Massachusetts Institute of Technology

Physics 8.03 Fall 2004

Problem Set 7

Due Friday, November 5, 2004 at 4 PM

Reading Assignment

French pages 274-280. Bekefi & Barrett: see assignment #6 pages 189-250; new reading, pages 208-251.

Problem 7.1 – Polarized radiation

Write down the electric field and associated magnetic field in vacuum for travelling plane waves. The amplitude of the electric vector is E_0 and the frequency is ω.

(a) The radiation is linearly polarized in the y-z plane at an angle of 45° with the y-axis, and it is travelling in the $+x$ direction. There are two solutions.

(b) The radiation is circularly polarized in the y-z plane, and it is travelling in the $+x$ direction. There are two solutions.

Problem 7.2 – Linear polarizers - Malus' law + absorption

In lectures an envelope was handed out that contains three linear polarizers of type HN30. If 100% linearly polarized light of intensity I_0 goes through an ide al polarizer (properly aligned), 100% linearly polarized light of intensity I_0 will emerge. Such a polarizer is called HN50.

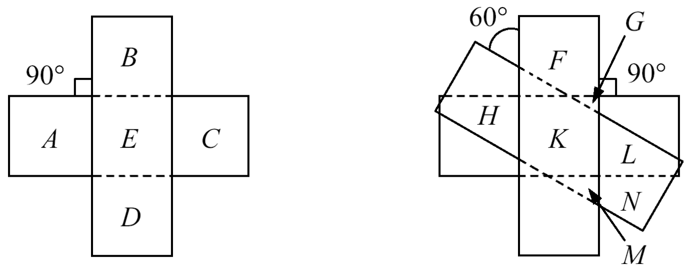
In reality, polarizers are not ideal, and they come in various types with different degrees of absor ption. Your polarizers are of type HN30. This means that if you do the experiment as described above, only 70% of I_0 emerges. Thus if we let the 100% linearly polarized light of intensity I_0 go through two of th em (both correctly aligned), only 49% of I_0 will emerge.

(a) Look through one of your linear polarizers at a light source. Then place a second linear polarizer on top of the first (same orientation). You will notice that the light intensity decreases.

Place two polarizers at right angle (see the figure on the left). Let us define the light intensity of the 100% linearly polarized light coming through the areas A, B, C and D as I_0. The light intensity through area E is now zero

Now stick your third linear polarizer between the two (see the figure on the right), and rotate this third polarizer in it's plane while keeping the original two at 90 degrees. The darkness at E wi ll disappear. Try it!!!

(b) What are the light intensities through the areas: F, G, H, K, L, M, and N? No tice the angle of 60 degrees. Take into account that your polarizers are of type HN30.



Problem 7.3 – Radiation from an accelerated charge

Do problem 4.1 from Bekefi, and Barrett. *Electromagnetic Vibrations, Waves and Radiation*. Cambridge, MA: The MIT Press, September 15, 1977. ISBN: 0262520478.

A charge, q, initially at rest, is given a brief acceleration along the z axis. Let this occur at the origin of the coordinate system at $t = 0$.

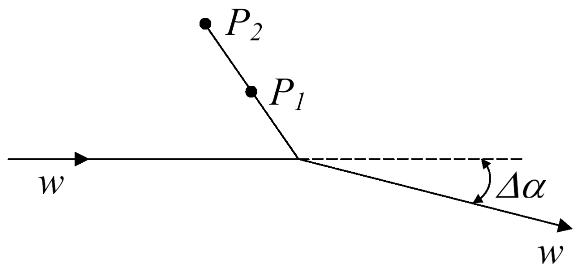
(a) Give the arrival times, the relative strengths, and specify the directions of the radiated electric field seen by three observers in the y-z plane at a large distance R from the origin. One observer is on the y-axis, one on the z-axis, and one is on a radius making an angle of 30° to the z-axis.

(b) Describe the orientation and magnitude of the associated radiated magnetic fields.

Problem 7.4 – Radiation from an accelerated charge

Do problem 4.2 from Bekefi, and Barrett. *Electromagnetic Vibrations, Waves and Radiation*. Cambridge, MA: The MIT Press, September 15, 1977. ISBN: 0262520478.

