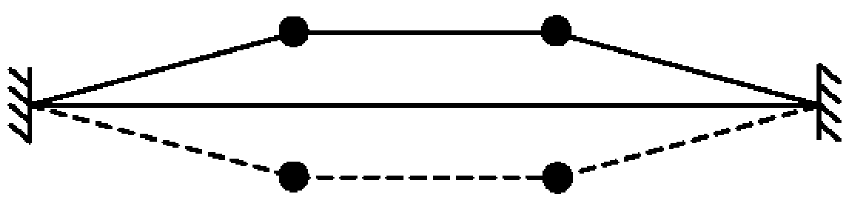
5 points
What is the angular frequency, ω, for very small amplitude transverse oscillations (i.e., in the vertical direction).



The vertical component of the tension in both strings is driving the mass back to equilibrium (see figure).

Newton's 2nd law: $m\ddot{y} = -2Ty/0.5L$.
Thus $\omega = 2\sqrt{T/mL}$.

3 points
We now place two masses (each of mass m) on the string (as shown below). The total length is now $1.5L$. Sketch the two masses as they are oscillating in the lowest frequency normal mode.



2 points
Would you expect this normal mode frequency to be higher or lower than the one you found for a single mass. Give your reasons. I am not asking you to calculate this frequency; your sketch should tell the story.

Look at the figure. The restoring force on each mass is now only due to one string (the tension in the horizontal string in the middle has no component in the y-direction). Thus the restoring force on each mass is lower than the restoring force in the case of a single mass. Thus this normal mode frequency (this is the lowest of the two normal modes) is lower than the frequency of the single mass system. (It must be lower by $\sqrt{2}$).
There is another way to look at this. Nothing changes in this mode if we replace the horizontal string by a massless rod. The net force (in the y-direction) on the two masses is now the same as it was for the one-mass system. However, this restoring force is now acting on a mass $2m$. This leads to the same conclusion.