Massachusetts Institute of Technology

Physics 8.03 Fall 2004

Problem Set 8

Due Friday, November 12, 2004 at 4 PM

Reading Assignment

Bekefi & Barrett pages 313-347, 356-385. This is a lot of reading!

Problem 8.1 – Doppler shifts of EM radiation ⇒ a black-hole X-ray binary

One star in an X-ray binary system (the donor, with mass m_1) is only detected in the optical band. The other (the accretor, with mass m_2) is only detected in X-rays. The orbits are circular, the radii are r_1 and r_2, respectively. The optical observers conclude from a close inspection of the optical spectrum that m_1 is approximately 30 times more massive than our sun (it is a super giant).

(a) Derive the orbital period T in terms of m_1, m_2, r_1, r_2, and G. Consult your 8.01 notes and/or watch 8.01 Lecture #23 on OCW.

A particular absorption line in the visible spectrum moves back and forth periodically (in a sinusoidal fashion) with a period of 5.6 days. The minimum and maximum observed wavelengths of the moving line are 499.75 nm and 500.25 nm, respectively. Assume that we observe the binary edge on.

(b) What is the speed of the donor in its circular orbit?

(c) Calculate r_1.

(d) Calculate r_2. Your calculations will be greatly simplified if you set up your equations in terms of r_2/r_1. You will find a third order equation in r_2/r_1. Only one solution is real. There are various ways to find a decent approximation for r_2/r_1: (i) trial and error using your calculator, (ii) plot the function, (iii) MatLab.

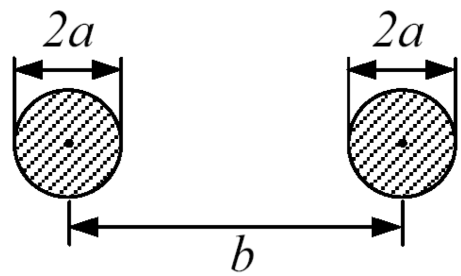
(e) Calculate the mass m_2 of the accretor.

Since the accretor must be compact (we observe a strong flux of X-rays) and because its mass is substantially larger than 3 times the mass of the sun (this is the maximum mass for a neutron star), it is very likely that the accretor is a black hole. A result somewhat similar to this simplified example was first published in 1972 by Bolton and independently by Webster and Murdin for the X-ray binary system Cyg X-1.

Problem 8.2 – Transmission line

Do problem 5.3 from Bekefi, and Barrett. *Electromagnetic Vibrations, Waves and Radiation*. Cambridge, MA: The MIT Press, September 15, 1977. ISBN: 0262520478.

A transmission line consists of two parallel wires each of radius a. The distance between the centers of the wires is b.



(a) Assuming that $b \gg a$, show that the capacity and inductance per unit length of the line are approximately given by

$$C_0 \simeq \frac{\pi \epsilon_0}{\ln(b/a)}$$

$$L_0 \simeq \frac{\mu_0}{\pi} \ln(b/a)$$

Notice that the units of C_0 are Farad/m (the same as ϵ_0). The units of L_0 are Henry/m (the same as μ_0).

(b) Using the results of part(a), compute the phase velocity v of a wave propagating on the line.

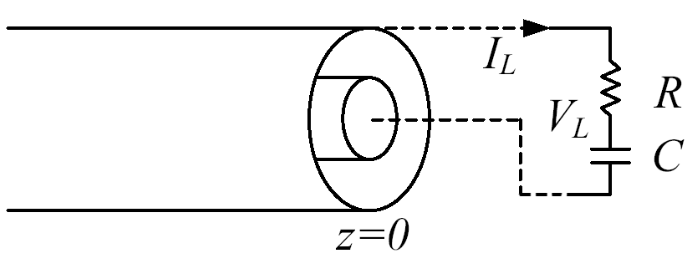
(c) Obtain an expression for the characteristic impedance Z_0.