A point charge $+q$ has been moving with constant velocity w along a straight line until the time $t = t_0$. In the short time interval from $t = t_0$ to $t = t_0 + \Delta t$, a force perpendicular to the trajectory changes the direction without changing the magnitude of the velocity. After the time $t = t_0 + \Delta t$ the charge again moves with velocity w along a straight line forming a small angle $\Delta\alpha$ with the initial trajectory.



(a) What is the direction of the electric field caused by the acceleration, at the distant point P_1?

(b) In what direction is the radiation intensity of the accelerated charge most intense?

(c) Where is it least intense?

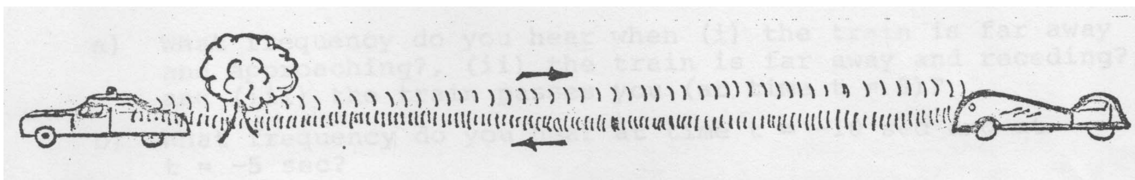
(d) Point P_2 is twice as far from the bend in the trajectory as P_1. By what fraction does the amplitude of the magnetic disturbance decrease as the radiation pulse moved from P_1 to P_2?

(e) What is the total energy radiated?

Make careful sketches in answering parts (a), (b), (c).

Problem 7.5 – Speed checked by radar

A radar beam ($\lambda = 3$ cm) is reflected off a car as shown in the figure (head-on reflection). The radar transmitter is <u>not</u> moving. The reflected beam is again received by the police; the frequency f' of this reflected signal is measured by a stationary receiver on the police car. From this the velocity v of the car is calculated. All this is done in a "black-box" in less than a small fraction of a second! A digital display tells the police officer your speed nicely converted into miles/hour.

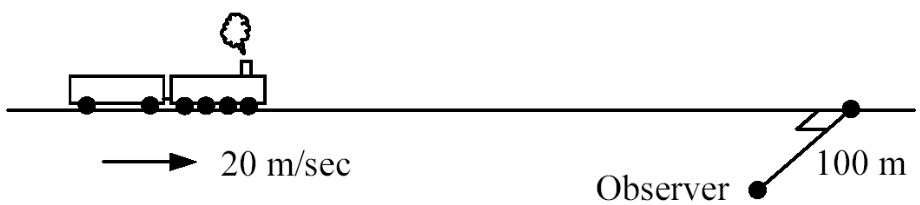


(a) What is the frequency of this radar transmitter?

(b) Give an equation that relates f, f' and v. (*Hint: first calculate the frequency that the car "sees", then send this signal back to the police.*)

Problem 7.6– Can't you hear the whistle blowing?

A train travels down a long straight track at a speed of 20 m/sec (~45 mph) blowing its whistle constantly (f = 1000 Hz). You, the observer, are standing at a point 100 m from the track (see figure). Define time $t = 0$ the moment that the train is closest to you.



(a) What frequency do you hear when (i) the train is far away and approaching?, (ii) the train is far away and receding?, and (iii) the train passes you (at time $t = 0$)?

(b) What frequency do you hear at time $t = -10$ sec and at $t = -5$ sec?

(c) Sketch the curve of the frequency that you hear versus time.

Problem 7.7 – Our expanding universe – simplified

The figure below illustrates the famous expanding balloon analogy for our universe. All of space is represented by the surface of the spherical balloon, and clusters of galaxies are represented by spots painted on this surface; the radius of the balloon corresponds to $R(t)$, the "radius" of the universe. As the balloon expands, the spots remain at constant angular separations (θ) from one another. Let the balloon expand at a constant rate.

