Useful Formulae

General differential equation for oscillators:

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = f \, \cos(\omega t)$$

has solutions

$$\begin{aligned}
x(t) &= A\,e^{-\frac{\gamma t}{2}} \cos\left(\sqrt{\omega_0^2 - \frac{\gamma^2}{4}}\; t + \alpha\right) + x_{ss}(t) && \omega_0 > \frac{\gamma}{2} \\
x(t) &= (A + B\,t)\; e^{-\frac{\gamma t}{2}} + x_{ss}(t) && \omega_0 = \frac{\gamma}{2} \\
x(t) &= A\,e^{-\Gamma_1 t} + B\,e^{-\Gamma_2 t} + x_{ss}(t) && \omega_0 < \frac{\gamma}{2}
\end{aligned}$$

where

$$\Gamma_{\frac{1}{2}} = \frac{\gamma}{2} \pm \sqrt{\frac{\gamma^2}{4} - \omega_0^2}$$

and

$$x_{ss}(t) = A(\omega)\, \cos(\omega t - \delta(\omega))$$

$$A(\omega) = \frac{f}{\left[(\omega_0^2 - \omega^2)^2 + \gamma^2\,\omega^2\right]^{1/2}} \qquad \tan\delta(\omega) = \frac{\gamma\,\omega}{\omega_0^2 - \omega^2}$$

Complex exponentials:

$$e^{j\,\theta} = \cos\theta + j\,\sin\theta \qquad \sin\theta = \frac{e^{j\,\theta} - e^{-j\,\theta}}{2\,j} \qquad \cos\theta = \frac{e^{j\,\theta} + e^{-j\,\theta}}{2}$$

Trigonometric Formulae:

$$\begin{aligned}
\sin(a+b) &= \sin a\,\cos b + \cos a\,\sin b \\
\cos(a+b) &= \cos a\,\cos b - \sin a\,\sin b \\
\sin a + \sin b &= 2\,\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right) \\
\sin a - \sin b &= 2\,\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right) \\
\cos a + \cos b &= 2\,\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right) \\
\cos a - \cos b &= -2\,\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)
\end{aligned}$$

Non-dispersive wave equation:

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \frac{1}{v^2}\ \frac{\partial^2}{\partial t^2}$$

where $v = \sqrt{\frac{T}{\mu}}$ for a string and $v = \sqrt{\frac{\kappa}{\rho}} = \sqrt{\frac{R\,T\,\gamma}{M}}$ for a gas.

Kinetic, potential energy and power:

$$\frac{dK}{dx} = \frac{1}{2}\,\mu \left(\frac{\partial y}{\partial t}\right)^2 \qquad \frac{dU}{dx} = \frac{1}{2}\,T \left(\frac{\partial y}{\partial x}\right)^2 \qquad P(t) = -T\left(\frac{\partial y}{\partial t}\right)\left(\frac{\partial y}{\partial x}\right)$$

Reflection and transmission coefficients:

$$R = \frac{v_2 - v_1}{v_2 + v_1} \qquad T = \frac{2v_2}{v_2 + v_1}$$