(d) The parallel wire transmission line is made from No. 12 wires (diameter 0.0808 inches) spaced 0.50 inches apart. Calculate $C_0,\ L_0,\ v$ and Z_0.

Problem 8.3 – Coaxial cable

A coaxial cable with characteristic impedance Z_0 is terminated by a series combination of a resistor and a capacitor. If a harmonic voltage wave is incident from the left, a reflected wave will be set up by the load. The resulting total voltage on the line will have the form

$$V(z,t) = V_i e^{j(\omega t - kz)} + V_r e^{j(\omega t + kz)}$$



(a) Write down an expression for the current $I(z,t)$ on the line.

(b) Find the relation between the complex voltage across the load, V_L, and the complex current into it, I_L.

(c) Find V_r in terms of V_i, ω, R, C, and Z_0 by matching the boundary conditions on voltage and current.

Comment: Notice that it is complex, indicating that the load can change both the amplitude and the phase of the reflected wave.

(d) Is your result in (c) consistent with the general relationship

$$\frac{V_r}{V_i} = \frac{Z_L - Z_0}{Z_L + Z_0} \ ?$$

(e) Sketch the amplitude and the phase of the reflected voltage wave as a function of frequency ω for the special case $R = Z_0$.

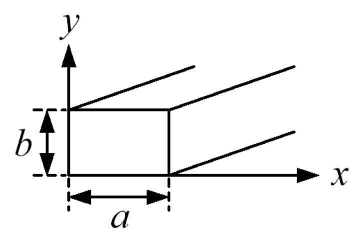
Problem 8.4 – Rectangular waveguide

Do problem 5.4 from Bekefi, and Barrett. *Electromagnetic Vibrations, Waves and Radiation*. Cambridge, MA: The MIT Press, September 15, 1977. ISBN: 0262520478.

A waveguide of rectangular cross section operates in the TE_{mn} mode with

$$E_y = E_{0y}\ sin(k_x x)\ cos(k_y y)\ cos(\omega t - k_z z).$$

The field distribution must satisfy the wave equation and boundary conditions at the faces of the guide tube.



(a) Using the wave equation, develop the necessary relationship between the frequency ω and the various wave numbers.

(b) Using boundary conditions at the faces $x = 0$ and $x = a$, show what restrictions on the wave numbers are required.

(c) Using boundary conditions at the faces $y = 0$ and $y = b$, show what restrictions on the wave numbers are required.

(d) Show that there is a minimum frequency for which propagation will occur and determine this for the TE_{mn} mode.

Problem 8.5 – Resonance cavity

Do problem 5.7 from Bekefi, and Barrett. *Electromagnetic Vibrations, Waves and Radiation*. Cambridge, MA: The MIT Press, September 15, 1977. ISBN: 0262520478.

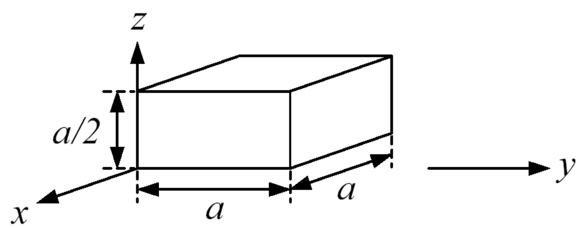
A copper box with dimensions as shown in the figure acts as a cavity resonator. The electric field

$$E_z = E_0\ sin(k_x x)\ sin(k_y y)\ sin(\omega t),\ \ E_x = E_y = 0$$

is a possible solution of the wave equation for this case.

(a) Find the lowest resonance frequency ω_1 and the corresponding free space wavelength λ_1.

(b) Find the next-to-lowest resonance frequency ω_2 and the corresponding free space wavelength λ_2.



Problem 8.6 – Radiation pressure

A perfectly reflecting mirror of mass $M = 1$ g hangs vertically from a wire of length $L = 10$ cm. It is illuminated with a constant laser beam of intensity 30 kW (a powerful laser!), incident normal to the surface of the mirror. What is the displacement of the mirror from its equilibrium position?