(a) Prove that

$$\left(\frac{\Delta s}{\Delta t}\right) = \left(\frac{1}{R}\frac{\Delta R}{\Delta t}\right)s \tag{1}$$

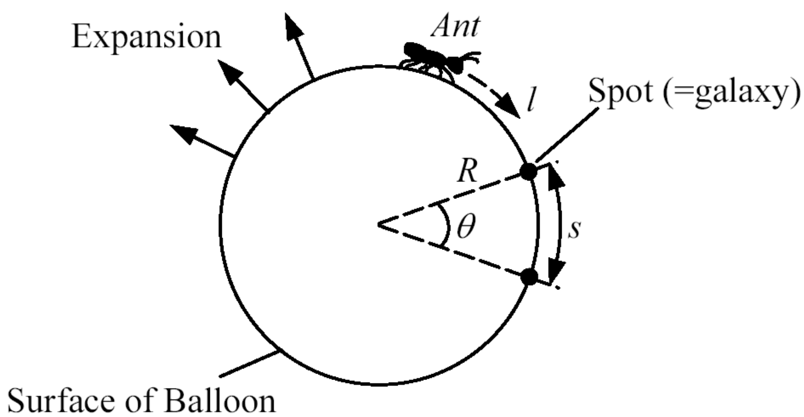
Here s is the separation between any two spots (on the surface) and $(\Delta s/\Delta t)$ is the speed of recession of one spot from another.

Note that eq. (1) is equivalent to Hubble's Law:

$$v = Hr \tag{2}$$

(b) What is H in terms of the symbols used in eq. (1)? Check that your answer is dimensionally correct; H in general is expressed in km/sec per Mpc.

The light photons from distant galaxies may be represented by "ants" crawling along the balloons surface at speed l.



(c) Show that for uniform cosmic expansion [i.e., $(\Delta R/\Delta t)$ = constant] there is a distance (s) from beyond which these ants cannot ever reach our galaxy (this distance is called the horizon)!

Let us now turn to the way we looked at our world a few years ago, before the discovery of "dark energy". Dark energy is responsible for the fact that the expansion of the universe, according to present believe, is accelerating, not decellerating. This mysterious dark energy is presently one of the "hottest" topics in all of physics. In what follows, pretend that there is no dark energy.

Consider an expanding (gravitating) gaseous sphere of uniform mass density ρ, with a total mass M and a radius $R(t)$. A gas particle at the surface of this sphere will move radially outward with velocity v so that

$$\frac{v^2}{2} = \frac{GM}{R} + constant \tag{3}$$

Here $v = \Delta R/\Delta t$ is the radial speed.

This must be a familiar equation from your 8.01 days. If the constant is positive then the expansion will never stop (too much kinetic energy); if the constant is negative then the expansion will ultimately stop and a collapse under the influence of gravitational forces will follow.

(d) Show that eq. (3) can be written in the form:

$$\left(\frac{1}{R}\frac{\Delta R}{\Delta t}\right)^2 = \frac{8\pi G\rho}{3} + \frac{2(constant)}{R^2} \tag{4}$$

In the following we will introduce a subscript 0 when we refer to our present time.

In the following assume $H_0 = 70\ \dfrac{\text{km/sec}}{\text{Mpc}}$ and $G = 6.7 \times 10^{-11}$ (SI unit).

(e) What is the minimum required density of our universe now (ρ_0) to make it closed?

Replace in eq. (4) ρ by $\frac{M}{\frac{4}{3}\pi R^3}$. Here M is the total mass of the universe.

(f) Show that in case of a flat universe the radius of the universe $R(t) \propto t^{2/3}$.

From this it follows immediately that $H = \frac{2}{3t}$ (show that) and consequently the age of our universe now $t_0 = \frac{2}{3H_0} \sim 9.3 \times 10^9$ years.

(g) Your modified eq. (4) relates $H^2 = \left(\frac{1}{R}\frac{\Delta R}{\Delta t}\right)^2$ with R. Show that Hubble's constant must have been larger in the past.

(h) Could H become negative? What consequence would that have for our "red-shifted" galaxies.